Volition regimes and news*

Alessandro Saccal†

University of Rome II "Tor Vergata"

Abstract

Confidence, sentiment, spirit, perception: what is one to make of such whencever beholding balanced growth path alterations and fluctuations thereat? How is one to specifically relate changes in economic fundamentals unto variations in the so-called market confidence and vice-versa? Why so much stress upon such latter factor and yet so much conundrum surrounding it? Howbeit, news and noise have been heralded to delineate it; with an empirical VAR therefor exploited and a micro-founded so-called DSGE model used therewith, three conclusions have from the literature summarily pervaded: there exist advance productivity shocks; they are not to be recovered empirically, for, however well devised, inherently undefinable; the noise dispersing them, withal demand oriented, may as well be seen as spurious; all such alongside the expected reinforcement whereby supply shocks are to truly be one’s history. Yet what about those spirits of the soul, those celebrated animal spirits of which Keynes so fervently wrote all through the 1930s? Are they truly so improbable? The present paper, within a single up to date micro-founded framework, is to attempt to bring forth a respectable reconciliation between such two main strands of literature upon the issue: spirits and tidings namely; volition regimes and news.

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†saccal.alessandro@gmail.com
1 Introduction

What is to be globally deduced about the actual role of market confidence within the orbit of business cycle fluctuations? Despite its general appeal, nay expedience, is one to capitulate before its oftentimes relentlessly professed cardinality amid the sphere of economic sciences or is one to yet remain well sceptical as to its influence upon the real activity? Is it of impact, of collision, of germaneness, is it even to exist, in fact? In essence, what is one to make of business or consumer confidence?

Appraise its reciprocity with regards to economic growth so as to invigorate it, it has been contended: for perturbations in the former alter indeed the latter and, in like manner, variations in the latter impinge upon the former. Restore it, stimulate it, improve it, strengthen it afresh, has in sum been but the doctrine; power, people, change, puissance the keywords; confidence, the truthful watchword. Indeed much accolade upon it has been not infrequently bestowed: from demagogic sentiment boosts through electoral campaigns, unto menacing admonitions amid induced periods of economic crisis; from mere consultancies worked by professional insiders, unto sheer reprehensions perpetrated by the same; from passive indifference unto active belligerence; from obliviousness to cognisance; from rumour to widespread opinion; from myth, to creed. Hence verily: whither confidence?

Literature upon the matter subdivides itself into three strands: animal spirits, contingent upon Keynes (1936); news, first attributable to Pigou (1927) and consumer search effort, whose origins are likewise traceable unto the 1920s’ Western European economic stream of thought. Revisitation of the both the former and the latter stances has been close to inexistent; in fact, Bai et al. (2011) aside, evidently most recent, as far as consumer search effort is concerned, being altogether candid, it has been rather scarce. Strides upon the theme of news have nevertheless been made with a considerable consistency. Whilst only scraping the issue of consumer confidence, at least initially, work upon the role of economic tidings and its consequent reverberations has been developed rather swiftly since fresh dissemination set forth by Cochrane (1994). More specifically, with the latter vindicating advance knowledge of productivity variations on the side of agents whilst consuming, via the use of impulse response functions originating from a short-run restricted bivariate SVAR(4) endowed with real output and consumption, the idea of predictable supply shocks began acquiring many a supporter.

Through the utilisation of yet another empirical SVAR, such a time consisting but of a stock price index and a measure of total factor productivity, Beaudry-Portier (2006) in fact corroborate the finding by providing evidence in favour long-run reactions in activity only upon shocks within itself. In other words, upon both long and short-run restrictions for the SVAR in question, total factor productivity is to react but upon long-run restricted self-induced perturbations. With such all as the case, Lorenzoni (2009)’s monograph, whereby, within a DSGE model, representative economic agents are devised to learn about noise-ridden productivity shocks somewhat in advance, definitively established the mentioned stream of literature amongst academia as but summarily contemporary and indeed worthy of active research endeavours. His crucial contribution unto the literature, in order not to suffer agents endure their a priori rational ignorance, withal reveals itself to but concern a signal extraction problem attendant upon them right afore their consumption choices; temporary and delusive output patterns, upon spurious news collisions, branded as demand shocks, noise therein, thence ultimately eventuate.
In such regard, the reasoning by which news and noise shocks are to summarily be viewed as but inseparable, within a strictly empirical VAR framework delineated by long-run restrictions, in light of the latter’s ex-ante orientation, is by Blanchard et al. (2012) expounded. More specifically, provided rational expectations on the side of agents, error acceptance prior to optimised expenditure is impossible; wherefore, considered the equivalence between the representative agent and the econometrician, all ex-ante restrictions to demarcate an underlying overt noise activity are to be itemised as but inconceivable. On the other hand, structural recovery of such real perturbations is therewith proclaimed as feasible. Be that as it may, Sims (2012) offers rationale in favour of how foreseen supply shocks, attendant upon productivity, are to neither be redeemed within structural model orbits. That is, modelled natural logarithmic production technology as a customary covariance stationary $AR(1)$ process augmented for news:

$$a_t = \rho_a a_{t-1} + v_{1t};$$

such that for

$$v_{1t} = v_{2t-1}$$
$$v_{2t} = \varepsilon_{at}$$

the following then hold

$$a_t = \rho_a a_{t-1} + \varepsilon_{at-1};$$

how are endogenous predetermined variables $v_{1t}$ and $v_{2t}$ to be empirically recovered throughout an hypothetical empirical analysis? In other words, how is one to admit an empirical $VAR(1)$ representation for the micro-founded model entailing those variables, itself to permit one to reconstruct real news perturbations, when the germane states are by definition unobservable? In synthesis, according to Sims (2012), unless be $v_{1t}$ and $v_{2t}$’s empirical obscurity, experimental abstruseness, factual non-traceability disposed of and apt measure thereof found, advance supply shocks on the side of productivity, commonly referred to as mere news, are destined to be ubiquitously branded but as empirically irrecuperable. So if errors be verified as irrecoverable structurally - for, verily, what is ever one to make of $v_{1t}$ and $v_{2t}$? - how more buried are they to appear empirically? Thus expounded is Blanchard et al. (2012).

Sims (2012) all the same sets forth an even apter contention: recover not structural shocks attendant upon news, but upon news signals. In order to obtain a clear depiction of the typology of structural perturbation underlying a reaction in activity upon jolt thereof or else, within a but empirical VAR framework, exploit an indicator instead. As the case with bond spreads, market, business or consumer confidence, typifying but two errors, news and noise, is
to wherefore act the proxy variable in question. Hence summarily explained confidence within
the news literature. Barsky-Sims (2012) bring forth exactly such an experiment: by assessing
impulse response functions in activity upon consumer confidence shocks, for the American
economy, and replicating them structurally via the use of a NK-DSGE model, not only are they
able to foretell to be activity patterns by considering such an arcane factor but simultaneously
succeed portraying the main drive behind such business cycle triggered trends. So-called
exogenous growth conducts are empirically beheld, that being, lack of reaction upon shock
ensued by non-reverting positive alteration a few quarters thence. The same are structurally
attributed unto pure advance productivity shocks, gainsaying noise’s active presence therewith.
The theory of demand shocks, nay of vexing demand shocks, appears thence dismantled.

In conclusion, by observing an empirical impulse response function dependant upon some
activity measure given an underlying perturbation in a sentiment signal, such as confidence
or the like, and by vindicating it structurally through some form of micro-founded modelling,
one is to ultimately speak in favour of a series of so-called exogenous economic shocks: to
wit, supply and demand shocks. Now, because literature seems to have drawn, unwillingly
perhaps, the simplistic and somewhat totalitarian conclusion by which non-reversions in activity
are to contradistinguish but news shocks, which, in turn, are to be solely attendant upon
productivity and hence branded as supply, and demand shocks only to concern instantaneous
reversions in activity to thence be labelled but as noise, the present monograph is to propound
a more comprehensive framework: one to amplify the amount of represented shocks such that
confidence’s signalling nature be then qualitatively refined.

Within a micro-founded DSGE model environment, labour augmenting confidence, in the
representative intermediate goods producing firm, is to be pictured as a volition-responsive
coaalence of three autoregressive processes: two news processes (beliefs and noise) and ini-
tiative. Two out of the three are to be devised as drifts, so to speak, within production
technology so as to affect it up to an extent contingent upon the processes’ stationarity:
initiative and beliefs. Initiative shocks, to be labelled as resoluteness or animal spirits\(^1\) ones
therewith, alongside advance productivity variations, by allowing TFP shocks to impinge both
upon their own covariance stationary process and upon a non-stationary initiative correlat-
ive, are to be engineered such that production technology be permanently altered upon jolts
therein. Random changes in the process for beliefs, in light of their ex-ante uncertainty, are
to instead be modelled so as to eventually bring about reverting trends in activity expansion.
Moreover, because noise alterations are exacted not to intrude upon production technology at
all, activity is to soar only in light of the labour augmentation effect through confidence and
not through a change in fundamentals.

The first fruits of the paper are to be twofold: on the one hand, confidence is to explicitly
be modelled as a signal of a multitude of structural shocks such that all approaches within
the literature hereto be recognised therein; on the other, the extent unto which confidence is
to labour augment activity is to be dependant upon a volition transmission mechanism such

\(^1\)In spite of what many an individual may have hereto desired to believe, the concept and expression by
the name of "animal spirits" dates back way longer than Baron Keynes’ epistemological writings regarding
the economic sciences. It may be found as far back as in Alighieri’s "Vita Nuova", a text by the Italian poet
concerning medieval courtly love customs. The locution is not so much to signify natural exuberance as in
irrational, or perhaps mercurial, conduct but rather to limn the dominated spirits attributable to stimuli but
of the anima, of the mind, of the soul: hence fully spiritual, fully volitive, truly of the conscious, of the wilful.
that be such responsiveness even inexistent perturbations within confidence would need to ultimately signal mere alterations within the process of production technology. But most of all, as bespoken by the title of the article itself, the present monograph is to coalesce two out of the three strands of literature upon the role of confidence within economic activity: initiative, beliefs, noise and advance productivity; animal spirits and news. In a nutshell, any conceivable empirical behaviour pursuant to activity measures upon confidence variations may, through the adoption of such framework, summarily be micro-founded.

2 News and volition framework

As erstwhile stated, for a stable trivariate SVAR(4) of the ensuing sort\(^2\)

\[
\begin{align*}
x_t &= \Pi_1 x_{t-1} + \ldots + \Pi_4 x_{t-4} + w_t \\
y_t &= \Gamma y_{t-1} + \varepsilon_t \\
y_t &= \Gamma y_{t-1} + D\eta_t \\
(I - \Gamma L)y_t &= D\eta_t \\
y_t &= \Gamma(L)^{-1}D\eta_t \\
y_t &= \sum_{j=0}^{\infty} \Gamma^j D\eta_{t-j}
\end{align*}
\]

endowed with quarterly data for the American economy to chronologically involve consumer confidence, real consumption and real output, at a forty quarters horizon \(j = 40\), has been by the literature affirmed to exhibit output responses upon confidence perturbations of the following two taints: short-run reaction followed by an immediate reversion and short-run non-reaction ensued by a long-run non-retreating trend. Respectively, the two conducts have been conjectured to be implicit signals of noise and advance productivity shocks, news, whereby an exogenous growth pattern has been contended to be the case within the second scenario.

Two more situations, yea shunned, are to arise to mind howbeit: short-run responsiveness and subsequent gradual long-run reversion, hence but outcome of a perturbation by the enduring yet not quite undying effects, as well as a yet short-run responsiveness although followed by a long-run definitive balanced growth path augmentation. Thus, summarily, let the four aforementioned patterns be encapsulated as ensues:

\[
\begin{align*}
SR \text{ responsiveness} &\rightarrow \text{immediate reversion}; \\
SR \text{ responsiveness} &\rightarrow LR \text{ reversion}; \\
SR \text{ irresponsiveness} &\rightarrow LR \text{ non-reverting responsiveness}; \\
SR \text{ responsiveness} &\rightarrow LR \text{ non-reversion}.
\end{align*}
\]

The fourth and second cases are to be, unto the undersigned’s mind, indeed clear personifications of but uncertain news shocks on the one hand and endogenous growth on the other. The latter is to refer to advance productivity shocks upon which agents are to rationally decide to

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\(^2\)Where \(\varepsilon_t = D\eta_t\), \(E[\varepsilon_t \varepsilon_t'] = \sum = DD’\) and \(E[\eta_t \eta_t'] = I\), for a lower triangular \((5 \times 5)\) \(D\) matrix, hold and \(|\lambda_{\Gamma(\lambda)}| < 1\) be such that characterising coefficients be square summable: \(\sum_{j=0}^{\infty} \text{tr}(\Gamma_j^j) < \infty\).
act and unto which they are to wilfully respond, hence indicating rational inaction upon such very perturbations in the exhibited exogenous growth coequal; whereas the former is to typify the topic of beliefs. Namely, shocks not to be read as advance productivity ones but as ones whose provenance is that of a distinct demand-oriented process, whereby news therein must needs concern events which themselves need not perennially alter the economy’s steady state, the balanced growth path, in light of their a priori natural uncertainty.

The Euro entry, for instance, has exemplified such nature right within the celebrated peripheral nations by the acronym of PIGS, wherein massive investments flowed in from abroad, seen the suddenly higher nominal interest rates imported via the adoption of the de facto German Mark; but as time elapsed realisations of no changes within fundamentals ultimately led to an inherent acclimatisation towards the former or perhaps non-spurious and original balanced growth path. Hence ample is to be the rationale even for beliefs shocks.

Consequently, within a micro-founded DSGE environment endowed with both real and nominal rigidities for the Northern American economy, the undersigned is to argue but in favour of the existence not only of ulterior shocks, whose nature may to some extent even revert the bizarre belief attendant upon demand shocks, but of a component which will crucially reflect both the hypothetical and lifelike reverberations to impinge upon the business cycle, given such shocks, hitherto within the literature nevertheless integrally neglected and eschewed: volition, to wit, action upon signal. The following flow diagram is to epitomise all redundant preludes.

![Flow Diagram]

What is to be deduced out of such a depiction? Action extent upon signal, \((v)\) volition degree namely, yea exercised upon the signal, to only yield confidence:

\[ \text{Signal}^v = \text{Confidence}. \]

Via more detail, the answer may expounded in a twofold fashion, but before advancing such reasonings pithy recollections of supply and demand shocks repercussions are to be sensed as becoming. That being:

\[ \text{supply} \rightarrow \text{alterations of the steady state}; \]
\[ \text{demand} \rightarrow \text{fluctuations about the steady state}. \]

\(^3\text{RW: random walk; CS: covariance stationarity; p: process; s: labour share; d: delayed drift; e: productivity error; v: volition degree; Signal: processes coalescence; Initiative: transmission mechanism; Beliefs: uncertain news; Noise: unfounded news; Technology: productivity/certain news.} \]
Because economies are to generally grow over time, economists have ultimately conjectured an economy’s steady state to be portrayed by the balanced growth path (BGP). Seldom are to be the instances wherein the latter’s antithesis, the balanced decline path (BDP), is to emerge, nevertheless definitely ones within the "Impossible Trio" literature. Be that as it may, let the conjoint response be set forth. Namely, on the one hand: confidence is to embed several a shock, supply and demand perturbations that being, and news are to also be demand shocks where the latter are to be even beneficial - that is, not that volatile nor too mercurial but rather soothing in their fluctuation about the steady state: with not so steep a slope. On the other, confidence is to both bespeak shock nature and to indicate volition degree. Wherefore, for $1 < \rho_{b,n} < 1$:

$$Confidence = Signal^\nu = [\text{noise}; \text{beliefs}; \text{initiative (productivity)}]^\nu$$

$$\Upsilon_t^\gamma = (n_t b_t s_t)^\gamma$$

where

$$n_t = \rho_n n_{t-1} + \varepsilon_{nt}$$
$$b_t = \rho_b b_{t-1} + \varepsilon_{bt}$$
$$s_t = s_{t-1} + \varepsilon_{at} + \varepsilon_{st}$$

More precisely, in such present framework, as visible, each perturbation is to retain its own process whereby news shocks are to be categorised as of supply origin when but contingent upon productivity ($\varepsilon_{at}$), upon a technology process that being, and of demand formation when dependant upon beliefs ($\varepsilon_{bt}$) and noise ($\varepsilon_{nt}$); respectively limning uncertain and unfounded news processes. Such demand shocks are to thence be seen as economically noxious when concerning the latter (noise) but as potentially desirable, for as erstwhile contended inherently reposeful in their fluctuating nature, when regarding the former (beliefs). The latter is to thus alter technology magnitude ($a_t$) - with a lag delay so as to allow for apprehension and appraisal of the tiding itself - the former isn’t:

$$a_t = \rho_a a_{t-1} + b_{t-1} + \varepsilon_{at}$$
$$n_t = \rho_n n_{t-1} + \varepsilon_{nt}$$
$$b_t = \rho_b b_{t-1} + \varepsilon_{bt}$$

Initiative ($\varepsilon_{at}$) shocks are to instead be construed as pure sentiment shocks. Ones by which man permanently resolves to ameliorate his thenceforth daily conduct such that positive reverberations upon economic activity be: willfulness; determination; intention; conscientiousness; resoluteness; resoluteness shocks indeed in fact. "That spontaneous urge to action rather than inaction to finally determine all our positive deeds" to put it à la Keynes - true animal spirits, of the soul.
In addition to all such, productivity shocks are to indelibly affect technology and output levels not via an undetermined or spontaneous fashion, of the innate and immanent as typically assumed whilst supinely modelling or devising random-walk technology processes; rather they, productivity variations, are to irreversibly alter technology degree but through that, yea infinite memory, adoption process by the name of initiative. In brief, productivity perturbations are to invariably transmit their innovation unto technology only by having first travelled through an infinite memory initiative adoption process: indeed one of infinite memory, for upon innovation intention to regress is hardly one.

Practically, it is to but be that present productivity collisions are to intrude upon present initiative and likewise the latter upon future technology, that being with a period delay: hence the "certain news" shock appellative within technology volume. Thus explained is the undersigned’s abjuration of non-stationary technology in favour of initiative; for, verily, all advance productivity alterations within technology are such for permanently affecting but the initiative of novel technology adoption and not technology per se:

\[
a_t = \rho_a a_{t-1} + b_{t-1} + s_{t-1} + \varepsilon_{at} \\
b_t = \rho b_{t-1} + \varepsilon_{bt} \\
s_t = s_{t-1} + \varepsilon_{at} + \varepsilon_{st}
\]

As a last remark upon this point it need be stated that the advance productivity shock to result within technology through the permanent albeit delayed initiative drift is by nature to be but a supply perturbation, yet it is through the infinite memory initiative effect that technology is to be everlastingly modified: thence, such supply shock is to be indissolubly effective but through a demand channel, that of enterprising initiative. Conclusively:

\[
\Upsilon_t = n_t b_t s_t \\
a_t = \rho_a a_{t-1} + b_{t-1} + s_{t-1} + \varepsilon_{at} \\
n_t = \rho n_{t-1} + \varepsilon_{nt} \\
b_t = \rho b_{t-1} + \varepsilon_{bt} \\
s_t = s_{t-1} + \varepsilon_{at} + \varepsilon_{st}
\]

3 Model

Households

Amid the presently simulated economy there is to exist a continuum of identical households whose utility need be enkindled by preferences attendant upon consumption \((C_t)\) and labour disservices \((L_t)\). The former is to be maximised whilst subject to two constraints: a budget constraint and the motion law for private capital. In more detail, the future discount factor with respect to the real interest rate is to be typified by \(\beta \in (0, 1)\); \(\frac{1}{\beta} = \frac{1}{\eta}, \frac{1}{\eta}\) are to instead respectively portray the elasticities of inter-temporal substitution and labour; consumption habits \((hC_{t-1})\) are to forbye characterise the model. Confidence \((\Upsilon_t)\), for a given volition
degree \((\gamma)\), to directly affect but consumption choices, is to moreover abide by the rules heretofore exposed. The budget constraint’s demand side is to, on the other hand, be constituted by homogenous units of: consumption \((C_t)\); investment \((I_t)\); newly-issued real governmental bonds \(\left(\frac{B_t}{P_t}\right)\), where \(P_t\) is to delineate the price level, and lump-sum taxes \((T_t)\). Its supply correlative is to thence set forth: labour provision compensation \((W_t)\); earned rent upon capital stock retention \(\left[R_tu_tK_{t-1} - \psi(u_t)K_{t-1}\right]\) for \(K_t\), with \(u_t\) to demarcate the capital utilisation rate and \(\psi(u_t)\) the capital utilisation function with properties \(\psi(1) = 0\) and \(\psi'' \geq 0\); obtained nominal interest rate upon past governmental bonds possession alongside principle repayments receipt\(^4\) \(\left(\frac{r^n_{t-1}B_{t-1}}{P_t}\right)\); firm dividends \((D_t)\) and lump-sum governmental subsidies, transfers \((TA_t)\). The second constraint, the motion law for private capital that being, is to instead feature capital depreciation \((\delta)\) as well as an investment adjustment cost \(F\left(\frac{I_t}{I_{t-1}}\right)\) with peculiarities \(F(1) = F'(1) = 0\) and \(F''(1) > 0\). Wherefore:

\[
\max_{\{C_t, I_t, L_t, K_t, u_t, b_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \kappa_t \left\{ \frac{\gamma_t(C_t - hC_{t-1})^{1-\sigma_C}}{1-\sigma_C} - \frac{\chi_tL_t^{1+\eta}}{1+\eta} \right\} \text{ s.t. } B \text{ and } C;
\]

where \(B = S_{RHS} - D_{LHS}\) and \(C = S_{RHS} - D_{LHS}\) are to respectively hold for

\[
C_t + I_t + b_t + T_t = W_tL_t + R_tu_tK_{t-1} - \psi(u_t)K_{t-1} + r^n_{t-1} \left(\frac{b_t}{\pi_t}\right) + D_t + TA_t
\]

\[
K_t = (1 - \delta)K_{t-1} + \left[1 - F\left(\frac{I_t}{I_{t-1}}\right)\right]I_{t}z_t.
\]

Six exogenous perturbations are to be furthermore embedded within the aforementioned representative household’s maximisation problem. Five of the six are but to respectively abide by five respectively distinct covariance stationary \(AR(1)\) processes, \(-1 < \rho < 1\) namely, two whereof already yea viewed, and one by an also beheld random walk, to be all howbeit expressed in natural logarithmic form, such that \(\varepsilon_{it} \sim i.i.d. \mathcal{N}(0, \sigma^2)\) for \(i = n, b, a, \kappa, \chi, z\) also hold\(^5\). Hence:

\[
\ln n_t = \rho_n\ln n_{t-1} + \varepsilon_{nt};
\]

Additionally, because \(b_t = \left(\frac{B_t}{P_t}\right)\), \(b_{t-1} = \left(\frac{B_{t-1}}{P_{t-1}}\right)\) and \(\pi_t = \frac{P_t}{P_{t-1}}\) it would then result that

\[
\frac{b_{t-1}}{\pi_t} = \frac{B_{t-1}}{P_{t-1}} \cdot \frac{P_t}{P_{t-1}}.
\]

\(^4\)See the "Intermediate good producers" section for more information upon \(\varepsilon_{at}\).
beliefs

\[ \ln b_t = \rho_t \ln b_{t-1} + \varepsilon_{bt}; \]

initiative

\[ \ln s_t = \ln s_{t-1} + \varepsilon_{at} + \varepsilon_{st}; \]

preferences

\[ \ln \kappa_t = \rho_t \ln \kappa_{t-1} + \varepsilon_{\kappa t}; \]

labour disutility

\[ \ln \chi_t = \rho_t \ln \chi_{t-1} + \varepsilon_{\chi t}; \]

investment-specific technology

\[ \ln z_t = \rho_t \ln z_{t-1} + \varepsilon_{z t}. \]

For \( U(\cdot) = \{ \frac{\gamma_t(C_t - hC_{t-1})^{1-\sigma_c}}{1-\sigma_c} - \frac{\chi_t \mu_t^{1+\eta}}{1+\eta} \} \), first order conditions pursuant to \( \mathcal{L}_t = E_0 \sum_{t=0}^{\infty} \beta^t \{ \kappa_t U(\cdot) + \lambda_t [\mathcal{B}] + \mu_t [\mathcal{C}] \} \) would consequently result to be:

\[ C_t : \quad \beta^t \{ \kappa_t \gamma_t (C_t - hC_{t-1})^{1-\sigma_c} + \lambda_t [-1] \} = 0 \;
\]

\[ \kappa_t \gamma_t (C_t - hC_{t-1})^{1-\sigma_c} = \lambda_t \]

\[ L_t : \quad \beta^t \{ -\kappa_t \chi_t L_t^{11} + \lambda_t [W_t] \} = 0 \;
\]

\[ \lambda_t W_t = \kappa_t \chi_t L_t^{11} \]
\[
\beta^t \{ \lambda_t[-1] + \mu_t \left[ z_t - \left( F \left( \frac{l_t}{l_{t-1}} \right) z_t + F' \left( \frac{l_t}{l_{t-1}} \right) \frac{1}{l_{t-1}} I_t z_t \right) \right] \} + \\
+ E_t \beta^{t+1} \left\{ \mu_{t+1} \left[ -F' \left( \frac{l_{t+1}}{l_t} \right) (-I_{t+1} I_t^{-2}) z_{t+1} I_{t+1} \right] \right\} = 0 \\
\mu_t \left[ z_t - \left( F \left( \frac{l_t}{l_{t-1}} \right) z_t + F' \left( \frac{l_t}{l_{t-1}} \right) \frac{1}{l_{t-1}} I_t z_t \right) \right] + \\
+ E_t \beta \left\{ \mu_{t+1} \left[ -F' \left( \frac{l_{t+1}}{l_t} \right) (-I_{t+1} I_t^{-2}) z_{t+1} I_{t+1} \right] \right\} = \lambda_t \\
\]

\[
K_t : \beta^t \{ \mu_t[-1] \} + E_t \beta^{t+1} \{ \lambda_t \left[ R_{t+1} u_{t+1} - \psi(u_{t+1}) \right] + \mu_{t+1}[(1-\delta)(1)] \} = 0 \\
E_t \beta \{ \lambda_{t+1} \left[ R_{t+1} u_{t+1} - \psi(u_{t+1}) \right] + \mu_{t+1}[(1-\delta)(1)] \} = \mu_t \\
\]

\[
u_t : \beta^t \{ \lambda_t \left[ K_{t-1} - \psi'(u_t)(1) K_{t-1} \right] \} = 0 \\
R_t K_{t-1} - \psi'(u_t)(1) K_{t-1} = 0 \\
\]

\[
b_t : \beta^t \{ \lambda_t \} + E_t \beta^{t+1} \left\{ \lambda_{t+1} \left[ \frac{r^p}{\pi_{t+1}} \right] \right\} = 0 \\
E_t \beta \left\{ \lambda_{t+1} \left[ \frac{r^p}{\pi_{t+1}} \right] \right\} = \lambda_t \\
\]

The ensuing motion laws would correspondingly thereafter arise:

labour supply

\[
\kappa_t Y_t^\gamma (C_t - hC_{t-1})^{-\sigma c} = \lambda_t \\
\lambda_t W_t = \kappa_t \lambda_t L_t^\nu \\
W_t = \lambda_t L_t^\nu Y_t^{-\gamma} (C_t - hC_{t-1})^{\sigma c} \\
\]

inter-temporal consumption

\[
\kappa_t Y_t^\gamma (C_t - hC_{t-1})^{-\sigma c} = \lambda_t \\
E_t \beta \left\{ \lambda_{t+1} \left[ \frac{r^p}{\pi_{t+1}} \right] \right\} = \lambda_t \\
\kappa_t Y_t^\gamma (C_t - hC_{t-1})^{-\sigma c} = E_t \beta \kappa_{t+1} Y_{t+1}^\gamma (C_{t+1} - hC_t)^{-\sigma c} \left[ \frac{r^p}{\pi_{t+1}} \right] \\
\]

inter-temporal investment

\[
\text{where } q_t = \frac{\mu_t}{\lambda_t} \text{ Tobin’s Q} \\
q_{t+1} = \frac{\mu_{t+1}}{\lambda_{t+1}} \text{ Tobin’s Q} \\
\]

11
\[
\mu_t \left[ z_t - \left( F \left( \frac{I_t}{I_{t-1}} \right) z_t + F' \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{1}{I_{t-1}} \right) I_t z_t \right) \right] + E_t \beta \left\{ \mu_{t+1} \left[ -F' \left( \frac{I_{t+1}}{I_t} \right) \left( -I_{t+1} I_t^{-2} \right) z_{t+1} I_{t+1} \right] \right\} = \lambda_t
\]

\[
\frac{\mu_{t+1}}{\lambda_{t+1}} \cdot \frac{\lambda_{t+1}}{\lambda_t} = \frac{q_{t+1} \pi_{t+1}}{r_t \beta}.
\]

\[
1 = q_t \left[ z_t - \left( F \left( \frac{I_t}{I_{t-1}} \right) z_t + F' \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{1}{I_{t-1}} \right) I_t z_t \right) \right] + E_t \left\{ \frac{q_{t+1} \pi_{t+1} z_{t+1}}{r_t \beta} \right\}
\]

capital pricing dynamics [where but by \( \lambda_t \) one must divide]

\[
E_t \beta \left\{ \lambda_{t+1} \left[ R_{t+1} u_{t+1} - \psi(u_{t+1}) \right] + \mu_{t+1} \left[ (1 - \delta)(1) \right] \right\} = \mu_t
\]

\[
\frac{\mu_{t+1}}{\lambda_{t+1}} \cdot \frac{\lambda_{t+1}}{\lambda_t} = \frac{q_{t+1} \pi_{t+1}}{r_t \beta}.
\]

\[
q_t = E_t \beta \left\{ \frac{\pi_{t+1}}{r_t \beta} \left[ R_{t+1} u_{t+1} - \psi(u_{t+1}) \right] + \frac{q_{t+1} \pi_{t+1} (1 - \delta)}{r_t \beta} \right\}
\]

**Final goods producers**

Final goods are to be by respective producers obtained via the utilisation of an intermediate goods continuum input, with said firms competing perfectly thereat. With the intermediate goods output \( [Y_t(i)] \) to be indexed by \( i \in (0, 1) \), the representative final goods producer is to wherefore employ the ensuing technology:

\[
Y_t = \left[ \int_{0}^{1} Y_t(i) \hat{\pi_t} di \right]^{\theta_t}.
\]

A price mark-up shock is to be therewith moreover regulated by a consuetudinary AR(1) process afresh personified by covariance stationarity, \( \varepsilon_{\theta_t} \sim i.i.d. \mathcal{N}(0, \sigma^2) \):

\[
ln \theta_t = \rho ln \theta_{t-1} + \varepsilon_{\theta_t}.
\]

Forthwith, profit maximisation, in accordance with the firm’s objectives, is to but trigger the following optimisation problem:

\[
\max_{Y_t(i)} P_t Y_t - \int_{0}^{1} P_t(i) Y_t(i) di \ s.t. \ Y_t
\]

\[
\Pi_{fl} = P_t \left[ \int_{0}^{1} Y_t(i) \hat{\pi_t} di \right]^{\theta_t} - \int_{0}^{1} P_t(i) Y_t(i) di
\]

The resulting first order condition would thence appear as thus:
Provided such, the intermediate goods demand could thereafter be unsnarled 6:

\[
\frac{P_t(i)}{P_t} = \left[ \int_0^1 Y_t(i) \frac{1}{\pi_t} \, di \right]^{\theta_t-1} \left[ Y_t(i) \frac{1}{\pi_t} \right]^{\theta_t-1} - P_t(i) = 0.
\]

Consequently, the domestic goods price index would be likewise computable:

\[
Y_t = \left[ \int_0^1 Y_t(i) \frac{1}{\pi_t} \, di \right]^{\theta_t}
\]

\[
Y_r = \left\{ \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{\theta_t} Y_t \left( \frac{1}{\pi_t} \right) \, di \right\}^{\theta_t}
\]

\[
1^{\frac{1}{\pi_t}} = \left\{ \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{\theta_t-1} \frac{1}{\pi_t} \, di \right\}^{\theta_t-1}
\]

\[
1 = P_t^{\frac{1}{\theta_t}} \int_0^1 \left[ P_t(i) \frac{1}{\pi_t} \right]^{\theta_t-1} \, di
\]

\[
P_t^{\frac{1}{\theta_t}} = P_t \int_0^1 P_t(i) \frac{1}{\pi_t} \, di
\]

\[
P_t = \left[ \int_0^1 P_t(i) \frac{1}{\pi_t} \, di \right]^{1-\theta_t}
\]

**Intermediate goods producers**

The intermediate goods market, to be characterised by monopolistic competition amongst participants, is to be limned by firms employing capital \([K_t(i)]\) and labour \([L_t(i)]\) respectively in exchange for salaries \((W_t)\) and capital returns \((R_t)\). Each enterprise, to be indexed by \(i \in (0,1)\), is to thus produce \(Y_t(i)\) differentiated output units through the following confidence \((Y_t(i))\) labour augmented emission technology:

\[
Y_t(i) = P_t(i) \theta_t \left[ \int_0^1 Y_t(i) \frac{1}{\pi_t} \, di \right]^{\theta_t} \left[ Y_t(i) \frac{1}{\pi_t} \right]^{\theta_t-1} - P_t(i) = 0.
\]
\[ Y_t(i) = a_t \tilde{K}_{t-1}(i)^{\alpha} [T_t L_t(i)]^{1-\alpha}. \]

Capital utilisation services, the simple product of capital stock and utilisation, are to be therewith represented by \( \tilde{K}_{t-1}(i) = \tilde{K}_{t-1}(i)u_t \). Confidence \( (Y_t^\gamma) \), with \( \gamma \) to act but as formerly analysed volition, is to on the other hand appear as the yet aforementioned coalescence of the three yea erstwhile discussed diverse AR(1) processes, \( \varepsilon_{it} \sim i.i.d. \mathcal{N}(0, \sigma^2) \) for \( i = n, b, s, a; \) two proceeding from covariance stationarity and one behaving in accordance with a unit root, respectively being:

**noise**

\[ \ln n_t = \rho_n \ln n_{t-1} + \varepsilon_{nt}; \]

**beliefs**

\[ \ln b_t = \rho_b \ln b_{t-1} + \varepsilon_{bt}; \]

**initiative**

\[ \ln s_t = \ln s_{t-1} + \varepsilon_{at} + \varepsilon_{st}. \]

Wherefore, for a set volition degree \( (\gamma) \), let confidence be fundamentally limned as follows:

\[ Y_t^\gamma = (n_t b_t s_t)^\gamma. \]

Production technology, for \( \varepsilon_{at} \sim i.i.d. \mathcal{N}(0, \sigma^2) \), is to correspondingly obey the previously scrutinised covariance stationary AR(1) process correlative:

\[ \ln a_t = \rho_a \ln a_{t-1} + \ln b_{t-1} + \ln s_{t-1} + \varepsilon_{at}. \]

Cost minimisation attendant upon firm \( i \) is to therefore concern the ensuing problem:
\[
\begin{align*}
\min_{\{L_t(i), K_{t-1}(i)\}} & \quad W_t L_t(i) + R_t \bar{K}_{t-1}(i) \quad \text{s.t.} \quad Y_t(i) \\
\Xi_t &= W_t L_t(i) + R_t \bar{K}_{t-1}(i) + \phi_t \{a_t \bar{K}_{t-1}(i)^{\alpha} [Y_t^\gamma L_t(i)]^{1-\alpha}\}.
\end{align*}
\]

First order conditions to be thereon elaborated would hence spell out:

\[
\begin{align*}
L_t(i) : & \quad W_t + \phi_t \{a_t \bar{K}_{t-1}(i)^{\alpha} Y_t^\gamma (1-\alpha)\} [Y_t^\gamma L_t(i)]^{\alpha} = 0; \\
K_{t-1}(i) : & \quad R_t + \phi_t \{a_t \alpha \bar{K}_{t-1}(i)^{\alpha-1} [Y_t^\gamma L_t(i)]^{1-\alpha}\} = 0.
\end{align*}
\]

According motion laws are to ultimately surface:

labour demand

\[
\begin{align*}
\frac{W_t}{R_t} &= \frac{\phi_t \{a_t \bar{K}_{t-1}(i)^{\alpha} Y_t^\gamma (1-\alpha)\} [Y_t^\gamma L_t(i)]^{\alpha-1}}{\phi_t \{a_t \alpha \bar{K}_{t-1}(i)^{\alpha-1} [Y_t^\gamma L_t(i)]^{1-\alpha}\}} \\
W_t &= \frac{R_t}{\alpha \bar{K}_{t-1}(i)^{\alpha-1} L_t(i)}; \\
L_t(i) &= \frac{(1-\alpha) R_t \bar{K}_{t-1}(i)}{\alpha W_t}.
\end{align*}
\]

average costs

\[
\begin{align*}
W_t &= (1-\alpha) a_t \bar{K}_{t-1}(i)^{\alpha} [Y_t^\gamma L_t(i)]^{\alpha} \left[\frac{Y_t^\gamma L_t(i)}{Y_t^\gamma L_t(i)}\right] \phi_t Y_t^\gamma \\
R_t &= \alpha a_t \bar{K}_{t-1}(i)^{\alpha-1} \left[\frac{\bar{K}_{t-1}(i)}{\bar{K}_{t-1}(i)}\right] [Y_t^\gamma L_t(i)]^{1-\alpha} \phi_t;
\end{align*}
\]

marginal costs

\[
\begin{align*}
W_t &= (1-\alpha) Y_t(i) \left[\frac{1}{Y_t^\gamma L_t(i)}\right] \phi_t Y_t^\gamma \\
Y_t^\gamma L_t(i) &= (1-\alpha) Y_t(i) \left[\frac{1}{W_t}\right] \phi_t Y_t^\gamma \\
R_t &= \alpha Y_t(i) \left[\frac{1}{R_t}\right] \phi_t \bar{K}_{t-1}(i) = \alpha Y_t(i) \left[\frac{1}{R_t}\right] \phi_t
\end{align*}
\]
\[
Y_t(i) = a_t \tilde{R}_{t-1}(i)^{\alpha} [Y_t^\gamma L_t(i)]^{1-\alpha} \\
Y_t(i) = a_t \left[ \frac{\alpha Y_t(i)^{\phi_t}}{R_t} \right]^{\alpha} \left[ \frac{(1-\alpha)Y_t(i)^{\phi_t} Y_t^\gamma}{W_t} \right]^{1-\alpha} \\
Y_t(i) = a_t R_t^{-\alpha} \alpha^\alpha Y_t(i)^{\alpha} \phi_t^{\alpha} (1 - \alpha)^{1-\alpha} Y_t(i)^{1-\alpha} \phi_t^{1-\alpha} Y_t^\gamma(1-\alpha) W_t^{\alpha-1} . \\
1 = \alpha^\alpha (1 - \alpha)^{1-\alpha} \phi_t^{1-\alpha} Y_t^\gamma(1-\alpha) W_t^{\alpha-1} \\
\phi_t = \frac{(1-\alpha)^{\alpha-1} R_t^\alpha W_t^{\alpha-1}}{\alpha^\alpha Y_t^\gamma(1-\alpha)}
\]

Inter-temporal aggregate price level dynamics

The monopolistically competitive intermediate goods market is to regard price setting rigidity.
In more detail, a sole randomly determined 1 – ξ firms fraction is to therein adjust prices in
each period; the remnant, ξ, is to instead implicitly index them in accordance with the previous
period’s price level but as follows:

\[
P_t(i) = \pi_{t-1}^\tau P_{t-1}(i)
\]

where \(\pi_t = \frac{P_t}{P_{t-1}}\) and \(\tau\) are to respectively depict the inter-temporal aggregate price level and
the overall price indexation degree. Interpreting such conduct alongside the domestic goods
price index is to but unravel a behaviour for the aggregate price level, namely:

\[
P_t = \frac{\left[ \int_0^1 P_t(i) \frac{1}{\pi_t^\tau} di \right]^{1-\theta_t}}{\int_0^1 P_t(i) \frac{1}{\pi_t^\tau} di}
\]

\[
P_t = \left[ \xi (\pi_{t-1}^\tau P_{t-1})^{1-\theta_t} + (1 - \xi)(P_t^*)^{1-\theta_t} \right]^{1-\theta_t}.
\]

The selection of price \(P_t^*(i)\), to thereby maximise the expected discounted sum of profits, is
to wherefore delineate the price adjusting firm’s actual optimisation problem:

\[
\max_{P_t^*(i)} \mathbb{E}_t \sum_{j=0}^{\infty} (\xi \beta)^j \sum_{h=0}^{\infty} \frac{\lambda_{i+h}}{\lambda_i} \left[ \prod_{k=0}^{j-1} \pi_i^\tau \left( \frac{P_t^*(i)}{\Pi_t(i)} \right)^{\frac{\theta_t}{1-\theta_t}} - \phi_{t+j} \right] Y_{t+j}(i)
\]

\[
s.t.
\]

\[
Y_{t+j}(i) = \left[ \prod_{k=0}^{j-1} \pi_i^\tau \left( \frac{P_t^*(i)}{\Pi_t(i)} \right)^{\frac{\theta_t}{1-\theta_t}} \right] Y_{t+j} .
\]

\[
\Pi_t = \mathbb{E}_t \sum_{j=0}^{\infty} (\xi \beta)^j \sum_{h=0}^{\infty} \frac{\lambda_{i+h}}{\lambda_i} \left[ \prod_{k=0}^{j-1} \pi_i^\tau \left( \frac{P_t^*(i)}{\Pi_t(i)} \right)^{\frac{\theta_t}{1-\theta_t}} - \phi_{t+j} \right] \left[ \prod_{k=0}^{j-1} \pi_i^\tau \left( \frac{P_t^*(i)}{\Pi_t(i)} \right)^{\frac{\theta_t}{1-\theta_t}} \right] Y_{t+j}
\]

The first order condition relative to would thence appear as thus:
Central bank and treasury

New-Keynesian hybrid Phillips relation
temporal aggregate price level closed form solution obtainment, commonly renowned as the advancement of the operative prerequisites for the log-linearised inter-temporal aggregate price level closed form solution obtainment, commonly known as the New-Keynesian hybrid Phillips relation.

\[ P_t^*(i) : \]
\[ E_t \sum_{j=0}^{\infty} (\xi \beta)^j \frac{\lambda_{t+j}}{\lambda_t} \left[ \prod_{k=0}^{j-1} \left( \frac{\pi_{t+k}^{\tau} P_{t+k}^{\tau}}{P_{t+j}^{\tau}} \right) \left( \frac{1}{1-\theta_t} \right) \right] P_t^*(i) \frac{\theta_t}{1-\theta_t} Y_{t+j} = \]
\[ = E_t \sum_{j=0}^{\infty} (\xi \beta)^j \frac{\lambda_{t+j}}{\lambda_t} \left[ \prod_{k=0}^{j-1} \left( \frac{\pi_{t+k}^{\tau} P_{t+k}^{\tau}}{P_{t+j}^{\tau}} \right) \left( \frac{1}{1-\theta_t} \right) \right] \left( \frac{\theta_t}{1-\theta_t} \right) Y_{t+j} \]

so that subsequent joint division by \( P_t^*(i) \frac{\theta_t}{1-\theta_t} \) alongside rearrangement for \( P_t^*(i) \) then spawn

\[ P_t^*(i) = \frac{E_t \sum_{j=0}^{\infty} (\xi \beta)^j \frac{\lambda_{t+j}}{\lambda_t} \left[ \prod_{k=0}^{j-1} \left( \frac{\pi_{t+k}^{\tau} P_{t+k}^{\tau}}{P_{t+j}^{\tau}} \right) \left( \frac{1}{1-\theta_t} \right) \right] \left( \frac{\theta_t}{1-\theta_t} \right) Y_{t+j} \}

The relative domestic price is to be furthermore defined as \( p_t^*(i) = \frac{P_t^*(i)}{P_t} \) and when \( \pi_{t+j} = \frac{P_{t+j}}{P_t} \)

be recollected, such that for \( k = j - 1 \), \( j = k + 1 \), \( \pi_{t+j} = \pi_{t+k+1} \) then be, one is to ultimately obtain:

\[ p_t^*(i) = \frac{A_t}{B_t} \]

In addition, to recursively express the above quotient as

\[ A_t = \lambda_t Y_t \theta_t \phi_t + \xi \beta E_t \left( \frac{\pi_t^{\tau}}{\pi_{t+1}} \right) \left( \frac{1}{1-\theta_t} \right) A_{t+1} \]
\[ B_t = \lambda_t Y_t + \xi \beta E_t \left( \frac{\pi_t^{\tau}}{\pi_{t+1}} \right) \left( \frac{1}{1-\theta_t} \right) B_{t+1} \]

is but equivalent to the advancement of the operative prerequisites for the log-linearised inter-temporal aggregate price level closed form solution obtainment, commonly renowned as the New-Keynesian hybrid Phillips relation:

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{\pi_{t-1}^{\tau}}{1+\beta \tau} + \frac{(1-\beta \xi)(1-\xi)}{(1+\beta \tau) \xi} (\phi_t + \theta_t). \]

Central bank and treasury

\footnote{Wherein capped variables are to represent percentage deviations from the steady state.}
A nominal interest rate \( r^n_t \) setting monetary policy rule is by the central bank to be pursued. More specifically, the latter is to target steady state deviations respectively dependant upon: former period’s nominal interest rate \( r^n_{t-1} \); inter-temporal aggregate price level \( \pi_T \), fluctuant target coequal \( \pi_{Tt} \), output \( Y_t \) as well as output and inter-temporal aggregate price level gaps, all alongside a nominal interest rate smoothing perturbation \( \varphi_t \) incarnating monetary collisions:

\[
\left( \frac{r^n_t}{r^n_{t-1}} \right) = \left( \frac{r^n_{t-1}}{r^n_{t-1}} \right)^\rho \left( \frac{\pi_t/\pi}{\pi_{Tt}/\pi_T} \right)^\phi \left( \frac{\pi_t/\pi}{\pi_{t-1}/\pi} \right)^\phi \left( \frac{Y_t}{Y_{t-1}/Y} \right)^\phi \left( \frac{Y_{t-1}/Y}{Y_{t-1}/Y} \right)^{1-\rho} e^{\varphi_t}
\]

The rule is to be obviously reliant upon two covariance stationary \( AR(1) \) processes, featured by the usual \( \varepsilon_t \sim i.i.d. N(0, \sigma^2) \) condition, hence being:

\[
\begin{align*}
\ln \pi_{Tt} &= \rho \ln \pi_{Tt-1} + \varepsilon_{\pi t} \\
\ln \varphi_t &= \rho \ln \varphi_{t-1} + \varepsilon_{\varphi t}
\end{align*}
\]

The treasury is to simultaneously consume erstwhile governmental bonds principle repayments and interest provisions \( \left( \frac{r^n_{t-1} B_{t-1}}{P_t} \right) \), lump-sum governmental subsidies, transfers namely \( (TA_t) \), and an exogenous stream of public governmental expenditure \( (G_t) \); altogether financed by collected lump-sum taxes \( (T_t) \) and newly-issued governmental bonds \( \left( \frac{B_t}{P_t} \right) \):

\[
\begin{align*}
\frac{r^n_{t-1} B_{t-1}}{P_t} + TA_t + G_t &= T_t + \frac{B_t}{P_t} \\
\frac{r^n_{t-1} B_{t-1}}{P_t} + TA_t + G_t &= T_t + \frac{B_t}{P_t} \\
\frac{r^n_{t-1} B_{t-1}}{P_t} + TA_t + G_t &= T_t + b_t
\end{align*}
\]

Said public expenditure \( (G_t = g_t s_t) \), inherently accounting but for fiscal shocks, is to lastly respect a customary covariance stationary \( AR(1) \) process, endowed with \( \varepsilon_{gt} \sim i.i.d. N(0, \sigma^2) \), but as follows:

\[
\ln g_t = \rho \ln g_{t-1} + \varepsilon_{gt}.
\]

**Market clearing and aggregation**

Gross market capital and labour, employed within the intermediate goods production sector, are to both act but as composites of individual household rented capital and furnished labour, enkindling perfect clearance thereat. Namely:
\[
K_{t-1} = \int_0^1 K_{t-1}(i)di \\
L_t = \int_0^1 L_t(i)di \\
Y_t = \int_0^1 Y_t(i)di
\]

so that the aggregate production function then appear as thus:

\[
Y_t = a_t \bar{K}_{t-1} \Gamma_t [Y_t^\gamma L_t]^{1-\gamma}.
\]

Conclusively, absent capital and labour offering and return, the simulated economy’s aggregate resources constraint is to arise but from the fusion between the representative household’s and the treasury’s respective budget constraints:

\[
Y_t = \left[ C_t + I_t + b_t + T_t + \psi(ut)K_{t-1} - r^n_{t-1} \left( \frac{b_{t-1}}{\pi_t} \right) - TA_t \right] + \\
+ \left[ r^n_{t-1} \left( \frac{b_{t-1}}{\pi_t} \right) + TA_t + G_t - T_t - b_t \right] \\
Y_t = C_t + I_t + \psi(ut)K_{t-1} + G_t
\]

**Equilibrium**

Provided fiscal and monetary policy rules respectively for

\[
G_t, T_t - TA_t, r^n_t,
\]

an equilibrium is to be:

an allocation

\[
\{ C_t, I_t, ut, b_t, D_t, Y_t, Y_t(i)_{i\epsilon(0,1)}, L_t, L_t(i)_{i\epsilon(0,1)}, K_t, K_t(i)_{i\epsilon(0,1)} \}
\]

and prices

\[
\{ r^n_t, R_t, W_t, p_t^*(i)_{i\epsilon(0,1)} \}
\]
such that:
- granted prices and a demand function for $Y_t(i)_{i \in (0,1)}$, subject to the rigid prices mechanism, the allocation then maximise the firms’ profits;
- all markets clear;
- all policy rules bear consistency with prices and allocations.

4 System solution

Numerical approximation, through the use of perturbation methods consisting in the log-linearisation of the germane motion laws about the steady-state, is to herewith set forth the prerequisites for model solution obtainment. Many a method are to be at one’s disposal in such latter regard, with the main ones being: Blanchard-Kahn (1980); Klein (2000); Christiano (2002) and Sims (2002). Such approaches, despite diversely, are to all lead unto model solution uniqueness, so that policy or welfare analysis be thereafter finally conducted. More specifically, upon log-linearisation of said motion laws, the resulting economic rules are to be cast into an algebraic system of linear equations, by the name of state equation, like unto a $VAR(1)$ process. Nominal variables $\{C_t, W_t, K_t, I_t, G_t, Y_t\}$ are to be nevertheless de-trended by $s_t$ beforehand, that being afore said log-linearisation, as ensues:

$$X_t = x_t s_t$$

where $X_t$ is to portray any amongst the nominal variables under scrutiny; howbeit, such all is to be worked so as to but dispose of the process for initiative’s inherent non-stationarity. Hereunder is to lie the entire set of log-linearised motion laws attendant upon the present model:

labour supply

$$\hat{w}_t = \hat{\chi}_t + \eta \hat{L}_t - \gamma \hat{\Upsilon}_t + \sigma_c (\hat{c}_t + \hat{s}_t) - \sigma_c h (\hat{c}_{t-1} + \hat{s}_{t-1}) - \hat{s}_t;$$

labour demand

$$\hat{L}_t = (1 + \omega) \hat{R}_t + \hat{k}_{t-1} + \hat{s}_{t-1} - \hat{w}_t - \hat{s}_t;$$

inter-temporal consumption
\[
\hat{c}_t = \frac{(1 - h)(\hat{c}_t + \gamma \hat{Y}_t)}{\sigma_c(1 + h)} + \frac{(1 - h)(E_t \hat{c}_{t+1} - \hat{c}_t^n)}{\sigma_c(1 + h)} + \frac{E_t(\hat{c}_{t+1} + \hat{s}_{t+1})}{(1 + h)} + \frac{h(\hat{c}_{t-1} + \hat{s}_{t-1})}{(1 + h)} - \hat{s}_t;
\]

nominal interest rate

\[
\hat{r}_t^n = \rho_r \hat{r}_{t-1} + (1 - \rho_r) \left[ \phi_\pi (\hat{\pi}_t - \bar{\pi}_{T_t}) + \phi_y (\hat{\pi}_t - \bar{\pi}_{t-1}) + \phi_y (\hat{\pi}_t - \bar{\pi}_{t-1}) + \phi_y (\hat{\pi}_t - \bar{\pi}_{t-1}) \right] + \varphi_t;
\]

inter-temporal aggregate price level

\[
\hat{\pi}_t = \frac{\beta E_t \hat{\pi}_{t+1}}{1 + \beta \tau} + \frac{\tau \hat{\pi}_{t-1}}{1 + \beta \tau} + \frac{(1 - \beta \xi)(1 - \xi)}{(1 + \beta \tau) \xi} (\hat{\phi}_t + \hat{\theta}_t);
\]

inter-temporal investment

\[
\hat{i}_t = \frac{\hat{q}_t}{(1 + \beta) \alpha} + \frac{\hat{s}_{t-1}}{(1 + \alpha)(1 + \beta)} + \frac{\hat{c}_{t-1}}{(1 + \beta)} + \frac{\beta E_t \hat{i}_{t+1}}{(1 + \beta)} + \frac{\beta E_t \hat{s}_{t+1}}{(1 + \beta)} + \frac{(1 - \beta) \hat{s}_t}{(1 + \beta)};
\]

capital pricing dynamics

\[
\hat{q}_t = \left[ \frac{(1 - \delta)}{(1 - \delta + R)} \right] E_t \hat{q}_{t+1} + \left[ \frac{R}{(1 - \delta + R)} \right] E_t \hat{R}_{t+1} - \hat{r}_t^n + E_t \hat{\pi}_{t+1};
\]

production

\[
\hat{y}_t = \hat{a}_t + \alpha \hat{k}_{t-1} + \alpha \omega \hat{R}_t + (1 - \alpha)(\gamma \hat{Y}_t + \hat{L}_t) - \hat{s}_t + \alpha \hat{s}_{t-1};
\]

confidence

\[
\hat{Y}_t = \hat{n}_t + \hat{b}_t + \hat{s}_t;
\]

marginal costs
\( \hat{\phi}_t = \alpha \hat{R}_t + (1 - \alpha) \hat{w}_t + (1 - \alpha) \hat{s}_t - \hat{a}_t - \gamma (1 - \alpha) \hat{Y}_t; \)

private capital

\( \hat{k}_t = (1 - \delta) (\hat{k}_{t-1} + \hat{s}_{t-1} - \hat{s}_t) + \delta (\hat{s}_t + \hat{i}_t); \)

aggregate resources constraint

\( \hat{y}_t = \left( \frac{c}{y} \right) (\hat{c}_t + \hat{s}_t) + \left( \frac{i}{y} \right) (\hat{i}_t + \hat{s}_t) + \left( \frac{k}{y} \right) R \hat{\omega} \hat{R}_t + \left( \frac{g}{y} \right) (\hat{g}_t + \hat{s}_t) - \hat{s}_t; \)

preferences shock

\( \hat{\kappa}_t = \rho_{\kappa} \hat{\kappa}_{t-1} + \epsilon_{\kappa t}; \)

labour disutility shock

\( \hat{\chi}_t = \rho_{\chi} \hat{\chi}_{t-1} + \epsilon_{\chi t}; \)

investment-specific technology shock

\( \hat{z}_t = \rho_z \hat{z}_{t-1} + \epsilon_{zt}; \)

fluctuant price mark-up shock

\( \hat{\theta}_t = \rho_\theta \hat{\theta}_{t-1} + \epsilon_{\theta t}; \)

noise shock
\[ \hat{n}_t = \rho_n \hat{n}_{t-1} + \varepsilon_{nt}; \]

beliefs shock

\[ \hat{b}_t = \rho_b \hat{b}_{t-1} + \varepsilon_{bt}; \]

initiative shock

\[ \hat{s}_t = \hat{s}_{t-1} + \varepsilon_{at} + \varepsilon_{st}; \]

production technology shock

\[ \hat{a}_t = \rho_a \hat{a}_{t-1} + \hat{b}_{t-1} + \hat{s}_{t-1} + \varepsilon_{at}; \]

fluctuant inter-temporal aggregate price level target shock

\[ \hat{\pi}_{Tt} = \rho_{\pi T} \hat{\pi}_{Tt-1} + \varepsilon_{\pi Tt}; \]

monetary shock

\[ \hat{\rho}_t = \rho_{\phi} \hat{\rho}_{t-1} + \varepsilon_{\phi t}; \]

fiscal shock

\[ \hat{g}_t = \rho_g \hat{g}_t + \varepsilon_{gt}. \]
Endogenous model variables are to be separated into two categories: past and future; with the former to encompass all ones present at times \( j \) and \( j-1 \) \( \forall j = t, t-1, \ldots \) and the latter all expected counterparts, yet within such periods. This is to imply that all preceding periods’ future variables are to but give rise to the necessity of expectational revisions usage, à la Sims (2002) namely, themselves to thence also be embedded within the system of log-linearised equations as relevant motion laws. In the present model, there are to thus be 23 past variables and 6 future correlatives, catalysing a total of 29 economic rules. Wherefore, cast all motion laws into a VAR(1) system of linear algebraic equations one is to summarily unravel:

\[
Q_{(n_x \times n_x)(n_x \times 1)} x_t = R_{(n_x \times n_x)(n_x \times 1)} x_{t-1} + S_{(n_x \times n_x)(n_x \times 1)} \varepsilon_t + T_{(n_x \times n_\eta)(n_\eta \times 1)} \eta_t;
\]

with matrix \( Q \) as incontrovertibly non-invertible. The so-called QZ matrices decomposition method is to be thence applied so that the condition for solution uniqueness, dictating expectational errors and states companion matrices unstable generalised eigenvalues equality, be then checked and, upon satisfaction thereof, the above state equation ultimately rewritten as thus:

\[
x_t = Ax_{t-1} + B \varepsilon_t.
\]

Given a thereupon available stable matrix \( A \), policy analysis, in this case via the evaluation of orthogonalised impulse response functions, withal limning sudden deviations from the economy’s inherent steady-state, is to be finally accomplished. In other words, provided \( |\lambda_{A(\lambda)}| < 1 \), \( L^i x_t = x_{t-i} \forall i = 0, 1, \ldots \) and a lower triangular \( D \) matrix attendant upon error positive-semidefinite variance-covariance matrix \( \Sigma = DD' \) such that \( \varepsilon_t = D \eta_t \) be, one is to bring pen to paper but as follows:

\[
(I - AL)x_t = B \varepsilon_t \\
x_t = A(L)^{-1}B \varepsilon_t \\
x_t = \sum_{j=0}^{\infty} A^j B \varepsilon_{t+j} \\
x_t = \sum_{j=0}^{\infty} A^j B D \eta_{t+j}
\]

with \( \frac{\partial x_t}{\partial \varepsilon_{t+j}} = A^j B D \forall \ j = 0, \ldots, \infty \) as model upon shock orthogonalised impulse response functions whenever plotted against time.

## 5 Calibration and policy analysis

Stochastic parameters, for the United States of America, are to be calibrated, in accordance with Smets-Wouters (2007)’s full information estimation methods application, but as follows:
<table>
<thead>
<tr>
<th>Stochastic parameter</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>2.45</td>
<td>Labour effort inverse elasticity</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>1.62</td>
<td>Relative risk aversion</td>
</tr>
<tr>
<td>$h$</td>
<td>0.69</td>
<td>Consumption habit</td>
</tr>
<tr>
<td>$\omega$</td>
<td>3.23</td>
<td>Capital utilisation adjustment cost inverse elasticity</td>
</tr>
<tr>
<td>$\rho_{\pi n}$</td>
<td>0.88</td>
<td>Nominal interest rate coefficient</td>
</tr>
<tr>
<td>$\phi_{\pi}$</td>
<td>1.48</td>
<td>Inter-temporal aggregate price level coefficient</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>0.08</td>
<td>Output coefficient</td>
</tr>
<tr>
<td>$\phi_{\pi g}$</td>
<td>0.24</td>
<td>Inter-temporal aggregate price level gap coefficient</td>
</tr>
<tr>
<td>$\phi_{y g}$</td>
<td>0.24</td>
<td>Output gap coefficient</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Future discount factor</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.66</td>
<td>Price rigidity indexation</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.87</td>
<td>Price rigidity probability</td>
</tr>
<tr>
<td>$\delta$</td>
<td>5.86</td>
<td>Investment adjustment cost elasticity</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Capital depreciation rate</td>
</tr>
<tr>
<td>$R$</td>
<td>0.0351</td>
<td>Steady-state return on capital ($\beta^{-1} - 1 + \delta$)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.24</td>
<td>Capital production share</td>
</tr>
<tr>
<td>$C$</td>
<td>0.65</td>
<td>Consumption to output share</td>
</tr>
<tr>
<td>$\sum K$</td>
<td>0.17</td>
<td>Investment to output share</td>
</tr>
<tr>
<td>$\sum Y$</td>
<td>6.8</td>
<td>Capital to output share ($\frac{K}{Y}\div\delta$)</td>
</tr>
<tr>
<td>$\sum \xi$</td>
<td>0.18</td>
<td>Government to output share</td>
</tr>
<tr>
<td>$\rho_k$</td>
<td>0.49</td>
<td>Preferences shock angle</td>
</tr>
<tr>
<td>$\rho_X$</td>
<td>0.99</td>
<td>Labour disutility shock angle</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.75</td>
<td>Investment-specific technology shock angle</td>
</tr>
<tr>
<td>$\rho_\theta$</td>
<td>0</td>
<td>Fluctuant price mark-up shock angle</td>
</tr>
<tr>
<td>$\rho_n$</td>
<td>0.67</td>
<td>Noise shock angle</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>0.95</td>
<td>Beliefs shock angle</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.822</td>
<td>Production technology shock angle</td>
</tr>
<tr>
<td>$\rho_{\pi_T}$</td>
<td>0.924</td>
<td>Fluctuant inter-temporal aggregate price level target shock angle</td>
</tr>
<tr>
<td>$\rho_{\phi}$</td>
<td>0</td>
<td>Monetary shock angle</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>0.98</td>
<td>Fiscal shock angle</td>
</tr>
<tr>
<td>$\gamma_i$</td>
<td>$i = H, M, L$</td>
<td>Volition degree</td>
</tr>
</tbody>
</table>

Summarily, hence, for a given volition degree ($\gamma$), upon confidence perturbations, diverse conducts in economic activity are to be expected, each reverberating an inherent change in its respective structural error: initiative ($\varepsilon_{it}$); production technology ($\varepsilon_{at}$); beliefs ($\varepsilon_{bt}$) and noise ($\varepsilon_{nt}$). Four herebefore mentioned scenarios are to be sought as replicable, to be synthesised as but ensues:

- "boom bust" cycle;
- long – lasting expansion;
- exogenous growth;
- endogenous growth.
To engender such, volition, heed to signal, action upon signal, is to be formulated as the master key. In more detail, three distinct volition degrees are to be devised, herewith propounded as three effectively distinct regimes: low; medium and high.

<table>
<thead>
<tr>
<th>Volition degree</th>
<th>Value</th>
<th>Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_H$</td>
<td>0.79</td>
<td>High</td>
</tr>
<tr>
<td>$\gamma_M$</td>
<td>0.28</td>
<td>Medium</td>
</tr>
<tr>
<td>$\gamma_L$</td>
<td>0.01</td>
<td>Low</td>
</tr>
</tbody>
</table>

Let it be emphasised that the process by which the selection of a befitting parameter value, attendant upon a specific volition degree, be done is not to be critiqued as worthy of full information or indirect inference estimation approaches, thereby relegating the present exercise to unfettered velleity. The sense of volition regimes is the one to depict, nay to portray, the potential scenarios within which a confidence centred economy, as it much seems to be for the present example, may well find itself across periods of time. Whatever the value, however untoward it may numerically be, resolution to act is to be either mild, inexistent or strong; resolution to act, all in all, is to be. Impulse response functions are to wherefore be gauged. Throughout a regime of low volition:

output responses upon beliefs (dotted), productivity (solid) and initiative (dashed) jolts are to be all exhibited. More specifically, an exogenous growth pattern is to be descried upon perturbations in the latter two processes; that is agents are to hesitate in acting upon the processes coalescence hence relinquishing anticipated growth opportunities. Responsiveness to noise (dashed-dotted) alterations is, on the other hand, to be but void, globally counterbalancing the evaluation. Hence, in synthesis, be volition low, responsiveness to the signal low, and be false news divulged, agents are not to spawn the customarily pernicious boom and bust cycle thereon; be real or uncertain news instead spread, or entrepreneurial initiative augment, anticipated growth is to be withal be forborne. Throughout a medium regime:
reactions in output to all four structural errors are to be personified. The trade-off between higher instantaneous convergence towards the eventually higher balanced growth path and a somewhat more conspicuous responsiveness to sheer noise is notwithstanding significant. More specifically, the adjustment phase following the boom upon a noise shock is to lack the habitually bothersome steepness to which agents are commonly averse, whilst an immediate action upon solid shocks is to appear to but concede instantaneous amelioration in overall output quantities. In a nutshell, whilst low volition, low signal heed, is to fully insure agents from being harmed by noise shocks, it is to annihilate all remote endogenous growth openings; in a context ruled by medium volition, endogenous growth is to be in reach and noise shocks not overly doleful. Lastly, throughout a high regime:

excessive heed to the signal is to eventuate as unwholesome and disadvantageous. A de
generative boom and bust cycle is to loom upon the agents be noise perturbed and indeed destabilising liquidation time fragments are to materialise themselves be even productivity or entrepreneurial initiative shocks the case. Beliefs alterations are, on the other hand, not to be as baneful: in fact, as compared with the prevenient scenarios, such over attention to the signal, and over reaction thereof, is to, because of the inevitably eventual fall verified as such but a posteriori, actually permit agents to extract the most out of an initial phase howbeit delineated by uncertainty. Basically, in spite of its eventually reverting trend instilled but by definition, but by nature, less of the otherwise foregone prosperity is to be thereon dispensed with.
6 Conclusion

The present monograph has set forth the structure by which spirits and news are now to be seen as both actively present within issues regarding fluctuations and confidence. News, being certain, uncertain and spurious, are yea to be present and are to take their effect, be they real, but through active intention. Volition regimes, upon signal the action, thereby labour augmenting final production, are to mushroom, to swell, to enlarge, to resound, the aforementioned economic induction. That is, be it supply shocks, through initiative passing, much heed upon signal is to unravel both immediate and, lo, non-reverting progression; be it beliefs, and yet high volition, long-lasting yet eventually dying is to be the reaction; be it noise, on the other hand, a boom and a bust is to be the depiction. Accordingly is conduct then going to behave for a medium volition and, behold, for an even lower degree of attention. In that case being but exogenous growth upon supply and initiative, reverting activity for consumer beliefs and no movement at all for the greatest and most frequent of all, for misinformation.
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