Modeling the Phillips curve with unobserved components

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Abstract

The relationship between inflation and the output gap can be modeled simply and effectively by including an unobserved random walk component in the model. The dynamic properties match the stylized facts and the random walk component satisfies the properties normally required for core inflation. The model may be generalized to as to include a term for the expectation of next period's output, but it is shown that this is difficult to distinguish from the original specification. The model is fitted as a single equation and as part of a bivariate model that includes an equation for GDP. Fitting the bivariate model highlights some new aspects of unobserved components modeling. Single equation and bivariate models tell a similar story: an output gap two per cent above trend is associated with an annual inflation rate that is one percent above core inflation.

KEYWORDS: Cycle; hybrid new Keynesian Phillips curve; inflation gap; Kalman filter, output gap.

1 Introduction

There has been something of a resurgence of interest in the Phillips curve in recent years; see the opening remarks to a recent conference at the Federal Reserve by Bernanke in 2007. A popular model is the hybrid New Keynesian Phillips curve (NKPC) in which the rate of inflation is assumed to depend on lagged inflation, the expected value of inflation in the next period and a measure of the output gap, such as detrended GDP; see Gali and Gertler (1999). The lagged variable is designed to capture inflation persistence. Formulations of this kind play a prominent role in dynamic stochastic general equilibrium (DSGE) models, such as the one described by Smets and Wouter (2003), and they are taken for granted in a good deal of macroeconomic modeling. Hence issues surrounding the specification have wider implications than just inflation.

The proposed model is a simple modification of the backward-looking Phillips curve in which lagged inflation is replaced by an unobserved random walk component. The role of the random walk component is to capture the underlying level of inflation. Since the output gap is stationary, the long-run forecast is the current expected value of the random walk. This is often taken as a definition of core inflation; see Bryan and Ceccheti (1994) and Cogley (2002).

The model can be extended to include a forward-looking term and as such it can be regarded as a modification of the hybrid NKPC. It is shown that inflation can be written as a linear combination of the expectation of core inflation, the output gap, the discounted sum of expectations of the output gap and a residual. Issues of identifiability for the hybrid NKPC are discussed in Nason and Smith (2006) and similar considerations arise here. When the discounted sum of expectations of the output gap depends only on current and (possibly) lagged values of the output gap, the model essentially reverts to its original specification. In these circumstances, it is difficult to produce convincing estimates of the coefficient of the forward-looking term.

The hybrid NKPC cannot adequately deal with nonstationarity[1] and is sometimes estimated after detrending, usually with a Hodrick-Prescott filter. The use of detrended inflation begs the question of what explains core inflation; see also Fukac and Pagan (2006). Gali and Gertler (1999, p.203) write 'Oddly, enough, however, the hybrid Phillips curve has met with rather limited success.' Their response is to use marginal cost. But the problem is that the dynamic specification of the hybrid NKPC is flawed and hence there is nothing odd about its failure. The proposed specification encompasses the hybrid NKPC and other models and in doing so exposes their shortcomings.

The papers by Kuttner (1994), Planas and Rossi (2004), Domenech and

[1]Furthermore GMM attempts to estimate the model with nonstationary series run into difficulties; see Mavroeidis (2005), Stock, Wright and Yogo (2002) and Pesaran (1987).

Gomez (2006) and Planas, Rossi and Fiorentini (2007) are related in that they use unobserved components in models linking inflation with the output gap. The inflation equation proposed here differs significantly from the ones in Kuttner (1994) and Planas and Rossi (2004) in that the lagged growth rate of GDP is dropped and a stochastic trend included. Note that the main motivation in these papers is to use the information in inflation to obtain better estimates of the output gap. This point is worth bearing in mind when bivariate estimation of our model is considered.

The statistical case for the proposed model is made in section 2 where univariate unobserved components methodology is used to examine the output and inflation gaps and the relationship between them. The model is set out in section 3 and extended to include a forward-looking term. Estimation is considered in section 4. Bivariate estimation makes use of some new capabilities in the STAMP package of Koopman *et al* (2007).

2 Preliminary modeling and stylized facts

It is useful to begin with an exploration of the relationship between inflation and output based on fitting univariate unobserved components (UC) models. We do not want to impose a tight 'theoretical' specification at the outset, particularly one involving a specific lag structure or a dogmatic statement of the way in which expectations enter the model. Instead we use UC models to yield a decomposition into persistent and transitory movements, the interpretation of which is informed by economic theory.

The model is developed with US data. Output is measured by the logarithm of quarterly real U.S. Gross Domestic Product (GDP), denoted y_t, while the (annualized) rate of inflation, π_t, is measured as the first differences of the quarterly CPI multiplied by four. We have data from 1947(1) to 2007(2), obtained for GDP from Department of Commerce (website: www.bea.gov) and, for CPI, U.S. Bureau of Labor Statistics (website: www.bls.gov). There is a case for starting estimation in 1952(1), since the early observations show quite extreme movements because of the Korean war. Models were also fitted using the GDP deflator to measure inflation. Although there are some differences, the overall message remains the same and the results are not reported.

2.1 Output gap

A trend-cycle model can be set up as

$$y_t = \mu_t + \psi_t + \varepsilon_t, \quad t = 1, ..., T \tag{1}$$

where μ_t is an integrated random walk,

$$\begin{array}{rcl} \mu_t & = & \mu_{t-1} + \beta_{t-1}, \\ \beta_t & = & \beta_{t-1} + \zeta_t, \end{array} \quad , \quad t = 1, ..., T \tag{2}$$

ψ_t is a stochastic cycle and ε_t is white noise. The stochastic cycle

$$\begin{bmatrix} \psi_t \\ \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \\ \kappa_t^* \end{bmatrix}, \quad t = 1, ..., T, \tag{3}$$

where λ_c is frequency in radians, ρ is a damping factor, with $0 \leq \rho < 1$, and κ_t and κ_t^* are two mutually independent white noise disturbances with zero means and common variance σ_κ^2. The reduced form is an $ARMA(2,1)$ process in which the autoregressive part has complex roots. The disturbances, ε_t, ζ_t, κ_t and κ_t^* are serially and mutually uncorrelated with variances σ_ε^2 and σ_ζ^2 for the irregular and slope and σ_κ^2 for the two cycle disturbances. The model is assumed to be Gaussian and is estimated by maximum likelihood (ML). The smoothed estimates of the cycle can serve as a measure of the output gap. When the model is fitted to US GDP, σ_ε^2 is estimated to be zero and so the cycle is the same as the detrended series. A smoother cycle can be produced by using the higher-order models proposed in Harvey and Trimbur (2003).

The Hodrick-Prescott (HP) filter is widely used for detrending time series in macroeconomics. The detrended series can be computed as the smoothed estimates of the irregular component in a model in which $y_t = \mu_t + \varepsilon_t$,where μ_t is an integrated random walk as in (2) and signal-noise ratio, $q = \sigma_\zeta^2/\sigma_\varepsilon^2$, is fixed at 1/1600 for quarterly data. The output gap series produced by HP detrending is close to the cycle series obtained by fitting the trend-cycle model. However, while the HP filter may be relatively efficient in the middle of a series, it is much less efficient at the end; see Mise, Kim and Newbold (2005). Furthermore a model is needed to produce forecasts.

2.2 Inflation

Cogley and Sargent (2007, section 2) argue that 'A consensus has emerged that trend inflation is well approximated by a driftless random walk'. A simple model is the random walk plus noise or local level model:

$$\pi_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2), \qquad t = 1, ..., T \tag{4}$$

$$\mu_t \;\;=\;\; \mu_{t-1} + \eta_t, \qquad \eta_t \sim NID(0, \sigma_\eta^2), \tag{5}$$

The disturbances, ε_t and η_t are serially and mutually uncorrelated and the notation $NID\,(0, \sigma^2)$ denotes normally and independently distributed with mean zero and variance σ^2. When σ_η^2 is zero, μ_t is a constant. The signal-noise ratio, $q = \sigma_\eta^2/\sigma_\varepsilon^2$, plays the key role in determining how observations should be weighted for prediction and signal extraction. Stock and Watson (2007) extend the model to allow the disturbances to be generated by stochastic volatility (SV) processes. This is not done here but it would be an option.

More generally, the *inflation gap,* the difference between inflation and core (trend) inflation, $\pi_t - \mu_t$, can be modeled as a stationary process. Cogley and Sargent (2007) focus on how its dynamics and volatility have changed over time. They do this by adding lagged inflation to the model and letting its coefficient evolve as a random walk. However, adding a lagged dependent variable to the model takes us in a different direction from the one in which we wish to go.

Figure 1 shows the smoothed components from a UC model in which a stochastic cycle and a stochastic seasonal, specified as in Koopman et al (2007), have been added to (4), that is

$$\pi_t = \mu_t + \psi_t + \gamma_t + \varepsilon_t, \qquad t = 1, ..., T.$$

If the inflation gap is estimated by the cycle it is somewhat smoother than the detrended (and seasonally adjusted) series because the irregular has been filtered out.

The above formulation in terms of additive components is preferred to one in which the dynamics of the inflation gap are picked up by a lagged dependent variable, that is

$$\pi_t = \mu_t + \alpha\pi_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim NID\left(0, \sigma_\varepsilon^2\right), \quad t = 1, ..., T. \tag{6}$$

The disadvantage of (6) is that the dynamics of the inflation gap are imposed on the underlying level (core inflation), since

$$\pi_t = (1 - \alpha L)^{-1}\mu_t + (1 - \alpha L)^{-1}\varepsilon_t, \quad t = 1, ..., T$$

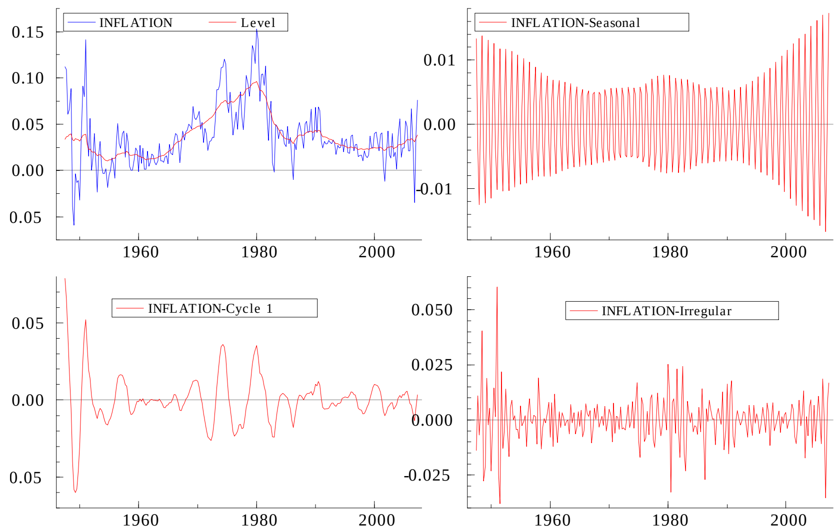

Figure 1: Inflation and its decomposition into stochastic level, cycle, seasonal and irregular.

2.3 Output gap and inflation gap

Figure 2 shows a joint plot of the cycles extracted from the models fitted in the last two sub-sections. It shows clearly why insisting on a model with time invariant dynamics is unwise. Output clearly leads inflation in the 1970s, primarily at the time of the two oil crises, but this is not the case later on.

3 Model specification

The analysis of the previous section suggests that inflation should be modeled so as to be integrated of order one, that is stationary in first differences. The output gap is stationary by construction. These properties need to be borne in mind when assessing the dynamic properties of various Phillips curve models.

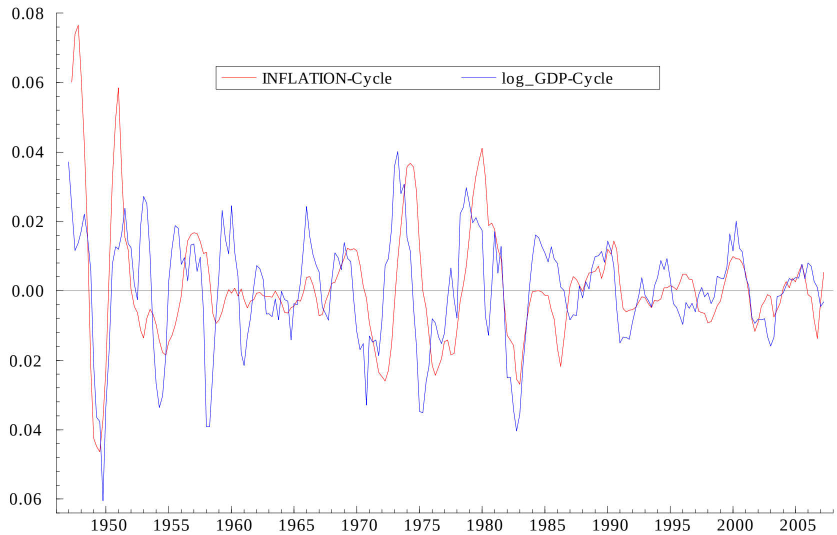

Figure 2: Smoothed estimates of the cycles obtained from univariate models for inflation and (log) GDP

3.1 A short history of the Phillips curve

The basic Phillips curve relating inflation to the output gap is, ignoring the constant,

$$\pi_t = \beta x_t + \varepsilon_t, \quad \varepsilon_t \sim NID\left(0, \sigma_\varepsilon^2\right), \quad t = 1, ..., T \tag{7}$$

where x_t is the output gap, such as detrended GDP. This model is inadequate for capturing dynamics and the initial response was to add a lagged dependent variable (Friedman-Phelps). This also proved inadequate for modeling behaviour and in the new Keynesian Phillips curve (NKPC) the lagged dependent variable was replaced by a future expectation to give

$$\pi_t = \gamma E_t(\pi_{t+1}) + \beta^* x_t + \varepsilon_t, \quad t = 1, ..., T \tag{8}$$

where $0 \leq \gamma \leq 1$. However, when expectations are based on this model, inflation depends exclusively on the discounted sequence of future output gaps. If x_t is assumed to be a stationary AR(1) process with coefficient ϕ, the model reverts to (7) with $\beta = \beta^*/(1 - \phi\gamma)$. If x_t were assumed to be AR(p) with for $p \geq 2$ it would introduce $p - 1$ lags of x_t into the equation and allow γ to be identified; see Nason and Smith (2006, p5) and Pesaran (1987, propositions 6.1 and 6.2). More generally, identifying information is only available when p is greater than the number of lags in the original equation.

The hybrid NKPC is

$$\pi_t = \alpha\pi_{t-1} + \gamma E_t(\pi_{t+1}) + \beta^* x_t + \varepsilon_t^* \tag{9}$$

Solving for $E_t(\pi_{t+1})$, as in Nason and Smith (2006, p5), and substituting in the original equation yields

$$\pi_t = \frac{\alpha}{1-\gamma\delta_1}\pi_{t-1} + \frac{\beta^*}{(1-\gamma\delta_1)\delta_2}\sum_{j=0}^{\infty}\left(\frac{1}{\delta_2}\right)^j E_t(x_{t+1+j}) + \frac{\beta^*}{1-\gamma\delta_1}x_t + \frac{1}{1-\gamma\delta_1}\varepsilon_t$$

where δ_1 and δ_2 are, respectively, the stable and unstable roots of $-L^{-1} + \gamma^{-1} - \alpha\gamma^{-1}L$. Identifiability issues are much the same as with the NKPC. Thus if x_t is AR(1), the model has the form of the original, (9). The rate of inflation is stationary if $|\alpha/(1-\gamma\delta_1)| < 1$. Setting $\alpha = 1-\gamma$, implies $\delta_1 = 1$ so that π_t is then nonstationary. However, writing the equation as $\Delta\pi_t = \beta x_t + \varepsilon_t$ makes it clear that what we have is not really a Phillips curve (though adding lags of x_t and incorporating a unit root could retrieve the situation). On the other hand, a stationary model could only fit the data if inflation were detrended first.

3.2 The unobserved components Phillips curve

Instead of using a lagged variable to capture inflation persistence, we might use an unobserved random walk so

$$\pi_t = \mu_t + \beta x_t + \varepsilon_t, \quad \varepsilon_t \sim NID\left(0, \sigma_\varepsilon^2\right), \quad t = 1, ..., T \tag{10}$$

where μ_t is as in (5). Since x_t is stationary, the long-run forecast is the current expected value of μ_t and so is a measure of core inflation. It is consistent with the univariate model fitted in sub-section 2.2, the difference being that the inflation gap is now explained by the output gap. The model captures backward price setting behaviour because it can be expressed as

$$\pi_t = E_{t-1}(\mu_t) + \beta x_t + \nu_t, \quad \nu_t \sim NID\left(0, \sigma^2\right) \tag{11}$$

where $\nu_t = \pi_t - E_{t-1}(\pi_t)$ is the innovation and $E_{t-1}(\mu_t)$ is weighted average of past observations, corrected for the effect of the output gap; in the steady state[2] this is an exponentially weighted moving average (EWMA), that is

$$E_{t-1}(\mu_t) = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j (\pi_{t-1-j} - \beta x_{t-1-j}) \tag{12}$$

and $\lambda = \left(-q + \sqrt{q^2 + 4q}\right)/2$. This is very different from just including a single lagged dependent variable, π_{t-1}.

The model can also be regarded as forward-looking. The conditions under which a model with an expectational term reverts to (10) are derived below. The fact that the forecast function for a random walk component is constant simplifies matters considerably.

The reduced form of (10) is an ARMAX model in which $\Delta\pi_t$ is equal to $\beta\Delta x_t$ plus a moving average disturbance term. The autoregressive distributed lag reduced form

$$\pi_t = \sum_{i=1}^{\infty} \alpha_i \pi_{t-i} + \beta x_t + \sum_{i=1}^{\infty} \beta_i x_{t-i} + \nu_t \tag{13}$$

where $\alpha_i = \lambda(1-\lambda)^i$ and $\beta_i = \beta\alpha_i, i = 1, 2, ...,$ can be obtained from the ARMAX model or directly from (11). The coefficients on the lags will typically be quite small with a slow decay. For example, if $q = 0.05$, then $\lambda = 0.2$. As will be seen later, this has implications for model selection and interpretation.

3.3 Expectations

Suppose the hybrid NKPC is modified by replacing lagged inflation by an unobserved random walk, μ_t^*, and setting its coefficient to $1-\gamma$, that is

$$\pi_t = (1-\gamma)\mu_t^* + \gamma E_t(\pi_{t+1}) + \beta^* x_t + \varepsilon_t^*, \quad t = 1, ..., T \tag{14}$$

with $0 \leq \gamma \leq 1$. Letting the coefficients of μ_t^* and $E_t(\pi_{t+1})$ sum to one is not a constraint in the way it is for the hybrid NKPC since a free coefficient would be absorbed in $\sigma^2_{\eta*}$, the variance of the disturbance driving μ_t^*, and so would not be identifiable.

[2] In practice the Kalman filter is initiated with a diffuse prior and it only approaches the steady-state asymptotically; see Harvey (1989). However, it simplifies matters to assume a steady-state at the outset.

Some algebraic manipulation, set out in appendix A, yields

$$\pi_t = E_{t-1}(\mu_t^*) + \gamma\beta^* \sum_{j=0}^{\infty} \gamma^j E_t(x_{t+1+j}) + \beta^* x_t + \nu_t, \tag{15}$$

showing that inflation decomposes into a linear combination of the expectation of core inflation, the output gap, the discounted sum of expectations of the output gap and a residual.

The future expectations can be removed as before by making an assumption about the process followed by x_t. Thus if x_t is AR(1) and $|\phi| < 1$, (25) becomes

$$\pi_t = \mu_t^\dagger + \frac{\gamma\phi\beta^*}{1-\phi\gamma} x_t + \beta^* x_t + \varepsilon_t^\dagger = \mu_t^\dagger + \frac{\beta^*}{1-\phi\gamma} x_t + \varepsilon_t^\dagger.$$

This cannot be distinguished from (10), but if x_t is a stationary AR(2), which corresponds approximately to the stochastic cycle used in (1), $E_t(x_{t+1+j})$, $j = 0, 1, 2, ...$, depends on x_t and x_{t-1}, and identification of γ may be possible. However, as with the hybrid NKPC, it is difficult to have confidence in the assumptions needed to separate out forward and backward effects. Furthermore the results in Rudd and Whelan (2005) lead them to conclude that '..forward-looking terms play a very limited role in explaining inflation dynamics.' Similarly Nason and Smith (2006) conclude that '..there is littel evidence of forward looking dynamics in U.S, U.K. and Canadian inflation.'

If extra information, beyond univariate predictions of the output gap, is used to form expectations, it needs to be taken on board. An interesting possibility is add the expected target inflation, π_{t+1}^*, of the Central Bank at time t. Thus

$$\pi_t = \mu_t + \beta x_t + \delta E_t(\pi_{t+1}^*) + \varepsilon_t.$$

Dossche and Everaert (2005) incorporate some elements of this approach into the hybrid NKPC by letting the expectational term in (9) be replaced by $E_t(\mu_{t+1}^\ddagger)$, where $\mu_t^\ddagger$ is a random walk that models the Central Bank's inflation target. Such a term could be included in (14) instead of $E_t(\pi_{t+1})$, but further assumptions would be needed to disentangle it from μ_t^*. In the absence of such assumptions we revert to (14) and the only issue is whether to interpret μ_t^* as (partly) reflecting the Central Bank's inflation target.

Note that if the Central Bank were to announce a commitment to setting the path of future output gaps to zero, the term in (24) containing the discounted sum of expectations of the output gap would disappear (and $\beta = \beta^*$) if agents expected that this would indeed be the case.

Inflationary expectations may also be obtained from surveys. Basistha and Nelson (2007) use the Michigan Consumer Survey in a UC model, but their model is quite different from ours; for example the trend and cycle are (negatively) correlated. They estimate β to be 0.29.

As a final point, note that the model proposed by Domencech and Gomez (2006) in their equation (8) is

$$\pi_t = (1-\gamma)\mu_t + \gamma\pi_{t-1} + \beta x_t + \varepsilon_t, \quad t = 1, ..., T, \tag{16}$$

where the notation has been adapted so as to be similar to that used here and there is only one lag on π_t to simplify the discussion. Setting $\gamma = 0$ gives (10), although if (10) is generalized, as is done later, so that $\pi_t - \mu_t - \beta x_t$ is a stationary process, rather than white noise, the two formulations are no longer nested; the reasons for preferring an additive formulation were discussed at the end of sub-section 2.2.

4 Estimation

This section reports the estimation of the model in (10), firstly as a single equation, with the output gap obtained by extracting a cycle from a univariate model of GDP, and secondly as a bivariate system in which the output gap and the inflation gap are modeled jointly. Models for fitted for two periods, one starting in 1947 and the other starting in 1986. After the mid 80s the rate of inflation is much lower and marks the beginning of what Sargent has called the 'Great Moderation.' The (edited) results for one of the bivariate models are shown in appendix B; others are available on request.

4.1 Single equation estimation

The model in (10) can be estimated easily using the STAMP package of Koopman et al (2007). Using data from 1947 suggests a model with lags of the output gap at one and four quarters with no contemporaneous effect. Estimating from 1952(1) makes very little difference. Figure 3 shows the components in a model of the form (10) extended to include a stochastic cycle and a seasonal, γ_t, as well as lags of x_t, that is

$$\pi_t = \mu_t + \psi_t + \gamma_t + \beta_1 x_{t-1} + \beta_4 x_{t-4} + \varepsilon_t, \qquad t = 1, ..., T \tag{17}$$

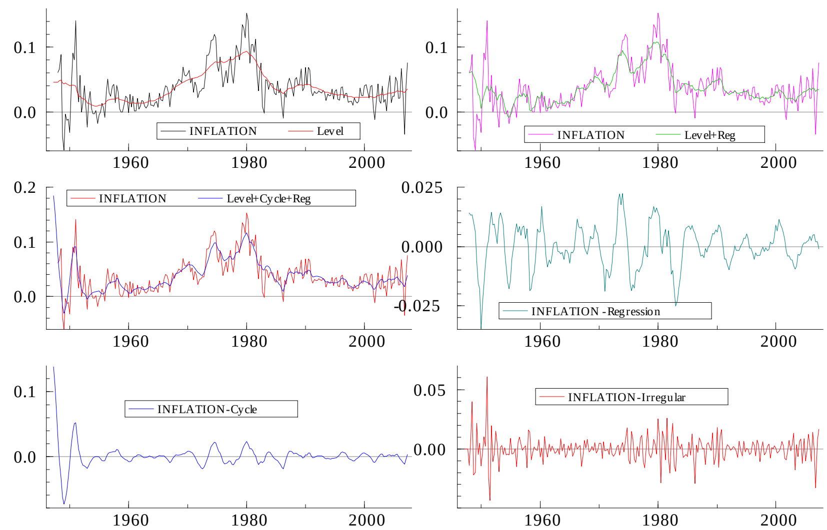

Figure 3: Components (excluding seasonal) in model relating inflation to (lagged) output gap

Including the cycle gives a slightly smaller equation standard error. More importantly perhaps, core inflation is much less erratic. The estimates of β_1 and β_4 are 0.50 (3.93) and 0.19 (1.51), with $t-$statistics' shown in parentheses. The diagnostics are not entirely satisfactory, but given the erratic movements in the 70s and the subsequent sharp fall in the early 80s this is not surprising. However, if we start in 1986(1), the diagnostics are much better, even without including a cycle. There is no evidence for lags beyond one and the contemporaneous and lagged one estimates of output gap coefficients, β_0 and β_1, are 0.11 (.34) and 0.39 (1.19). In fact a contemporaneous output gap provides a good fit, the estimate of β being 0.49 with a t-statistic of 3.56. Using the output gap series estimated from 1947 gives an estimate of 0.42.

Multi-step forecasts from the end of 1997 are shown in figure 4. The movements, which are conditional on the output gap, are not big but the higher inflation around 2000 is picked up. The volatility of the series in recent years has made accurate forecasting of any one quarter difficult, a

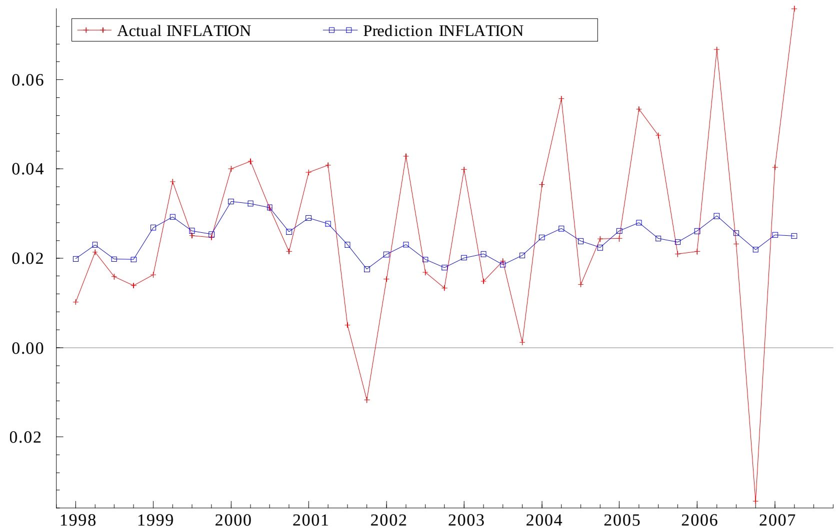

Figure 4: Multi-step predictions made from the end of 1997, conditional on the output gap estimated from the full sample

point made by Stock and Watson (2007).

If filtered estimates of the output gap are used in the equation estimated from 1986, the diagnostics are still satisfactory, but the coefficients on the output gap are small and insignificant. Using the predictive (one-step ahead) filter, obtained starting in 1947, the coefficient on the current output gap is 0.16(0.38). With the contemporaneous filter it is 0.09(0.26).

Autoregressive distributed lag models, (13), are often used to make forecasts based on the Phillips curve; see, for example, Orphanides and van Norden, (2002). However, the estimates are erratic and difficult to interpret. For a data set beginning in 1986, the results of with four lags are as shown in table 1. There is no clear interpretation to the pattern. Using the 'Autometrics' option in the PC Give module[3] in Oxmetrics only gives lags at one and three only and the negative coefficient in lag 3 of the output gap is puzzling, to say the least. Note that the seasonally adjusted CPI series was used to create the inflation series since using unadjusted data with fixed seasonal dummies would have resulted in the changing seasonal pattern be-

[3]Doornik and Hendry (2007)

coming incorporated into the lag structure. The estimates from the full data also show no clear pattern.

	Lag	Coefficient	t-statistic	Autometrics
Dependent variable	1	.29	2.41	.25 (2.42)
	2	-.10	-.86	-
	3	.40	3.22	.34 (3.09)
	4	-.08	-.62	-
Output gap	0	-.10	-.29	-
	1	.49	.97	.57 (2.70)
	2	.38	-75	-
	3	-.71	-1.40	-.47 (-2.27)
	4	.06	.15	-

Table 1 Estimates of coefficients in an unrestricted autoregressive distributed lag model for inflation.

4.2 Bivariate model

Rather than first estimating the output gap from a univariate model for GDP, inflation and GDP may be modeled jointly as

$$\left[\begin{array}{c} \pi_t \\ y_t \end{array}\right] = \left[\begin{array}{c} \mu_t^{\pi} \\ \mu_t^{y} \end{array}\right] + \left[\begin{array}{c} \psi_t^{\pi} \\ \psi_t^{y} \end{array}\right] + \left[\begin{array}{c} \varepsilon_t^{\pi} \\ \varepsilon_t^{y} \end{array}\right] \tag{18}$$

where μ_t^{π} is a random walk, as in (5), and μ_t^{y} is an integrated random walk, as in (2). These two stochastic trends are independent of each other. The irregular disturbances may be correlated and are assumed to have a covariance matrix, $\mathbf{\Sigma}_{\varepsilon}$. A seasonal component can be added to the model and in the estimates reported seasonal effects were included in the equation for inflation. The stochastic cycles are modeled as 'similar cycles', as in Harvey, Trimbur and van Dijk (2007), so that if $\boldsymbol{\psi}_t = (\psi_t^{\pi}, \psi_t^{y})'$, then

$$\left[\begin{array}{c} \boldsymbol{\psi}_t \\ \\ \boldsymbol{\psi}_t^{*} \end{array}\right] = \left[\rho \left(\begin{array}{cc} \cos\lambda_c & \sin\lambda_c \\ \\ -\sin\lambda_c & \cos\lambda_c \end{array}\right) \otimes \mathbf{I}_2\right] \left[\begin{array}{c} \boldsymbol{\psi}_{t-1} \\ \\ \boldsymbol{\psi}_{t-1}^{*} \end{array}\right] + \left[\begin{array}{c} \boldsymbol{\kappa}_t \\ \\ \boldsymbol{\kappa}_t^{*} \end{array}\right], \quad t = 1, ..., T, \tag{19}$$

where $\boldsymbol{\kappa}_t$ and $\boldsymbol{\kappa}_t^{*}$ are 2×1 vectors of the disturbances such that $E(\boldsymbol{\kappa}_t \boldsymbol{\kappa}_t') = E(\boldsymbol{\kappa}_t^{*} \boldsymbol{\kappa}_t^{*'}) = \mathbf{\Sigma}_{\kappa}$, where $\mathbf{\Sigma}_{\kappa}$ is an 2×2 covariance matrix, and $E(\boldsymbol{\kappa}_t \boldsymbol{\kappa}_t^{*'}) = \mathbf{0}$. Because the damping factor and the frequency, ρ and λ_c, are the same in all series, the cycles in the different series have the same autocorrelation functions. As in the univariate models, the trend, cycle, seasonal and irregular components are assumed to be mutually independent.

A simple transformation of the bivariate similar cycle model allows the cycle in inflation to be broken down into two independent parts, one of which depends on the GDP cycle, that is $\psi_t^{\pi} = \beta\psi_t^{y} + \psi_t^{\pi\dagger}$, where $\beta = Cov(\psi_t^{\pi}, \psi_t^{y})/Var(\psi_t^{y}) = Cov(\kappa_t^{\pi}, \kappa_t^{y})/Var(\kappa_t^{y})$ and $\psi_t^{\pi\dagger}$ is a cyclical component specific to inflation. Substituting in the inflation equation in (18) gives

$$\pi_t = \mu_t^{\pi} + \beta\psi_t^{y} + \psi_t^{\pi\dagger} + \varepsilon_t^{\pi}$$

If the cycle disturbances κ_t^{π} and κ_t^{y} are perfectly correlated, the above expression corresponds to (10) if ψ_t^{y} is set to x_t. However, (10) could be extended to include a stochastic cycle, as was done in (17).

The model estimated over the full period works quite well - but the case for a contemporaneous relationship is undermined by the lag structure identified from figure 2. With data from 1986(1), the diagnostics are much better, although there is still some residual serial correlation in inflation; see appendix. But more to the point, a contemporaneous relationship is reasonable. The model was also fitted with GDP lagged one period, but the fit was found to be very similar.

The almost perfect correlation, 0.998, means that the implied equation for π_t is effectively as in (10). The correlation matrix of the cycle gives an estimate of β equal to 0.52. The two left-hand panels in figure 5 show the effect of the output gap on inflation.

5 Conclusion

The simple unobserved components Phillips curve is parsimonious and provides a good fit to the data. It embodies core inflation and has dynamic properties consistent with the data. The model can be extended to include forward-looking behaviour but, as in all such model, identification is difficult.

An important part of the unobserved components statistical methodology is the use of graphical output for assessing whether extracted components have a meaningful interpretation. In this case a preliminary analysis based on univariate time series models gives a clear indication of a relationship that has changed over time and a warning that the uncritical use of lagged dependent variables may be problematic.

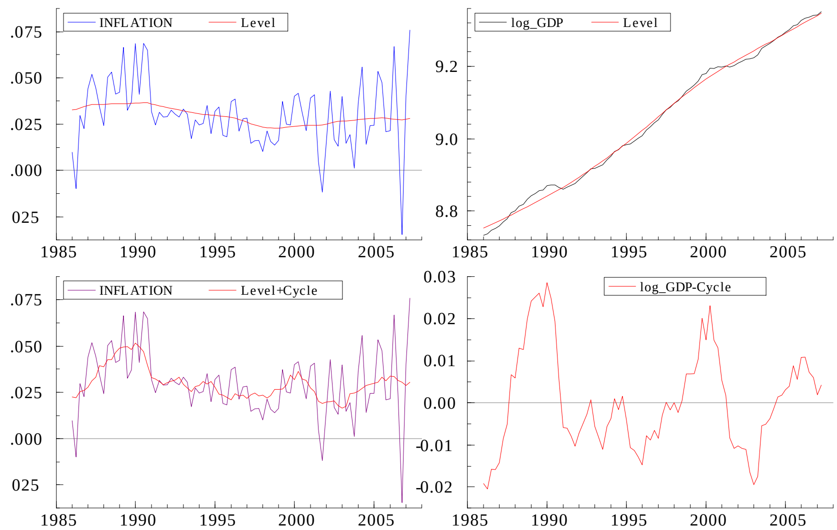

Figure 5: Smoothed components from a bivariate model for GDP and inflation

A bivariate model allows the output and inflation gaps to be estimated simultaneously, and it could be argued that this makes the best use of the available data. If the model were to be further extended, common trends - and hence co-integration - could easily be imposed on variables such as GDP and investment.

The models seem fairly stable when estimated using quarterly data from the mid-eighties and are robust to minor changes in the specification. Single equation and bivariate models tell a similar story: an output gap two per cent above trend is associated with an annual inflation rate that is one percent above core inflation. There is some evidence for a lag of one quarter, but this may not be stable. Core inflation has been relatively stable, not moving far from 3%.

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Appendix A - Expectations in the UC model

Shifting (14) forward one time period and taking expectations at time t gives

$$E_t(\pi_{t+1}) = (1-\gamma)E_t(\mu^*_{t+1}) + \gamma E_t(\pi_{t+2}) + \beta^* E_t(x_{t+1}) \tag{20}$$

as $E_t(\pi_{t+2}) = E_t E_{t+1}(\pi_{t+2})$ by the law of iterated expectations. Substituting in (14) yields

$$\pi_t = (1-\gamma)\mu^*_t + \frac{(1-\gamma)\gamma}{1-\gamma L^{-1}} E_t(\mu^*_{t+1}) + \frac{\gamma\beta^*}{1-\gamma L^{-1}} E_t(x_{t+1}) + \beta^* x_t + \varepsilon^*_t,$$

Since μ^*_t is a random walk, $E_t(\mu^*_{t+j}) = E_t(\mu^*_t)$, for $j = 1, 2, ..$, and so

$$\pi_t = (1-\gamma)\mu^*_t + \gamma E_t(\mu^*_t) + \gamma\beta^* \sum_{j=0}^{\infty} \gamma^j E_t(x_{t+1+j}) + \beta^* x_t + \varepsilon^*_t \tag{21}$$

Now consider the equation

$$\pi_t = a_t \mu^*_t + b_t E_{t-1}(\mu^*_t) + c_t z_t + d_t \varepsilon^*_t \tag{22}$$

where a_t, b_t, c_t and d_t are non-stochastic and $z_t = \gamma\beta^* \Sigma \gamma^j E_t(x_{t+1+j}) + \beta^* x_t$. The Kalman filter can be applied to this equation since $E_{t-1}(\mu^*_t)$ is known at time $t-1$ (and we continue to assume that x_t and its future expectations are known). The updating equation linking $E_t(\mu^*_t)$ to $E_{t-1}(\mu^*_t)$ is

$$E_t(\mu^*_t) = E_{t-1}(\mu^*_t) + k_t(\pi_t - E_{t-1}(\pi_t))$$

where k_t is the Kalman gain and $E_{t-1}(\pi_t) = (a_t + b_t)E_{t-1}(\mu^*_t) + c_t z_t$. Substituting in (21) and re-arranging gives

$$\pi_t = \frac{1-\gamma}{1-\gamma k_t}\mu^*_t + \frac{\gamma(1-(a_t+b_t)k_t)}{1-\gamma k_t} E_{t-1}(\mu^*_t) + \frac{(1-\gamma k_t c_t)z_t}{1-\gamma k_t} + \frac{1}{1-\gamma k_t}\varepsilon^*_t$$

Comparing with (22), it can be seen that $a_t = (1-\gamma)/(1-\gamma k_t)$, $b_t = (\gamma - \gamma k_t)/(1-\gamma k_t)$, $c_t = 1$ and $d_t = 1/(1-\gamma k_t)$. Note that $a_t + b_t = 1$ and so

$$\pi_t = \frac{1-\gamma}{1-\gamma k_t}\mu^*_t + \frac{\gamma(1-k_t)}{1-\gamma k_t} E_{t-1}(\mu^*_t) + z_t + \frac{1}{1-\gamma k_t}\varepsilon^*_t \tag{23}$$

In the steady-state, $k_t = \lambda$. An equivalent (reduced form) model can be obtained by taking expectations in equation (23) at time $t-1$ to give

$$\pi_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j (\pi_{t-1-j} - z_{t-1-j}) + z_t + \nu_t, \tag{24}$$

where ν_t is the innovation, $\pi_t - E_{t-1}(\pi_t)$ and λ depends on the signal-noise ratio for the unobserved components, which is $(1-\gamma)^2\sigma^2_{\eta*}/\sigma^2_{\varepsilon*}$. The same equation can be obtained from a model with measurement equation

$$\pi_t = \mu_t^{\dagger} + z_t + \varepsilon_t^{\dagger} \tag{25}$$

where the signal-noise ratio is $(1-\gamma)^2\sigma^2_{\eta*}/\sigma^2_{\varepsilon*}$. The MSEs will be the same if both variances are divided by $1-\gamma\lambda$.

Writing (24) may be written as in as (15).

Appendix B Output from STAMP 8 for bivariate model

A full explanation of the output can be found in Koopman et al (2007).

```
Estimation done by Maximum Likelihood (exact score)
 The databased used is US2007.in7
 The selection sample is: 1986(1) - 2007(2)
 The dependent vector Y contains variables:
INFLATION      log_GDP
 The model is:  Y = Trend + Seasonal + Irregular + Cycle

Log-Likelihood is 786.57 (-2 LogL = -1573.14).
Prediction error variance/correlation matrix is
            INFLATION     log_GDP
INFLATION     0.00024     0.07683
log_GDP       0.00001     0.00002

Summary statistics
                INFLATION     log_GDP
 T                 86.000      86.000
 d                 4.0000      2.0000
 std.error       0.015398   0.0046256
 Normality         10.381    0.019840
 H(28)             1.2380     0.82929
 DW                1.6741      2.1718
 r(1)             0.12075   -0.095475
 q                 7.0000      6.0000
 r(q)             0.14403   -0.021789
```

```
 Q(q,q-d)           8.7686      6.4688
 Rs^2              0.36595     0.11430

 Variances of disturbances in Eq INFLATION:
                   Value  (q-ratio)
Level       2.79672e-006  (0.01765)
Seasonal    1.70226e-006  (0.01074)
Cycle       4.26745e-006  (0.02693)
Irregular    0.000158439  (  1.000)

 Variances of disturbances in Eq log_GDP:
                   Value  (q-ratio)
Slope       2.90808e-007  ( 0.5250)
Cycle       5.53937e-007  (  1.000)
Irregular       0.000000  ( 0.0000)


Cycle variance/correlation matrix:

            INFLATION     log_GDP
INFLATION  4.267e-006      0.9984
log_GDP    8.260e-006  1.604e-005

Cycle other parameters:

Period              30.77785
Period in years      7.69446
Damping factor       0.95330
```