Sacrifice Ratio or Welfare Gain Ratio? Disinflation in a DGSE monetary model

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Abstract

The aim of this paper is to see how successful the current workhorse DSGE model of business cycle fluctuations can be in replicating the stylized facts above, *without resorting to any kind of irrationality or imperfect credibility/information.* We will show that: (i) a credible disinflation cause a prolonged slump; (ii) the SR resulting from the model simulations are well within the range of the estimated SRs in the literature; (iii) the model exhibits the property that SR decreases with average inflation; (iv) the second kind of dynamic adjustment path is perfectly coherent with credible disinflation in the CEE model of the business cycle. Finally, we perform a rigorous welfare evaluation of the costs of a disinflation, constructing a welfare based SR.

Contents

1 Introduction

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Christiano, Eichenbaum, and Evans (2005) (CEE)

2 Empirical evidence on disinflations

This section briefly reviews the empirical evidence to illustrate the basic facts characterizing a disinflation. Most of the empirical literature on disinflations focused on the so-called *Sacrifice Ratio* (SR, henceforth) defined as the ratio of the total cumulative percentage output loss to the size of the disinflation. Hence, for a given disinflation episode the SR measures the output costs per unit of decrease in trend inflation.

Broadly speaking, the empirical literature employed three alternative approaches to estimate the SR. The first one is based on Phillips curve estimates. Using this methodology, Gordon and King (1982) obtained a wide range of estimates ranging from

0 to 8; more recently, however, Andersen and Wascher (1999) provided a comprehensive analysis for 19 industrialized countries, showing that SR estimates are subject to a great deal of variation across time periods and model specification. Moreover, Andersen and Wascher (1999) reported that the average SR has increased from 1.5 to about 2.5 as the average rate of inflation decreased from the '80s to the '90s, because lower average inflation rates cause a flattening of the aggregate supply curve. A result, which is similar in spirit, is contained in J.Filardo (1998), that highlights the importance of non-linearities in the estimation of SRs, as the estimates of the slope of the Phillips curve are quite different between periods when the economy is booming and periods when the economy is weak. Finally, Cuñado and Gracia (2003) estimated the SR for EMU countries to range from 0.55 to 1.96. As Andersen and Wascher (1999), also Cuñado and Gracia (2003) found that the lower the rate of inflation, the larger the sacrifice ratio, due to non-linearities in the Phillips Curve.

A second type of approach to estimate the SR is based, instead, on the analysis of the single disinflation episodes. This strategy was first popularized by Ball (1994), that applied it to 65 episodes in 19 moderate-inflation OECD countries from 1960 to 1991. The estimates of Ball (1994) vary from 1.8 to 3.3. Analyzing the Volcker disinflation in the years 1982-1985 in US, Mankiw (1999) arrived at an estimate of 2.8, higher than the one of 1.8 by Ball (1994), that in turn is anyway very close to the more recent estimate of 1.7 by Erceg and Levin (2003). Ball (1994) also showed that the SR is negatively related to the speed of disinflation. Zhang (2005) generalized Ball (1994) approach showing that the estimates of the SR are larger when long-lived effects on output are taken into account. Moreover, Zhang (2005) presented evidence for a negative relationship between SR and the level of inflation at the beginning of the disinflation, and between SR and the speed of the disinflation. Quite interestingly, the significance of these two relationships disappears when the regression features both the initial inflation level and the speed of the disinflation. This results suggests that the data are not able to distinguish between the two competing effects, because of collinearity.

This second approach has been criticized by Andersen and Wascher (1999) and Cecchetti and Rich (2001), that advocates the use of more structural models. Andersen and

Wascher (1999) used structural wage and price equations in one of their model specifications. Cecchetti and Rich (2001) employed a structural VAR methodology on quarterly US data for the period 1959-1997, using three different identification models and estimating the SR over a one-to-five-year horizon. The estimates varying considerably from 1 to nearly 10 and are quite imprecise.[1] More recently, Durand et al. (2007) performed a study along the same lines for twelwe EMU countries, finding values of SR ranging from 0.23 to 0.75. Interestingly, despite employing a structural VAR methodology rather than one based on Phillips Curve estimates, also Durand et al. (2007) found evidence in favour of a negative realtionship between the average level of inflation and the SR. Finally, Collard, Fève, and Matheron (2006) and Fève, Matheron, and Sahuc (2007) used SVAR analysis, and Fève, Matheron, and Sahuc (2007) provided an estimate of SR for the Eurozone of 4.26.

This third approach based on structural estimation and SVAR methodology provides also a description of the dynamic effects of a disinflation. The impulse responses in Cecchetti and Rich (2001), Collard et al. (2006) and Fève et al. (2007) undoubtedly show that after a disinflation output declines and eventually turns back, while inflation decreases permanently. There are, however, differences in the size and the timing of the effects of a disinflation across model specifications, identification assumptions and data sets. In particular, two different cases seem to emerge. In the first one, inflation is quite sticky, and it actually increases on impact, before declining sluggishly afterwards to its permanently lower value. In this case output enters a deep and very protracted (more than 5 years) recession. This kind of impulse responses is identified by Cecchetti and Rich (2001) (in one of their model specification) and by Collard et al. (2006), both for US data. The second case, instead, exhibits a quick fall in inflation, that declines abruptly on impact, then surges mildly (since the adjustment path is oscillatory) and eventually converges to its lower new long-run value. The response of output is similar to the previous case, but the recession is smaller in size and less prolonged. Also for output

[1] A striking results that conveys an idea of the large confidence bands is that a 90% confidence interval foir teh sacrifice ratio includes 0 for all the three models estimated by Cecchetti and Rich (2001).

the adjustment exhibits small damped oscillations. This kind of impulse responses is shown by Cecchetti and Rich (2001) in their benchmark model specification for US data, and by Fève et al. (2007) for Eurozone data. It is important to note that in all cases the paths of consumption and hours worked qualitatively follow the one of output (see Collard et al. (2006) and Fève et al. (2007)).[2] Finally, it is difficult to assess superneutrality from this literature, that is, the long-run effects on output of a permanent reduction in inflation. As often in the literature, Cecchetti and Rich (2001) employs Blanchard and Quah (1989) typical identifying restriction of no long-run effects of aggregate demand shock on output. However, when this restriction is not imposed, it is not granted that output goes back exactly to its initial level (see Collard et al. (2006) and Fève et al. (2007)).

The two cases of disinflation dynamics described above can be easily reconciled by appealing to imperfect credibility (see Erceg and Levin (2003)). Under imperfect credibility expected inflation adjust sluggishly and therefore, given some version of the New Keynesian Phillips Curve, also inflation is persistent and the disinflation effects are more long-lasting. Recall that SVAR analysis necessary implies taking an average across disinflation episodes. Hence, it simply may be that different data sets hide different degrees of credibility, and different identification schemes weights them differently, generating biased results through one of the two possibilities.

Summing up, it seems fair to say that there is a consensus, corroborated by robust empirical analysis employing different estimation methodology on the following facts:

(i) a disinflation cause a loss in output;

(ii) the estimate of this loss, call SR, varies across countries an time perios, but it seems that a palusible range can be between between 0.23 and 4.26, with most of the estimates lying between 1 and 3;

(iii) the SR decreases with average inflation;

(iv) the dynamic adjustment path of inflation can be inertial or quick.

The aim of this paper is to see how successful the current workhorse DSGE model

[2]Both Collard et al. (2006) and Fève et al. (2007) performed a comprehensive robustness analysis to establish the robustness of these impulse response paths.

of business cycle fluctuations can be in replicating the stylized facts above, *without resorting to any kind of irrationality or imperfect credibility/information.* We will see that: (i) a credible disinflation cause a prolonged slump; (ii) the SR resulting from the model simulations are well within the range of the estimated SRs in the literature; (iii) the model exhibits the property that SR decreases with average inflation; (iv) the second kind of dynamic adjustment path is perfectly coherent with credible disinflation in the CEE model of the business cycle. Finally, we perform a rigorous welfare evaluation of the costs of a disinflation, constructing a welfare based SR.

3 The CEE Model

The basic setup for our experiment is a medium-scale macroeconomic model, obtained by augmenting the standard New Keynesian model with nominal and real frictions as in CEE. Since the model is exactly the one described in Schmitt-Grohé and Uribe (2004) (SGU henceforth), p. 4-23, and it is now a standard workhorse for monetary DSGE models of the business cycle[3], we will present it briefly, while the interested reader can refer to SGU for the details. Its main features are: (i) *Households*: habit persistence in consumption, money in the utility function, each household comprises all the type of labors and owns capital stock, sticky wages a la Calvo; (ii) *Firms*: Cash-in-advance constraint on wage payments, monopolistic competition, price stickiness a la Calvo, standard Cobb-Douglas production function plus a fixed cost to guarantee zero profit in equilibrium, variable capacity utilization, adjustment costs in investment; (iii) Government expenditure is financed through lump-sum taxes and seigniorage.

Household

There is a continuum of infinitely-lived households whose expected intertemporal utility function is given by

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u \left(c_t - b c_{t-1}; h_t^s; m_t^h \right) \right\}. \tag{1}$$

[3] Various versions of the analogous basic model are provided also in Schmitt-Grohé and Uribe (2005, 2007). Other empirical papers use analogous set-up, such as Smets and Wouters (2003).

where E_0 defines the mathematical expectation operator conditional on the information set available at time 0, β is the subjective discount factor, function $u\left(c_t - bc_{t-1}; h_t^s; m_t^h\right)$ is well-behaved and increasing in consumption c_t and money holdings m_t^h, while decreasing in hours worked h_t^s. Preferences display habit in consumption levels, measured by the parameter b.

There is a continuum of final goods indexed by $i \in [0, 1]$, that are aggregated in the usual CES consumption bundle c_t

$$c_t = \left[\int_0^1 c_{it}^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where the parameter η indicates the elasticity of substitution between different varieties of goods. The standard household problem defines the optimal demand of good i, given by $c_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} c_t$, where P_t is the general price index given by $P_t = \left[\int_0^1 P_{it}^{1-\eta} di\right]^{\frac{1}{1-\eta}}$.

There is a continuum of labour services h_{jt}, $j \in [0, 1]$, that are combined according to the following technology

$$h_t^d = \left[\int_0^1 h_{jt}^{\frac{\tilde{\eta}-1}{\tilde{\eta}}} dj\right]^{\frac{\tilde{\eta}}{\tilde{\eta}-1}}, \tag{3}$$

where $\tilde{\eta}$ is the elasticity of substitutions of labour types. The standard cost minimization problem for the firms yield the labour-specific demand function given by $h_{jt} = \left(\frac{W_{jt}}{W_t}\right)^{-\tilde{\eta}} h_t^d$, where W_{jt} is the wage paid to labor type j and W_t is a wage index defined as $W_t = \left[\int_0^1 W_{it}^{1-\tilde{\eta}} di\right]^{\frac{1}{1-\tilde{\eta}}}$. The total labor supply is found by integrating labour-specific demand functions, to obtain h_t^s

$$h_t^s \equiv \int_0^1 h_{jt} dj = h_t^d \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj. \tag{4}$$

Agents owns physical capital k_t that depreciates at rate δ. The capital accumulation equation is

$$k_{t+1} = (1-\delta) k_t + i_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right], \tag{5}$$

where the function S introduce the adjustment cost on investment and satisfies the properties that $S(1) = S'(1) = 0$, $S''(1) > 0$. The model features also variable capacity utilization of physical capital, denoted by u_t,. The cost of capital then depends on the degree of utilization and it is given by $a(u_t)$. Agents rent capital to firms at a real interest rate r_t^k and decide also over the utilization rate. There are complete markets for state contingent assets, such that all agents choose the same level of consumption.

Household first order conditions are hence given by

$$u_{c_t}\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) + u_{c_t}\left(c_{t+1} - bc_t; h_{t+1}^s; m_{t+1}^h\right) = \lambda_t \tag{6}$$

$$u_{h_t}\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) = -\lambda_t \frac{w_t}{\tilde{\mu}_t} \tag{7}$$

$$q_t = \beta \frac{\lambda_{t+1}}{\lambda_t}\left[q_{t+1}(1-\delta) + r_{t+1}^k u_{t+1} - a(u_{t+1})\right] \tag{8}$$

$$q_t \lambda_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - \left[S_i\left(\frac{i_t}{i_{t-1}}\right)\right] i_t\right] - \beta q_{t+1}\lambda_{t+1} S_i\left(\frac{i_{t+1}}{i_t}\right) i_{t+1} = \lambda_t \tag{9}$$

$$a_{u_t}(u_t) = r_t^k \tag{10}$$

$$u_{m_t^h}\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) + \beta \frac{\lambda_{t+1}}{\pi_{t+1}} = \lambda_t. \tag{11}$$

Wages are sticky a la Calvo, and $1-\tilde{\alpha}$ is the probability of being able to reset wages next period. If wages can not be re-optimized, the CEE model assumes that wage are anyway updated according to past inflation, such that: $w_{j,t+1} = w_{j,t}\pi_t^{\tilde{\chi}}$ where $\tilde{\chi}$ is the degree of indexation to past inflation. Define $\tilde{w}_t$ as the optimal wage set every period t. The union chooses the optimal wage maximizing its the utility function given by equation (2), subject to demand of labour in the specific market $h_{jt} = \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} h_t^d$ and the probability of not being able to re-optimize in future periods. The resulting first order condition is

$$E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \lambda_{t+s}\left(\frac{\tilde{w}_t}{w_{t+s}}\right)^{-\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)^{\tilde{\eta}} \left[\frac{\tilde{\eta}-1}{\tilde{\eta}} \frac{\tilde{w}_t}{\prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)} - \frac{w_{t+s}}{\tilde{\mu}_{t+s}}\right] = 0. \tag{12}$$

All the reset optimal wages are identical in all labour markets.

Firms

Each good is produced by a firm which monopolistically supply its own variety using a production technology of the form

$$z_t F(k_{it}, h_{it}) - \psi,$$

where z_t is an aggregate technology factor common across firms, and ψ represents a fixed cost of production. The production function $F(k_{it}, h_{it})$ is well-behaved and it's the same across firms. Final goods can be used for consumption, investment, public expenditure and to pay cost of capital utilization. Each firm faces the following demand function

$$y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} y_t, \tag{13}$$

where

$$y_t = c_t + i_t + g_t + a(u_t) k_t. \tag{14}$$

Firms rent capital from the households on a competitve market, and must pay a fraction ν of wages at the beginning of the period by cash. Therefore their money demand function is

$$m_{it}^f = \nu w_t h_{it} \tag{15}$$

The firms' problem is then to maximize the expected value of future profits, under their demand function (13) and the cash-in-advance constraint (15). The first order conditions with respect to capital and labour services are

$$mc_{it} z_t F_{k_{it}}(k_{it}, h_{it}) = r_t^k \tag{16}$$

$$mc_{it} z_t F_{h_{it}}(k_{it}, h_{it}) = w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right]. \tag{17}$$

Since F is homogeneous of degree one, equation (16) and equation (17) imply that all firms have the same marginal costs and aggregation across firms is straightforward.

Prices are sticky a la Calvo. Every period each firm can choose a new price of its own good with a probability $1 - \alpha$. As for wages, also the prices that can not be resetted optimally, are automatically updated according to past inflation, such that: $P_{it} = P_{it-1}\pi_{t-1}^{\chi}$, where χ is the degree of price indexation. The first order condition for the optimal price is

$$E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \alpha^s \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} y_{t+s} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi}}\right)^{\eta} \left[\frac{\eta - 1}{\eta} \frac{\tilde{P}_t}{P_t} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right) - mc_{i,t+s}\right] = 0. \tag{18}$$

Again, all the reset optimal prices are identical for all goods.

The Government

Government expenditure is financed through lump-sum taxes and seigniorage

$$g_t = \tau_t + m_t - \frac{m_{t-1}}{\pi_t}. \tag{19}$$

where m_t denotes real money balances, and $\pi_t \equiv P_t/P_{t-1}$ is the (gross) inflation rate at time t. Government minimizes the costs of acquiring the composite good, hence given public expenditure, government's absorption of a single type of good is $g_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} g_t$.

To close the model we just need to specify how monetary policy operates. In the paper, we study the effects of various disinflation experiment enegneered by monetary policy under two different assumptions. In the first case, monetary policy uses as instrument the monetary aggregates (e.g., as in the Volcker disinflation) and thus it controls the rate of growth of money supply. That is

$$M_t = (1 + \Pi_t^*) M_{t-1}, \tag{20}$$

where Π_t^* is the rate of growth of the money supply, which represents also the central bank's inflation target, because obviously, in steady state the rate of inflation is equal to the rate of growth of money supply.

In the second case, instead, monetary policy uses the nominal interest rate as instrument. We then can define a standard interest rate targeting rule, as

$$\frac{1 + i_t}{1 + i^*} = \left(\frac{1 + \Pi_t}{1 + \Pi^*}\right)^{\phi}, \tag{21}$$

where $0 < \beta < 1$ is the subjective discount factor, $1 + i^* = \frac{1+\Pi^*}{\beta}$ is the long-run nominal interest rate, and ϕ is a constant strictly greater than one.

In both cases, a disinflation will then be envisaged as a permanent step reduction in Π^*.

Equilibrium

The model equilibrium conditions are

$$
\begin{aligned}
\text{Money market} &: \quad m_t = m_t^h + m_t^f \\
\text{Labor market} &: \quad h_t^s = \int_0^1 h_{it}^d di \\
\text{Capital market} &: \quad \int_0^1 k_{it} di = u_t k_t \\
\text{Good } i \text{ market} &: \quad z_t F\left(k_{it}, h_{it}\right) = \left(c_t + g_t + i + a\left(u_t\right) k_t\right) \left(\frac{P_{it}}{P_t}\right)^{-\eta} \\
\begin{array}{c}\text{Aggregate}\\ \text{Goods market}\end{array} &: \quad z_t h_t^d F\left(\frac{u_t k_t}{h_t^d}, 1\right) = \left(c_t + g_t + i + a\left(u_t\right) k_t\right) \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di
\end{aligned}
$$

where $s_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta}$ is the price dispersion generated by price staggering, causing a wedge between aggregate supply and aggregate absorption. Similarly wage staggering gives rise to wage dispersion, given by $\tilde{s}_t \equiv \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj$, see (4).

Functional forms and calibration

As in SGU, we assume the following functional forms

$$
\begin{aligned}
u\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) &= \ln(c_t - bc_{t-1}) - \frac{\phi_0}{2} h_t^2 + \phi_1 \frac{\left(m_t^h\right)^{1-\sigma_m}}{1-\sigma_m} \\
F\left(u_t k_t, h_t^d\right) &= \left(u_t k_t\right)^{\theta} \left(h_t^d\right)^{1-\theta} \\
S\left(\frac{i_t}{i_{t-1}}\right) &= \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 \\
a\left(u_t\right) &= \gamma_1\left(u_t - 1\right) + \frac{\gamma_2}{2}\left(u_t - 1\right)^2 .
\end{aligned}
$$

Calibration is also as in SGU, that follows CEE's estimation results. The parameters values are listed in Table 1.

3.1 Long-run Effects

The model calibration assumes full indexation of prices and wages to previous period inflation rate. Nonetheless, money in the CEE model is non-superneutral. This is due to the cash-in-advance constraint on the firms payment of the wage bill. As explained in CEE, this assumption makes the marginal cost to depend on the nominal interest rate, and this is crucial for the model to match the empirical impulse response function.

However, the hypothesis that firms pay wages before production affects not only the short-run dynamics but also the steady state: it causes real cost of inflation. Indeed, the higher the level of trend inflation, the larger the labor costs for the firms; hence, *ceteris paribus*, the lower the wage paid to workers. In response, households reduce their labor supply, so that employment falls. Firms in turn decrease their capital stock, because labor and capital are complements in the production function. Eventually, the level of output decreases.

In the CEE model, the steady state real wage is equal to

$$w = \frac{mc\,(1-\theta)\,z\left(\frac{uK}{h^d}\right)^{\theta}}{1+\nu\frac{R-1}{R}}. \tag{22}$$

It is easy to show that the full indexation assumptions implies that the steady state real marginal cost depends only on the elasticity of substitution between goods in the usual CES consumption aggregator in the utility function (i.e., mc equals to the inverse of the mark-up, thus $mc = \frac{\eta-1}{\eta}$). In other words, the marginal cost is independent from the rate of steady state inflation. It follows that also the capital-labor ratio is independent from the rate of inflation, since so are both the marginal cost and the cost of capital. (22) shows that the steady state cost of labor, instead, is a function of the nominal interest rate, given the cash in advance constraint on firms. The wage is decreasing in R, and hence in π, since $R = \pi/\beta$ and $\frac{R_t-1}{R_t} = 1 - \beta/\pi$. However, $w\left[(1-1/R)\,\nu+1\right]$ does not depend on π, as also the relative cost of the two productive inputs : $w_t\left[(1-1/R)\,\nu+1\right]/r_t^k$ (and indeed the capital-labor ratio is constant). It is also easy to show that in a steady state equal with full indexation

$$h = \frac{b+\sqrt{b^2+4a\tilde{c}w}}{2a}; c = \frac{-b+\sqrt{b^2+4a\tilde{c}w}}{2}; y = z_t\left(\frac{k}{h^d}\right)^{\theta} h_t^d - \psi$$

where $a, b, \tilde{c}$ are terms independent from the steady state inflation rate, while only w depends on the rate of inflation as in (22). Steady state employment, consumption and output are therefore all decreasing in the rate of inflation. Note that without the cash in advance on firms, i.e., $\nu = 0$, money would be superneutral.

Even assuming full prices and wages indexation, therefore, the long-run Phillips Curve is not vertical, meaning that a disinflation would produce a long-run increase in output. Given CEE calibration these effects are rather minor: a permanent 1% reduction in inflation implies roughly a 0,1% increase in steady state output.[4]

The fact that the rate of growth of money has real effects in the long-run implies that it will also affects the long-run welfare level of the representative household. In particular, the household's utility function is the sum of three components: consumption, employment and real money balances. The steady state values of all the three arguments of the utility function decreases with long-run inflation. The effects on welfare is thus ambiguous. Given our calibration, however, positive long-run inflation would entail welfare costs, because the decrease in consumption affects utility more than the decrease in employment (while the real money balances effect is only marginal). This model thus implies long-run welfare gains from a disinflation. These effects are also quite small: a permanent reduction in inflation implies roughly a 0,07% increase in steady state consumption for each point of inflation.

It is important to stress that assuming full indexation in prices and wages we are cancelling possible real effects arising from the nominal rigidities. It is well-known that a positive steady state inflation rate increases steady state price and wage dispersion in the absence of full indexation and that price or wage dispersion causes an inefficiency loss on aggregate production, due to the non-linearity of the CES aggregators (e.g., Ascari, 2004, SGU). In other words, with partial wage and/or price indexation the real effects of long-run inflation, and thus also the effects on welfare, would be much larger.

4 Cold-turkey disinflation

We begin our analysis studying the cold-turkey (CD henceforth) disinflationary monetary policies in the non linear CEE model. A CD disinflation experiment can be described as follows. Let the economy be in a steady state characterized by a *positive* inflation target, denoted by Π_{H}^{*}, which is believed to last forever. At time $t = 0$, the central bank in charge of monetary policy unanticipatedly, credibly, permanently and

[4]Moreover, these effects are approximately linear, in the sense that the output gains are insensitive to the starting inflation level.

without further surprises lowers the inflation target from $\Pi^*_{\rm H}$ to $\Pi^*_{\rm L}$, with $\Pi^*_{\rm L} < \Pi^*_{\rm H}$.

A CD disinflationary monetary policy can be implemented in two ways: either through a money supply rule as (20) or through an interest rate rule as (21). In the former case, the central bank varies the growth rate of nominal money supply, while in the latter case the central bank disinflates by changing the inflation target in the interest rate targeting rule.

Using in turn each of the two operative strategies described above, we ask if a credible disinflation in the model can replicate the main stylized facts reported in Section 2. In particular we address the following issues:

1. What is the dynamic transitional path after a disinflation? Particularly, does the CD disinflation entails recessionary effects?
2. if so, how large are the output costs of a disinflation, i.e., the SR?
3. does the disinflation size, i.e., $\Delta_{\rm H,L} \equiv \Pi^*_{\rm H} - \Pi^*_{\rm L}$, plays any role?
4. do initial and final values of steady state inflation matter?

To answer questions 2), 3) and 4), we consider two customary measures, largely used in the empirical literature: the sacrifice ratio (henceforth, SR) and the discounted sacrifice ratio (henceforth, β-SR).[5]

The sacrifice ratio. The SR, which is the commonly used measure in the empirical literature, is defined as the cumulative output loss that the economy has to bear to achieve a sustained reduction of inflation. in our theoretical model, the SR over a time horizon of T quarters is calculated as

$$SR = -\frac{1}{\Delta_{\rm H,L}} \sum_{t=0}^{T} \left(\frac{Y_t - Y_L}{Y_L} \right), \qquad (23)$$

[5] In the next section, we will also propose a new *welfare-based indicator* to measure the cost of disinflation in rigorous welfare terms. Generally speaking, the SR and the β-SR can be viewed as measures of the short-(medium) term output costs of disinflation, while the welfare-based SR measure both the transitional effects and the long-run welafre costs of diinflation.

where $\Delta_{\mathrm{H,L}} \equiv \Pi^*_{\mathrm{H}} - \Pi^*_{\mathrm{L}}$ represents the disinflation size and Y_L is the steady state level of output in the new regime (notice that $\Pi^*_L < \Pi^*_H$ implies $Y_L > Y_H$). A value of 2 for the SR means that in order to achieve a permanent reduction of inflation rate of 2 percentage points the economy has to "sacrifice" a cumulative output loss of 4 per cent (relative to steady state).

The discounted sacrifice ratio. Following Gordon and King (1982), the discounted sacrifice ratio, discounts at the rate β the future output deviations from Y_L,

$$\beta\text{-SR} = -\frac{1}{\Delta_{\mathrm{H,L}}} \sum_{t=0}^{T} \beta^t \left(\frac{Y_t - Y_L}{Y_L} \right). \tag{24}$$

4.1 CD disinflation under money targeting rule

Figure 1 illustrates the dynamic adjustment of some key variables after a CD disinflation under the money targeting rule, as in equation (20). Each panel reports transition paths for different initial steady inflation regimes, namely, $\Pi^*_{\mathrm{H}} = \{2\%, 4\%, 6\%, 8\%\}$.[6]

Does the CD disinflation entails recessionary effects? No matter the initial steady inflation rate, a CD *fully credible* disinflation under money targeting rule comes with a notable recession. There is an initial hump-shaped output downturn followed thereafter by a small boom. Eventually, output converges to the new steady state through dying oscillations. Regarding other key variables, inflation abruptly drops first giving rise to a long-lasting deflation. Real money balances gradually build up while the nominal interest rate decreases. The ex ante real interest rate increases at the beginning and then reverts towards steady state.

To understand the adjustment paths depicted in Figure 1 and the effects of varying Π^*_{H}, it is useful to focus first on the disinflation from 2% to zero.

At time zero, when the central bank halts the nominal money supply, only a fraction of intermediate firms receives the Calvo signal to re-optimise prices. Acknowledging the new inflation regime and the forthcoming output contraction (necessary to bring down the inflation rate), optimising intermediate firms lower prices. Those firms that instead

[6] Throughout, transition paths of variables reported in figures are expressed in percentage deviations from the new steady state.

do not receive the Calvo signal mechanically raise prices as for full indexation to previous period's inflation rate, i.e., Π_{H}^{*}. As shown in figure 1, of these two opposing pricing decisions the former dominates, causing the aggregate price index to decrease relatively to previous period's. Inflation rate *overshoots* the zero target, unexpectedly boosting real money balances while pushing downwards the nominal interest rate. However, the ex-ante real interest rate rises considerably reflecting the deep expected deflation in place in the next period. Consequently, households postpone consumption and investment spending. The economy then enters a recession.

In the following period, both optimizing and non-optimising intermediate firms lower prices. The former do so because of the persistent weakening of aggregate demand, because habit in consumption and investment adjustment costs indeed make households to respond only gradually to the rise of the real interest rate. The latter lower prices as for the full indexation to the *deflation rate* occurred in the previous period. Thus, the deflation exacerbates, the ex-ante real interest rate peaks up and output reaches the bottom. In the following periods, the real interest rate slowly reverts its path and as soon as it goes below its long-run equilibrium level the economy enters a temporary boom. The dynamic pattern of inflation qualitatively mirrors that of output and at last reaches the new steady state after about 28 periods.

Disinflations starting from higher inflation levels, i.e., $\Pi_{\mathrm{H}}^{*} = \{4\%, 6\%, 8\%\}$, qualitatively exhibit the same kind of adjustment dynamics.

As a consequence, the CEE model does not exhibit inflation persistence, following a permanent shock to the rate of growth of money. Past inflation indexation is a somewhat ad hoc feature of the model introduced to match the kind of inflation persistence that impulse response functions show in VAR studies after a temporary monetary shock. It is thus important to note that, instead, full prices and wages indexation to past inflation do not imply an inertial response to inflation to a permanent monetary shock. Moreover, the data do not have a clear pattern of inflation behavior in this case (see the discussion in Section 2).

It is important to note, however, that even if inflation falls largely on impact, this does not prevent a deep and prolonged recession. In other words, the workhorse DSGE monetary model of the business cycle imply a slump after a codl-turkey disinflation without the need neither of imperfect credibility (Ball, 1995), nor of learning or adaptive expectations (Erceg and Levin, 2003), nor of other kinds of price contracting arrangements (Mankiw, 2001).

How costly is the CD disinflation under money targeting rule? Table 1 reports the theoretical values of SR and β-SR calculated for $T = 28$, i.e., the number of periods inflation takes to converge to the new inflation target. Interestingly, the values of SR and β-SR are approximately equal to 2.8, a value well within the empirical estimates of the SR in the literature (and exactly equal to the one estimated for the Volcker disinflation by Mankiw, 1999). Accordingly, to permanently lower the steady inflation rate (equivalently, the inflation target), say, from 2% to zero the economy has to sacrifice a cumulative output loss (in deviation from the new steady state) of 5.6%.

Does the size of disinflation matter? Looking at the plots in Figure 1, *neither* the overall transmission mechanism *nor* the timing of recession and boom alternation are affected by higher initial steady inflation rates. Interestingly enough, however, the initial steady inflation rate remarkably affects the amplitude of fluctuations of key variables during the transition towards $\Pi^*_{\mathrm{L}} = 0$. In general, higher initial steady inflation rates yields substantially more macroeconomic volatility. Table 2 reports output and inflation variances, computed during the transition period. Output variance substantially increases from 0.52 for $\Pi^*_{\mathrm{H}} = 2\%$ to 1.66, 3.15 and 4.92 for $\Pi^*_{\mathrm{H}} = 4\%, 6\%, 8\%$, respectively. Similarly, inflation variance moves from 1.08 for $\Pi^*_{\mathrm{H}} = 2\%$ to 8.74 for $\Pi^*_{\mathrm{H}} = 8\%$. Thus, output and inflation variances tend to linearly rise with the initial steady inflation rate. Intuitively, higher values of Π^*_{H} urge optimising intermediate firms to lower prices more, thus yielding a larger drop of inflation[7] and a more pronounced rise of the ex-ante real interest rate.

Referring back to Table 1, the disinflation size, however, only marginally affects the values of the sacrifice ratios. There is indeed a negative relation between the initial

[7]Notice that not in all case the price reduction operated by optimising firms is enough to lower the aggregate price index and lead to a deflation. For instance, this happens for $\Pi^*_H = 6\%$ and $\Pi^*_H = 8\%$. And in this cases, one can also see that after the disinflation, real money balances decrease.

steady inflation rate and the values of the sacrifice ratios, but however is quantitatively negligible. For different values of Π^*_{H}, the values of SR and β-SR lie in the range 2.9-2.7. Hence, varying the size of the disinflation produces an almost proportional rescaling of the output transition paths, that is reflected in the output and inflation volatility, but then leaves essentially unchanged the sacrifice ratios.

Finally, do initial and final values of inflation target matter? A robust feature of many empirical analysis is that the SR decreases with the starting inflation level. To study the effects, if any, of varying the initial and final values of steady inflation rate, namely, the pair $(\Pi^*_{\mathrm{H}}; \Pi^*_{\mathrm{L}})$, we experiment CD disinflations of *fixed size.* For $\Pi^*_{\mathrm{H}} = \{4\%, 6\%, 8\%\}$ we thus investigate disinflationary policies aimed at achieving $\Pi^*_{\mathrm{L}} \equiv \Pi^*_{\mathrm{H}} - 2\%$. The bottom (left) panel of Table 1 reports the sacrifice ratios. It turns out that, in line with the empirical evidence, the model delivers lower SR for higher inflation levels. For example, lowering the inflation target from 4% to 2% entails a sacrifice ratio of 2.2, that is 0.8 points lower than the value reported for the CD disinflation from 2% to zero. The values of the sacrifice ratio further decrease when the initial steady inflation rate rises: the SR, for example, drops to 1.8 and 1.6 for $\Pi^*_{\mathrm{H}} = 6\%$ and $\Pi^*_{\mathrm{H}} = 8\%$, respectively. The same is true for the β-SR. In sum, CD disinflation of fixed size under money supply targeting rule has non trivial (or asymmetric) effects on the sacrifice ratio.

4.2 CD disinflation under interest rate targeting rule

We now turn to analyse the effects of CD disinflations under the interest rate targeting rule, as in equation (21). Figure 3 illustrates the dynamic adjustment paths of some key variables for different initial steady inflation rates, namely, $\Pi^*_{\mathrm{H}} = \{2\%, 4\%, 6\%, 8\%\}$.

Is the CD disinflation recessionary? Also under the postulated contemporaneous interest rate rule CD disinflation comes with a notable recession, the intensity of which worsens as the initial steady inflation rate increases. Although the transmission mechanism is for many aspects similar to the one at work under money supply targeting rule, see Figure 1, there however a number of qualitative and quantitative differences worthy to emphasize.[8]

[8] These results are somewhat sensitive to the specification of the Taylor rule. In particular, letting

Qualitatively. Firstly, the CD disinflation under interest rate targeting rule involves an immediate rise of nominal interest rate. As one might expect, the disinflation is initiated by a monetary policy contraction. Even though the new steady state equilibrium level of nominal interest rate is lower (Fisher equation), the central bank increases the nominal interest rate to raise the real interest rate, which then leads to lower aggregate demand and lower inflation rate. As consumption and investment adjust only gradually following a hump-shaped path, the central bank already in the first period cuts the nominal interest rate. Nonetheless, the ex-ante real interest rate remains above steady state for several periods, ensuring the endurance of the recession. Secondly, under the interest rate targeting rule inflation rate converges to steady state following a gradual and largely monotonic path. There is a deflationary spell between the sixth and tenth period, but it is quantitatively negligible. This is in stark contrast to what is shown in Figure 1, in which the transition to the zero steady inflation rate occurs through a notable and long-lasting deflation.

Quantitatively. Firstly, CD disinflation under the interest rate yields, generally speaking, less macroeconomic volatility than under the money supply targeting rule. This is visually evident for output when comparing figures 1 and 3. Indeed, the bottom values of output during the initial recession under money supply targeting are roughly three times larger than under interest rate targeting. This is largely confirmed in Table 1, where output volatility under interest rate targeting not only is remarkably lower than under money supply targeting but is also less sensitive to the initial steady inflation rate. Secondly, CD disinflation under interest rate targeting is accomplished in 15 periods. This is approximately half of the time taken under money supply targeting.

Finally, Figure 3 also shows the growth rate of money supply implied by the interest rate rule. The growth rate of money supply (Π_t^*) is retrieved using the transformation $\Pi_t^* = \frac{m_t}{m_{t-1}}\pi_t$. As illustrated, the path of Π_t^* is somewhat unusual. At time zero the growth rate of nominal money supply suddenly falls and then right away rises overshooting the initial steady level. Only afterwards, the growth rate of money supply gradually

the central bank to also care about the output gap may yield deflation and overall more volatility in the adjustment of the economy.

decreases towards equilibrium through dying oscillations. Notice that the earliest behaviour of Π_t^* is due to the initial rise of the nominal interest rate, which temporarily depresses real money balances.

The important message here is that the way monetary policy implements a disinflation matters: with respect to disinflating by targeting the monetary aggregates,.following an interest rate targeting rule greatly decreases the output cost of disinflation, i.e., SR, by an order of magnitude going from one half to one third. The intution rests on the different path of the money supply implied by the two monetary policy strategies. The new steady state implies an higher value for the real money balances. Under a strict monetary tragteting strategy, this has to be accomplished by a *decrease* in the price level, implying a *deflation* that can be attainable only through a deep recession in output. Under an interest rate targeting strategy, instead, the higher demand for money can be partly satiated by an increase in the money supply in the second period. The price level, thus, need just to slow down (and not to decraese) to reach the new higher steady state level of m.

How costly is the CD disinflation under the interest rate targeting? Table 1 reports the values of sacrifice ratios calculated for $T = 15$. Not surprisingly, the costs of disinflation are significantly lower than under money supply targeting. Both the values of SR and β-SR are equal to one, that is again within the estimated range reported in Section 2.

Does the size of disinflation matter? Paralleling the case under money supply targeting, the disinflation size seems irrelevant for the values of the sacrifice ratios. Also in this case, higher initial steady inflation rates tend to re-scale the dynamic adjustment path output leaving the sacrifice ratio essentially unchanged.

Finally, do initial and final values of trend inflation matter? As in the previous section, to address this question we run CD disinflations of fixed size. The bottom right panel of Table 1 shows that the values of the sacrifice ratios decreases also in this case, but only marginally.

5 Disinflation timing

We now relax some of the assumptions underlying the mechanics of the CD disinflation. In particular, we look at two cases: gradualism and anticipations. In the former case, the central bank implements a gradual reduction of the inflation target towards the new inflation target over a certain time window. In the latter case, instead, the central bank announces today a CD disinflation forthcoming in a given future date.

5.1 CD *vs.* gradual disinflation

At time zero the central bank implements a gradual reduction of the inflation target Π^* to be completely accomplished in k periods. In particular, the central bank sets the time-varying inflation target path to fulfill the following stair function,

$$\Pi_t^* = \Pi_{t-1}^* - \frac{\Pi_{\mathrm{H}}^* - \Pi_{\mathrm{L}}^*}{k}, \text{ with } k \geqslant 1, \tag{25}$$

for $t = 0, 1, ..., k-1$ and given that $\Pi_{-1}^* = \Pi_{\mathrm{H}}^*$. Notice that k can be thought as the *disinflation speed*: the lower k, the faster the reduction of inflation target. Clearly, the CD disinflation attains for $k = 1$.

Figures 4 and 5 illustrate dynamic adjustment paths of output and inflation for different disinflation speed, i.e., $k = \{0, 4, 8, 12\}$ and different initial level of steady inflation, i.e., $\Pi_H^* = \{2\%, 4\%, 8\%\}$.

Is the gradual disinflation recessionary? Both under money supply and interest rate targeting rule, gradual disinflation leads to an initial hump-shaped recession, then followed by a small, but persistent, expansion of output. While it is true that for a given k, higher initial steady inflation rate increases output volatility, more gradual disinflationary policy schemes yields the opposite effect. Table 2 shows, in fact, that for $k = 4$, 8, 12 output variability decreases to a larger extent under the money supply targeting and to a lesser extent under interest rate targeting. In the former case, it must be said, that output variability, in absolute terms, is already much smaller. These results hinge on the fact that operating a gradual disinflation makes optimising intermediate firms to lower prices less. Consequently, inflation adjusts more gradually, the ex ante real interest rate increases more moderately and the contraction of output follows less severe.

Interestingly, the effects of gradualism has ambiguous effects on inflation variability. Under money supply targeting, more gradual disinflation distinctly dampens the variance of inflation. Under interest rate targeting, however, inflation variability is lowest for $k = 4$, taking into account also $k = 0$, and remarkably increases for $k = 8$, 12.

How costly is the gradual disinflation? Table 3 shows that gradualism unambigously reduces the output costs of a disinflation, i.e. SR, with respect a CD disinflation both under money supply and interest rate targeting. However, while under money supply targeting more gradual disinflation to zero inflation steady state monotonically reduces the sacrifice ratios, this is not the case under interest rate targeting. In the former case, Table 3 shows that moving from CD to 1-year gradual disinflation both SR and β-SR decrease by roughly 0.6, regardless of the disinflation size. A further 0.6 reduction of sacrifice ratios is gained when moving from 1-year to 2-year gradual disinflation, while it becomes 0.2 from 2-year to 3-year. Under interest rate targeting, instead, the reduction of the values of the sacrifice ratios are not monotonic in k. Referring back to Table 3, the sacrifice ratios decrease up until $k = 8$, and then start rising.

Does the size of disinflation matter? As shown in Table 3, with gradual disinflationary monetary policies the size of disinflation does matter for the sacrifice ratio. Quite interestingly, for a given k the values of the sacrifice ratios are *negatively* correlated with the size of disinflation. To say, it is less costly to gradually disinflate the economy starting off from relatively higher initial steady inflation rates. From a quantitative standpoint, these effects are more noticeable under money supply targeting rule.

Finally, do initial and final values of trend inflation matter? As before, for a given size of disinflation, disinflation costs are decreasing with the initial level of inflation whatever the monetary policy strategy for *given gradualism*, i.e., a given value of k. However, for both monetary policies and for each starting inflation level, it seems there is an optimal degree of gradualism, since SR starts increasing with gradualism after $k = 8$.

To sum up, gradualism surely reduces SR with respect to an immediate disinflation, but it seems that the relationship between SR and gradualism is not monotonic. Moreover, for a given gradualism, both the size of the disinflation and the starting inflation level reduce SR.

5.2 Unanticipated versus anticipated CD disinflation

Another common case to look at is the *anticipated CD disinflation*: in this case, the central bank announces at time $t = 0$, and it is perfectly believed, a CD disinflation plan that will take place only after k periods ahead. We consider CD disinflations implemented after 1, 2 and 3 years from the announcement period.

The adjustment paths under money targeting rule and interest rate rule are illustrated in Figures 6 and 7, respectively. Under money targeting rule, the effects of announcing the future intention to disinflate the economy has considerable effects both qualitatively and quantitatively. It holds true, and it is quite robust, that there is an initial recession, followed then by a boom. The main effect of delaying the implementation of the CD is mainly reflected in the intensity of the recession and boom. Indeed, the more delayed is the disinflation the smaller are the fluctuations of output, whatever the disinflation size. Even more remarkable, when compared to Figure 1, are the effects of the adjustment path of inflation. In this case, delaying the implementation of the disinflation greatly reduces the deflation, and for more than 2-year announcement the deflation disappears. Intermediate firms' start slowly adjusting their prices *from the moment of the announcement and before the actual implementation of the disinflation.* Hence, inflation gradually reduces, the response of the ex-ante real interest rate is more muted, then the recession is milder. Table 3, however, highlights the fact that delaying the implementation of the CD disinflation does not result in a monotonic reduction of the sacrifice ratios. As a matter of fact, although announcing in advance the disinflation considerably reduces both the SR and the β-SR compared to the unanticipated case, the benefit of delaying are relative stronger in the case of the 2-year announcement. After that both the sacrifice ratios increase. As illustrated in Table 2, these effects holds for any disinflation size and for any starting level of inflation, for a given disinflation size.

Figure 7 and 8, dsiplay the case of an interest rate rule. The effects of announcing a future CD disinflation are for several reasons more surprising. Indeed, as expected, delaying the CD disinflation by one year has more stabilizing effects compared to the unanticipated policy. The intensity of the recession and the following boom are considerably decreased. A longer pre-announcement period, however, tends to have destabilising effects. This is actually an artifact of the peculiar experiment, where the central bank continues targeting the old inflation rate during the announcement period. Then, immediately after the announcement, intermediate firms start decreasing the prices, and this leads monetary policy to decrease the nominal interest rate, because inflation is lower than the old target, that still enters the Taylor rule. This counteracts the increase in the real interest rate and tends to stimulate output, thus reducing the initial slump. The nominal and the real interest rate reaches a minimum in the period before the implementation of the disinflationary policy. Then, in the implementation period, both the real and the nominal interest rate jump upward because the inflation target suddenly drops.Then they both monotonically decrease towards the new steady state. This "artificial" sudden reversal and zig-zagging behavior of the nominal interest rate can causes a peculiar adjustment dynamics, especially if the annoucement period is long. Note that in the case of 2 years pre-announcement, the real interest rate decreases on impact after the announcement causing a boom in output. In any of the cases, however, the effects on output are quantitatively small.

Possibly it is not sensible to assume that a pre-announced disinflationary policy should follow a strict Taylor rule based on inflation targets that are already announced to be abandoned in the near future. However, an alternative assumption open the door to many possibilities and, then, would call for the normative prescription: what is the optimal anticipated disinflationary policy? This is however beyond the scope of this paper and will be the sbuject of future work.

Gradualism versus anticipation. Finally, the results in Table 2 and 3 suggest that an announced disinflation might perform better relative to a gradual disinflation, if the pre-announcement period is not too long. One-year credibly anticipated policies deliver the lower both SR and $\beta - SR$ for both monetary targeting and interest rate targeting rules. When the pre-announcement period gets longer then corresponding gradualist policies exhibit lower SR. This is not surprising, since it would be a quite strange policy the one that anticipates a change in the policy in the future, but then it does not modify its current behavior which is actually hampering the transition.

6 A welfare based measure of the cost of disinflation

As already noted in Gordon and King (1982), the output loss from disinflation does not by itself contain policy implications. A careful assessment must be made of the welfare cost of lost output *and* the welfare benefits of lower inflation. On this latter point, the recent monetary policy literature has abundantly and convincingly emphasized the reasons why achieving full price stability is desirable (see Woodford, 2003 and the references therein). One notable advantage of working with structural model is that they provide a natural metric to evaluate the overall welfare implication of disinflation: the representative household's value function. Since we are dealing with a DSGE model, we can easily calculate a welfare based measure of the cost of disinflation, rather than an empirical based one, as the sacrifice ratio. Paralleling the way SR is built, a measure of the loss in welfare caused by the implementation of the policy is given by the difference between the level of the value function in period 1, and the level of the value function if the policy was not implemented, that is, the starting steady state value. So a *microfounded sacrifice ratio* could be defined as

$$MSR = -\frac{1}{\Delta_{\mathrm{H,L}}} \left(V_1 - V_H \right) \tag{26}$$

where $V_H =$ starting steady state value function, and $V_1 =$ value function the first period after the implementation of the disinflationary policy. Note that, as in the standard sacrifice ratio definition, $MSR > 0$, if $V_1 - V_H < 0$, that is, if a disinflation brings about a welfare loss, and vice versa. It is important to note that V_1 includes both the transition dynamics and the long-run effects.

The consumption equivalent measure

A policy maker is interested in the welfare cost of implementing a disinflationary policy, but given that the utility function is not cardinal, a measure based on V is not very revealing. The difference $(V_1 - V_H)$ can, as usual, be expressed in terms of consumption equivalent units. The consumption equivalent measure is then defined as that constant fraction of consumption that households should give away in each period in the starting steady state, in order to obtain the same level of value function that households would get if the disinflationary policy is implemented. Note that this is a true measure of the costs of disinflation in terms of consumption: indeed it measures how much households have to suffer in terms of consumption loss, in order to reduce the inflation rate permanently of a certain amount.

Finally, this measure is very easy to get. The starting initial value function is

$$V_H = E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(c_t - bc_{t-1}) - \frac{\phi_0}{2} h_t^2 + \frac{m_t^{h^{1-\sigma_m}}}{1-\sigma_m} \right], \tag{27}$$

that in steady state this reduces to

$$V_H = V_{ss0} = \frac{1}{1-\beta} \left[\ln\left((1-b)\bar{c}\right) - \frac{\phi_0}{2} \bar{h}^2 + \frac{\bar{m}^{h^{1-\sigma_m}}}{1-\sigma_m} \right]. \tag{28}$$

Given the value of V_1 from our simulations, then we just need to solve for what constant fraction of steady state consumption households should give away in each period in the starting steady state to obtain the same level of value function as V_1. This ends up to find the solution for λ in the following equation

$$V_1 = \frac{1}{1-\beta} \left[\ln\left((1-b)\bar{c}(1-\lambda)\right) - \frac{\phi_0}{2} \bar{h}^2 + \frac{\bar{m}^{h^{1-\sigma_m}}}{1-\sigma_m} \right], \tag{29}$$

where λ measure exactly that constant fraction. The consumption equivalent measure is thus simply given by

$$\lambda = 1 - \exp\left[(1-\beta)(V_1 - V_H)\right]. \tag{30}$$

Finally, the proposed *welfare based sacrifice ratio* measure is[9]

$$\mathbb{W}{-}SR = \frac{\lambda}{\Delta_{\mathrm{H,L}}} \tag{31}$$

[9]Note that there is no minus in front of $\frac{1}{\Delta_{\mathrm{H,L}}}$ to mantain a positive sign for a loss. Indeed, if $V_1 - V_{ss} < 0$, that is disinflation brings about a welfare loss, then $\lambda > 0$, and vice versa.

The last columns of Table 1 reports $\mathbb{W}-SR$ in the case of CD disinflations under both a money targeting and an interest rate targeting rule. $\mathbb{W}-SR$ ***is negative: the disinflation is welfare improving.***[10] *Therefore, we should speak of welfare gain ratio, rather than of sacrifice ratio, as in the empirical literature.* The long-run gain prevails on the short run costs. We think this is an interesting result: the empirical literature focuses only on the short-run cost of a disinflation in terms of output, but it neglects (or denies) any long-run effect (gain or loss). We demonstrate that in a medium scale DSGE monetary model of the business cycle, instead, a disinflationary policy is welfare improving. The size of $\mathbb{W}-SR$, however, is small: under both polices the welfare gain is equivalent to an extra 0.06% of consumption each period.

Actually, the results are possibly even more striking, if we disentangle the short-run costs of a disinflation during the adjustment dynamics and the long-run gains due to price stability. In a standard medium scale DSGE macro model, considered as a benchmark model in the literature, a CD disinflation entails a large and prolonged recession, as basic intuition would predict. The sacrifice ratio, moreover, are in line with the empirical evidence. The short-run costs of such a painful adjustment path are, however, insignificant. The order of magnitude must be given by the difference between $\mathbb{W}-SR$ and the long-run gains, that is, roughly about 0.01% of initial consumption. Following the same reasoning above:

(i) the consumption equivalent long-run costs are equal to[11]

$$\lambda_{LR} = 1 - \exp\left[(1-\beta)(V_L - V_H)\right] \tag{32}$$

where V_H (V_L) is the value function in the high (low) inflation steady state, and the same costs or gains can be expressed per unit of diminished inflation to yield *a long-run welfare based sacrifice ratio*

[10] Note that this qualitative result does not depend on the inclusion of real money balances in the utility function. We can also calculate a similar measure without taking into account the gain in utility coming from an increase in real money balances in the new steady state. The measure would then about 2/3 of the reported one.

[11] Note that we use a coherent definition as above also for the long-run SR. Indeed, if $V_L - V_H < 0$ (that is if disinflation brings about a welfare loss) then $\lambda > 0$, and vice versa.

$$\mathbb{W}{-}SR\_LR = \frac{\lambda_{LR}}{\Delta_{\mathrm{H,L}}}; \qquad (33)$$

(ii) *a short-run welfare based sacrifice ratio* is then given by

$$\begin{aligned} \mathbb{W}{-}SR\_SR &= \frac{\lambda - \lambda_{LR}}{\Delta_{\mathrm{H,L}}} = \\ &= \frac{\exp\left[(1-\beta)(V_L - V_H)\right] - \exp\left[(1-\beta)(V_1 - V_H)\right]}{\Delta_{\mathrm{H,L}}}; \end{aligned} \qquad (34)$$

Table 4 shows the long-run gains and the short-run costs of a CD disinflation policies for different experiment under the two monetary policy rules. Obviously, the long-run gains, i.e., $\mathbb{W}{-}SR\_LR$, do not depend on the monetary policy strategy. The analysis of the short-run costs, instead, show that disinflating through an interest rate targeting rule is always less costly, for any kind of experiment considered in Table 4. The difference is anyway very tiny, relatively insensitive to the kind of experiment, and it approximately amounts to a 0.002% of consumption in each period. Regarding CD disinflation, the size of the disinflation matters only for the long-run gains, while short-run costs seems insensitive to it. Moreover, there are "decreasing returns to scale to disinflating", in the sense that , $\mathbb{W}{-}SR\_LR$, i.e., the long-run gains per percentage point of diminished inflation, decreases, almost linearly, with the size of the disinflation. Both long-run gains and short-run costs are, instead, decreasing with respect to the initial level of inflation, for a given size of disinflation. The long-run effects, however, quantitatively dominates, such that the gains from disinflating decreases with the starting level of inflation.[12]

As in Table 4, Table 5 looks at the effects of anticipated and gradualist policies on our welfare based measure of the sacrifice ratio. As for the case of output based sacrifice ratios, the benefits of a gradual disinflation are monotonic in the case of a monetary rule, while they are not monotonic in the case of an interest rate rule. Furthermore, anticipation increases the gains from disinflating only up to a point, which is two year in the case of a monetary rule and one year in the case of an interest rate rule. In

[12] Disinflation from 8% to 6% exhibits a higher (more costs) $\mathbb{W}{-}SR$ than disinflating from 6% to 4%. In a sense, starting from a high inlfation level, there are "increasing marginal returns to stepwise disinflation".

any case, these effects are really marginal. The main message from the two Tables is that a disinflation is going to be welfare improving of the order of an increase of initial consumption of 0.06-0.07% each period per point of diminished inflation, ***no matter how the disinflation is implemented (monetary policy strategy, gradualism, anticipation).***

This stands in sharp contrast with the consensus view about the effects of a credible disinflation. What is the intuition for these results? To illustrate it, let's take the case of a CD disinflation implemented through a monetary rule. Figure 9 displays the path of consumption and employment, expressed in deviation from final steady state, together with value of the utility function. The CD disinflation induces a prolonged recession that cause both consumption and employment to be below their new (and higher) steady state value for some periods. Consumption and employment, however, has opposite effects on the utility function of the representative agent. It follows, therefore, that the net effects of the recession on the utility of the representative agent is ambiguous. Indeed, the decrease in consumption dominates in the impact period, dragging the utility function down, but then already in the second period the effects of the dynamics of employment takes over, and the utility function is above its new higher long-run value. Moreover, it will stay there for all the periods of the recession. This is because the drop in employment is bigger in percentage terms, and slightly more sluggish. It follows that the positive effect of employment is quite effective in counterbalancing the negative effect of lower consumption. Overall the transition, thus, entails a short-run cost, as shown above, but of a negligible order of magnitude. Finally, also the value of the utility function without counting the real money balances term is visualized in Figure 9, so to make clear that the role of the real money balances term in the utility function in the above results is nil.

This result obviously should not be taken as face value. Indeed, in the dynamics of the "average" representative consumer are hidden very different situations. In particular, as known, a representative agent framework can not be taken into account the fact that some people may suffer a very big drop in utility during recession because they loose their jobs: such a composition effect is missing by construction. We believe, however, our results show two important points. On the one hand, they cast shadow on using these DSGE models for welfare evaluation without "inspecting the mechanism". In particular, the ranking between different monetary policy rules or the optimal policy problems are bound to be based on mechanism similar to ours. On the other hand, recall that if there are complete markets (and agents are the same ex-ante), then all the agents will have the same marginal utility from consumption. Hence, our results simply show once again that, if the economy could provide an efficient risk-sharing between the agents (either through good capital markets, or some public welfare system), then disinflation, in particular, and recession, in general, are less a problem than we normally think.

7 Conclusions

[to be done]

8 References

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9 Tables

Calibration		
β	$1.03^{-0.25}$	Time discount rate
θ	0.36	Share of capital
ψ	0.5827	Fixed cost (guarantee zero profits in steady state)
δ	0.025	Depreciation of capital
η	1	Elasticity of substitution of different varieties of goods
$\tilde{\eta}$	6	Elasticity of substitution of labour services
α	21	Probability of not setting a new price each period
$\tilde{\alpha}$	0.6	Probability of not setting a new wage each period
b	0.64	Degree of habit persistence
ϕ_0	1.1196	Preference parameter
ϕ_1	0.5393	Preference parameter
σ_m	10.62	Intertemporal elasticity of money
κ	2.48	Investment adjustment cost parameter
χ	1	Price indexation
$\tilde{\chi}$	1	Wage indexation
γ_1	0.0324	Capital utilization cost function parameter
γ_2	0.000324	Capital utilization cost function parameter
z	1	Steady state value of technology shock

Table 1. Calibration

		Money supply rule (T=28)			Interest rate rule (T=15)		
Π_H^*	Π_L^*	SR	β-SR	$\mathbb{W}$-SR	SR	β-SR	$\mathbb{W}$-SR
2%	0%	2.94	2.96	-6.40	1.06	1.04	-6.59
4%	0%	2.85	2.85	-6.29	1.05	1.03	-6.49
6%	0%	2.78	2.77	-6.20	1.04	1.02	-6.39
8%	0%	2.73	2.72	-6.12	1.03	1.01	-6.31
4%	2%	2.22	2.23	-6.26	1.03	1.02	-6.39
6%	4%	1.89	1.90	-6.12	1.01	1.00	-6.23
8%	6%	1.69	1.71	-5.99	0.99	0.98	-6.08

Table 1: Costs of cold turkey disinflation

10 Figures

			Money suppy targeting rule (T=28)							Interest rate targeting rule (T=15)						
			2→0	4→0	6→0	8→0	4→2	6→4	8→6	2→0	4→0	6→0	8→0	4→2	6→4	8→6
cold-turkey		σ^2_Y	0.52	1.66	3.15	4.92	0.29	0.21	0.17	0.05	0.22	0.48	0.84	0.05	0.05	0.04
disinflation		σ^2_Π	1.08	3.20	5.79	8.74	0.61	0.45	0.36	0.51	2.05	4.59	8.12	0.49	0.47	0.45
	k=4	σ^2_Y	0.32	0.96	1.68	2.45	0.16	0.10	0.07	0.04	0.15	0.34	0.58	0.03	0.03	0.03
	k=8	σ^2_Y	0.17	0.45	0.73	0.99	0.07	0.04	0.03	0.02	0.08	0.19	0.33	0.02	0.02	0.01
Gradualism	k=12	σ^2_Y	0.10	0.26	0.40	0.54	0.04	0.03	0.02	0.02	0.10	0.22	0.38	0.02	0.02	0.02
	k=4	σ^2_Π	0.67	1.92	3.46	5.24	0.35	0.26	0.22	0.50	2.01	4.49	7.97	0.48	0.46	0.45
	k=8	σ^2_Π	0.33	0.99	1.96	3.33	0.21	0.20	0.20	0.66	2.64	5.91	10.50	0.64	0.63	0.60
	k=12	σ^2_Π	0.20	0.83	2.06	4.04	0.22	0.27	0.30	0.98	3.89	8.79	15.60	0.95	0.96	0.89
	k=4	σ^2_Y	0.18	0.47	0.77	1.05	0.07	0.03	0.02	0.023	0.09	0.20	0.35	0.02	0.02	0.02
	k=8	σ^2_Y	0.04	0.09	0.14	0.20	0.01	0.01	0.01	0.01	0.07	0.16	0.29	0.01	0.01	0.01
Anticipation	k=12	σ^2_Y	0.02	0.09	0.19	0.34	0.02	0.02	0.02	0.06	0.24	0.52	0.90	0.05	0.05	0.04
	k=4	σ^2_Π	0.38	1.14	2.19	3.58	0.22	0.20	0.20	0.52	2.09	4.70	8.32	0.50	0.48	0.46
	k=8	σ^2_Π	0.25	1.31	3.51	7.02	0.41	0.50	0.54	1.35	5.39	11.92	20.96	1.32	1.23	1.14
	k=12	σ^2_Π	0.66	3.25	8.18	15.59	0.93	1.02	1.04	2.78	11.04	24.17	42.22	2.74	2.52	2.29

Table 2: Measures of output and inflation volatility.

			Money suppy targeting rule (T=28)							Interest rate targeting rule (T=15)						
			2→0	4→0	6→0	8→0	4→2	6→4	8→6	2→0	4→0	6→0	8→0	4→2	6→4	8→6
Gradualism	k=4	SR	2.32	2.12	1.96	1.83	1.63	1.35	1.19	0.95	0.93	0.92	0.91	0.91	0.90	0.88
	k=8	SR	1.78	1.52	1.34	1.21	1.22	1.03	0.93	0.86	0.85	0.84	0.83	0.83	0.82	0.81
	k=12	SR	1.55	1.32	1.16	1.06	1.16	1.06	1.00	1.05	1.04	1.03	1.02	1.02	1.01	1.00
	k=4	β-SR	2.33	2.13	1.96	1.84	1.65	1.36	1.20	0.93	0.92	0.91	0.89	0.91	0.89	0.87
	k=8	β-SR	1.79	1.53	1.35	1.21	1.23	1.04	0.93	0.84	0.83	0.82	0.81	0.80	0.80	0.79
	k=12	β-SR	1.55	1.31	1.16	1.05	1.16	1.05	0.99	1.01	1.00	0.99	0.99	0.98	0.98	0.97
Anticipation	k=4	SR	1.76	1.51	1.33	1.20	1.17	0.92	0.79	0.79	0.78	0.78	0.77	0.77	0.76	0.76
	k=8	SR	1.13	0.94	0.84	0.78	0.87	0.80	0.78	0.90	0.90	0.90	0.90	0.88	0.85	0.84
	k=12	SR	1.35	1.28	1.28	1.29	1.36	1.39	1.41	1.80	1.77	1.74	1.72	1.76	1.68	1.61
	k=4	β-SR	1.77	1.52	1.34	1.21	1.18	0.93	0.79	0.77	0.76	0.76	0.75	0.76	0.75	0.75
	k=8	β-SR	1.13	0.94	0.83	0.78	0.87	0.79	0.77	0.86	0.86	0.86	0.86	0.84	0.82	0.81
	k=12	β-SR	1.31	1.25	1.24	1.25	1.31	1.34	1.36	1.72	1.69	1.66	1.64	1.68	1.60	1.54

Table 3: Sacrifice ratios: gradual and anticipated disinflations

Π_H^*	Π_L^*	Long-Run Welfare Gain (10^{-2})	Monetary Rule	Short-Run Welfare Cost (10^{-2})	$\mathbb{W}$-SR (10^{-2})
2%	0%	-7.40	MS	1.00	-6.40
			IR	0.81	-6.59
4%	0%	-7.30	MS	1.00	-6.29
			IR	0.81	-6.49
6%	0%	-7.20	MS	1.00	-6.20
			IR	0.81	-6.39
8%	0%	-7.11	MS	1.00	-6.12
			IR	0.80	-6.31
4%	2%	-7.18	MS	0.91	-6.26
			IR	0.79	-6.39
6%	4%	-7.00	MS	0.87	-6.12
			IR	0.76	-6.23
8%	6%	-6.82	MS	0.83	-5.99
			IR	0.74	-6.08

Table 4: Costs of cold turkey disinflation: Short-run Welfare Cost and Long-run Welfare Gain

			Money suppy targeting rule (10^{-2}) (T=28)							Interest rate targeting rule (10^{-2}) (T=15)						
Gradualism			2→0	4→0	6→0	8→0	4→2	6→4	8→6	2→0	4→0	6→0	8→0	4→2	6→4	8→6
	k=4	W-SR	-6.42	-6.36	-6.28	-6.20	-6.32	-6.18	-6.04	-6.61	-6.50	-6.41	-6.33	-6.41	-6.24	-6.09
	k=8	W-SR	-6.51	-6.42	-6.35	-6.27	-6.36	-6.21	-6.07	-6.62	-6.51	-6.42	-6.34	-6.42	-6.25	-6.10
	k=12	W-SR	-6.53	-6.45	-6.37	-6.30	-6.37	-6.21	-6.07	-6.61	-6.50	-6.41	-6.33	-6.41	-6.24	-6.09
Anticipation	k=4	W-SR	-6.51	-6.42	-6.35	-6.27	-6.37	-6.22	-6.08	-6.63	-6.52	-6.43	-6.35	-6.43	-6.26	-6.11
	k=8	W-SR	-6.57	-6.48	-6.40	-6.32	-6.39	-6.23	-6.08	-6.62	-6.51	-6.42	-6.34	-6.42	-6.25	-6.11
	k=12	W-SR	-6.57	-6.47	-6.38	-6.30	-6.37	-6.20	-6.05	-6.57	-6.46	-6.37	-6.29	-6.37	-6.21	-6.06

Table 5: Welfare gains:gradual and anticipated disinflations

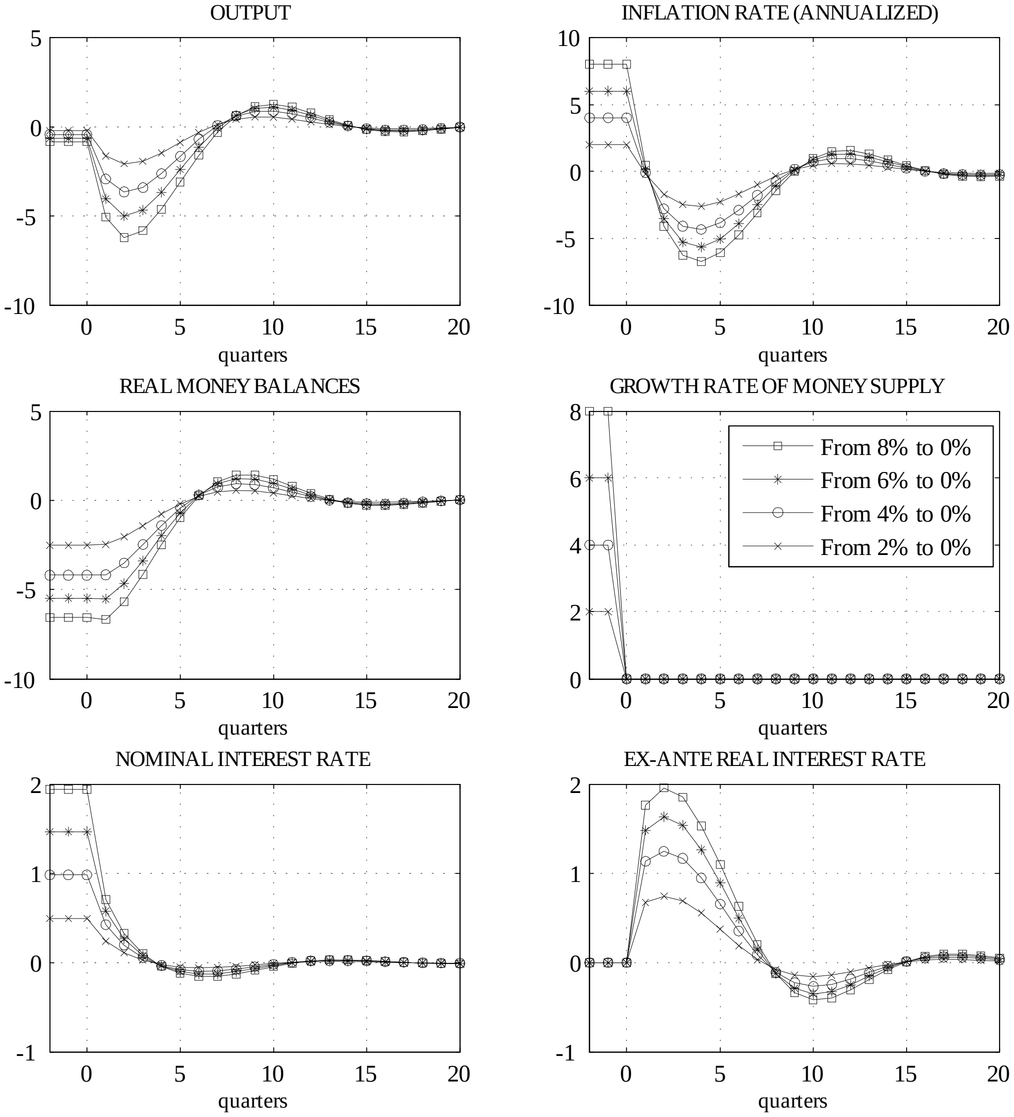

Figure 1: Cold-turkey disinflation under money supply rule. Transition paths are expressed in percentage deviations from the new steady state.

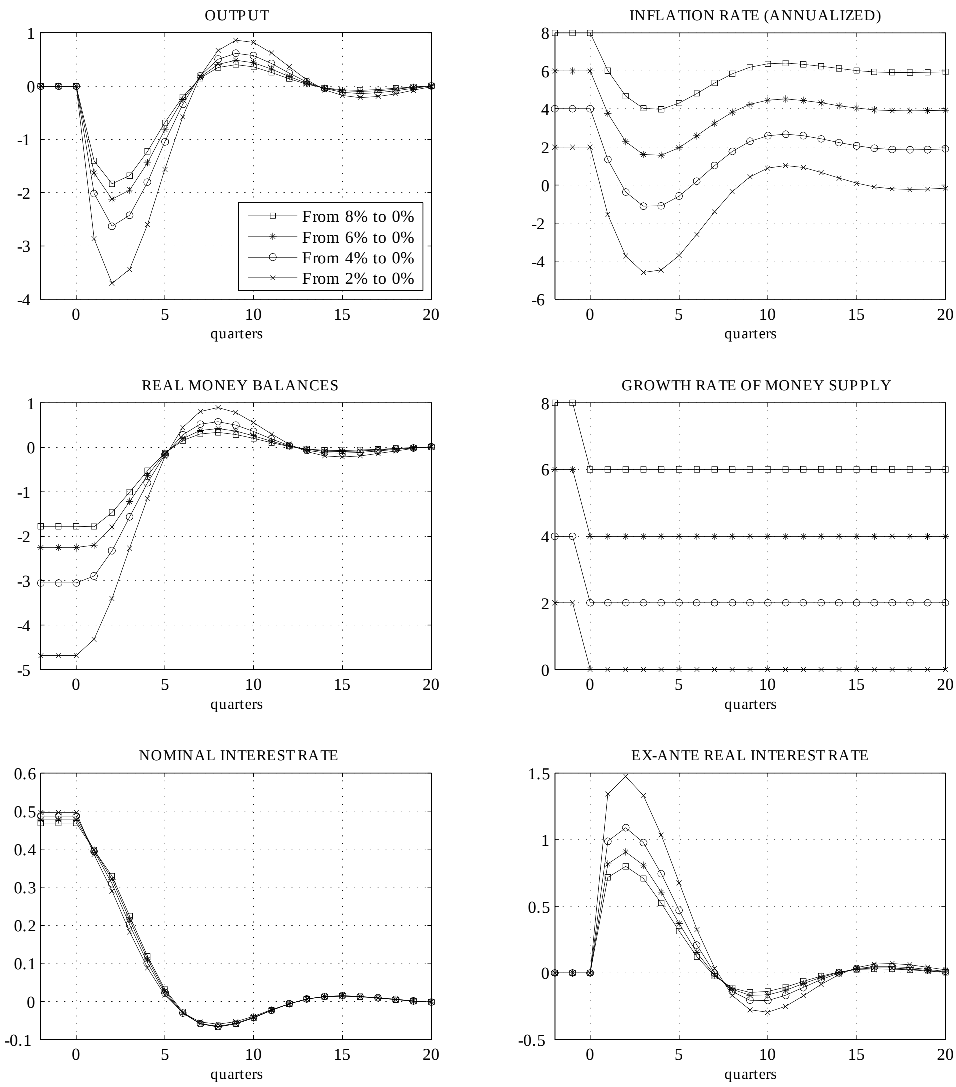

Figure 2: Cold-turkey disinflation under money supply rule for a fixed 2% disinflation size. Transition paths are expressed in percentage deviations from the new steady state.

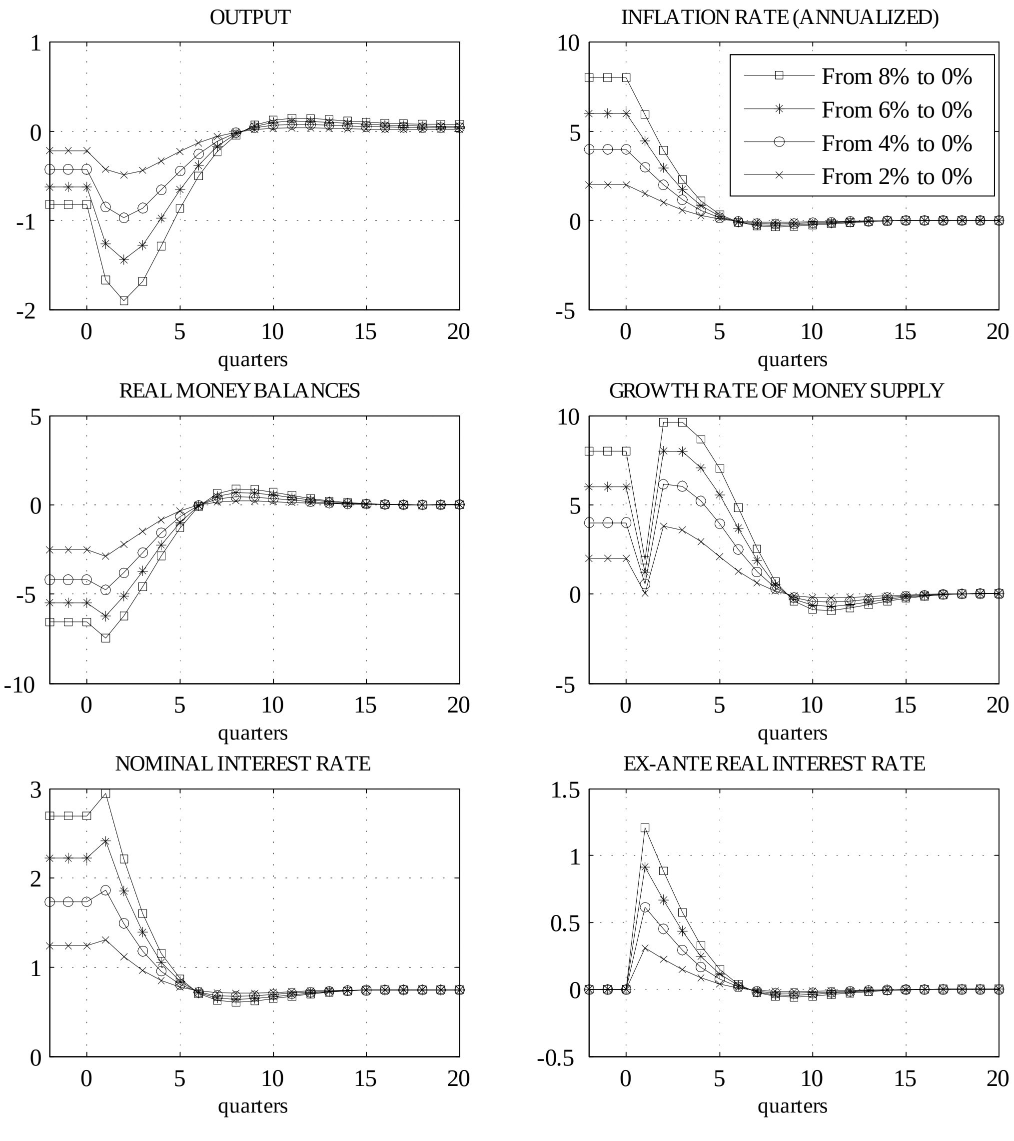

Figure 3: Cold-turkey disinflation under interest rate rule. Transition paths are expressed in percentage deviations from the new steady state.

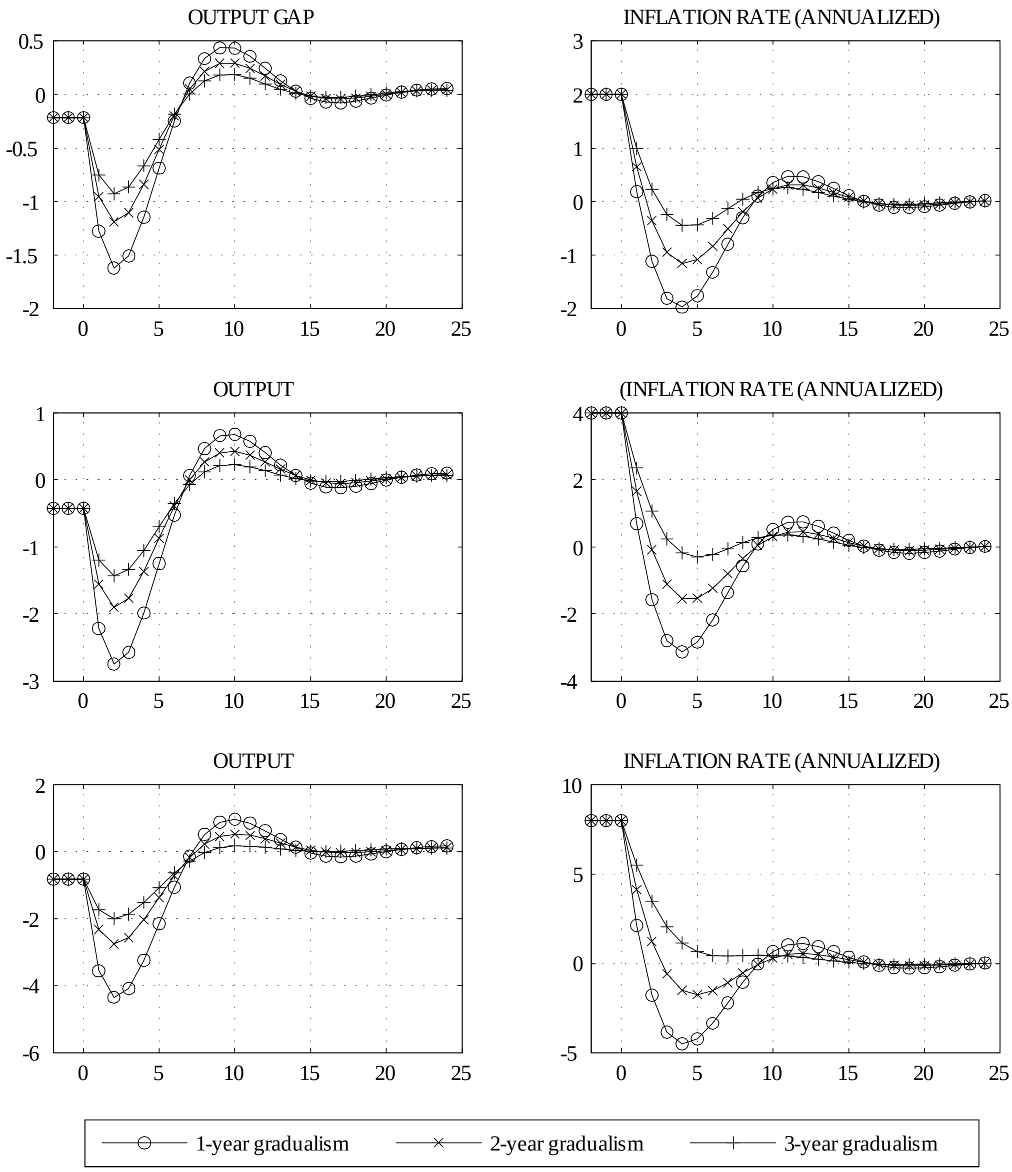

Figure 4: Gradual disinflation under money supply rule. First line from 2% to 0; second line: from 4% to 0; third line: from 8% to 0. Transition paths are expressed in percentage deviations from the new steady state.

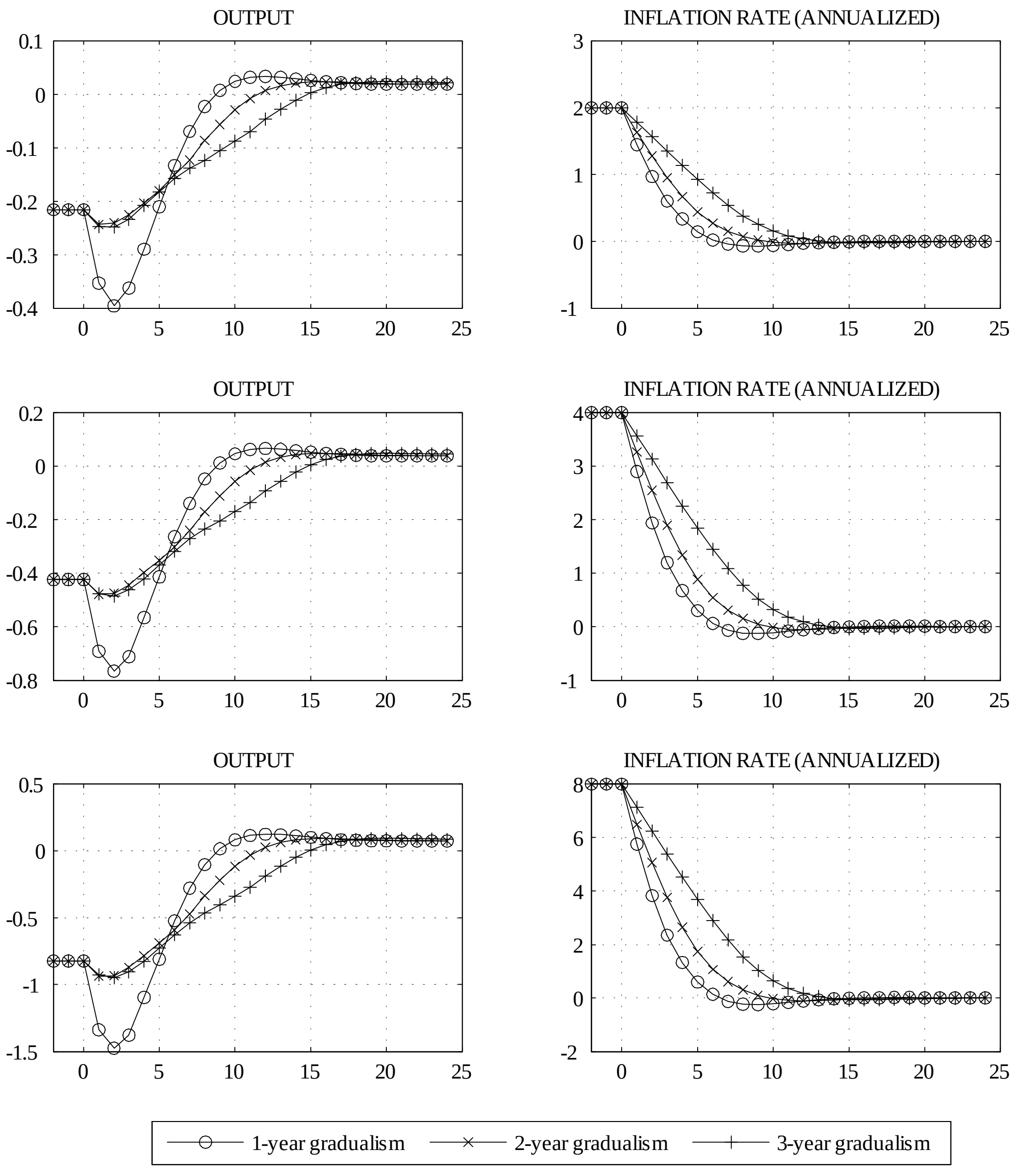

Figure 5: Gradual disinflation under interest rate targeting rule. First line from 2% to 0; second line: from 4% to 0; third line: from 8% to 0. Transition paths are expressed in percentage deviations from the new steady state.

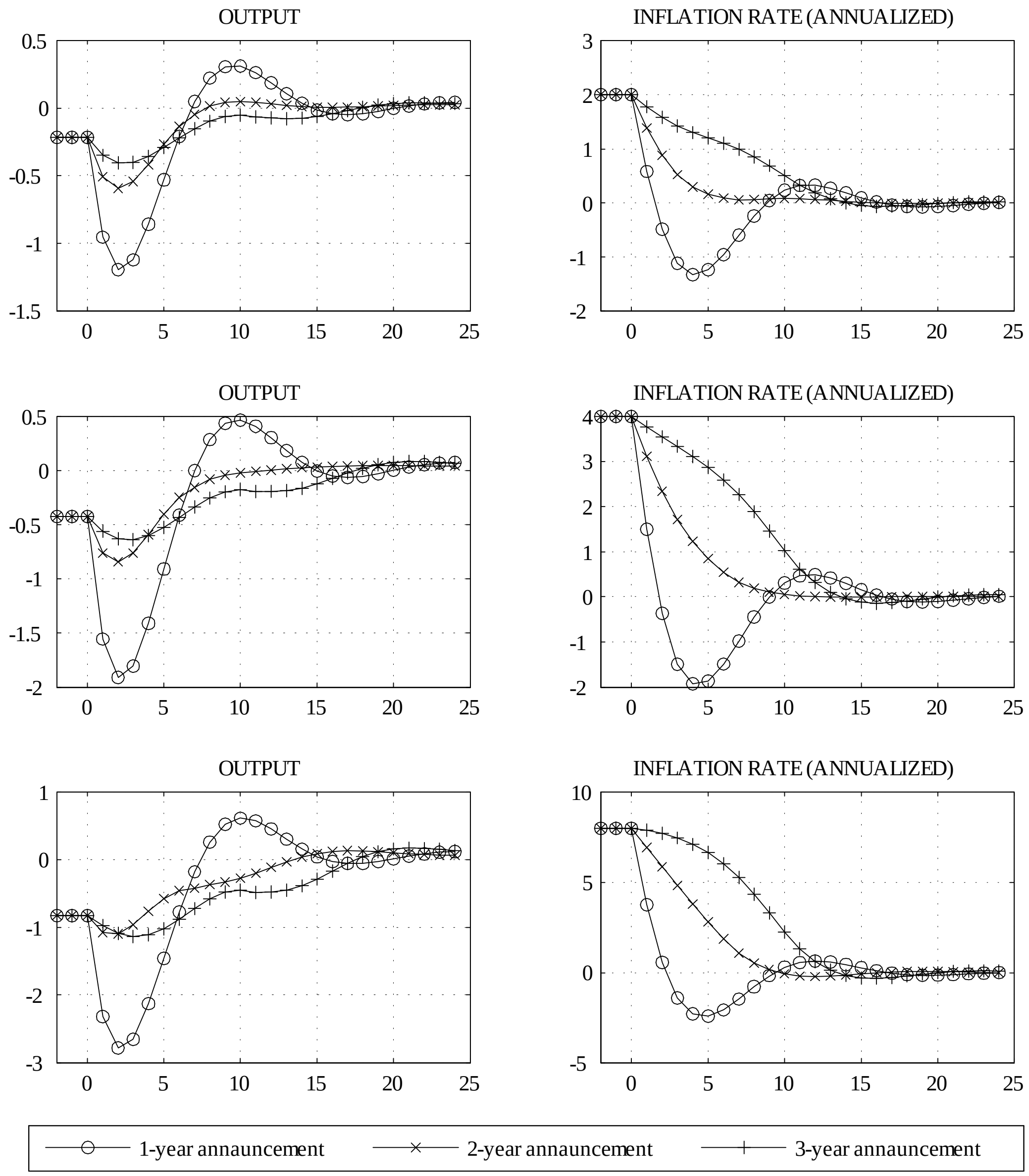

Figure 6: Anticipated disinflation under money supply rule. First line from 2% to 0; second line: from 4% to 0; third line: from 8% to 0. Transition paths are expressed in percentage deviations from the new steady state.

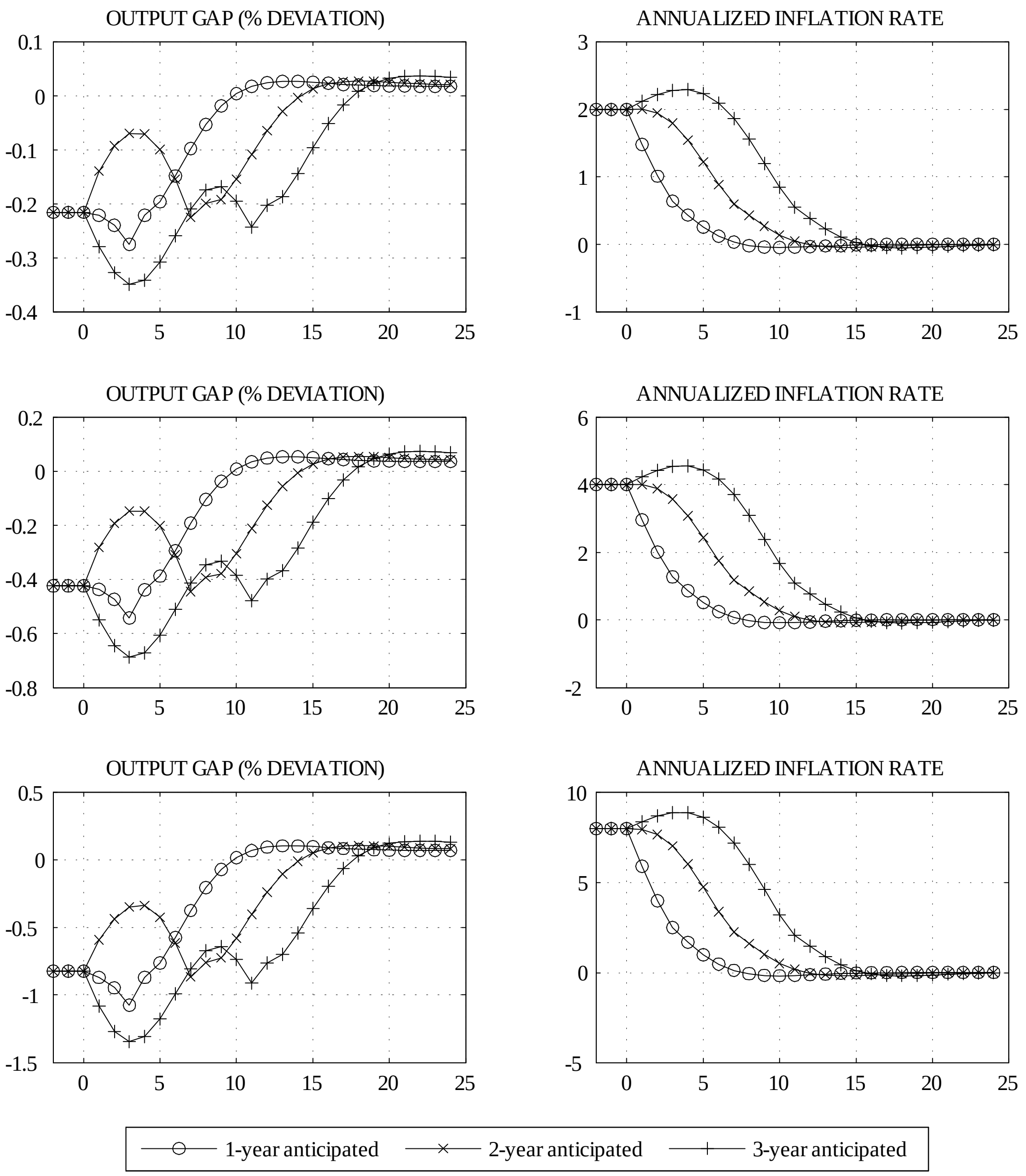

Figure 7: Anticipated disinflation under interest rate rule. First line from 2% to 0; second line: from 4% to 0; third line: from 8% to 0. Transition paths are expressed in percentage deviations from the new steady state.

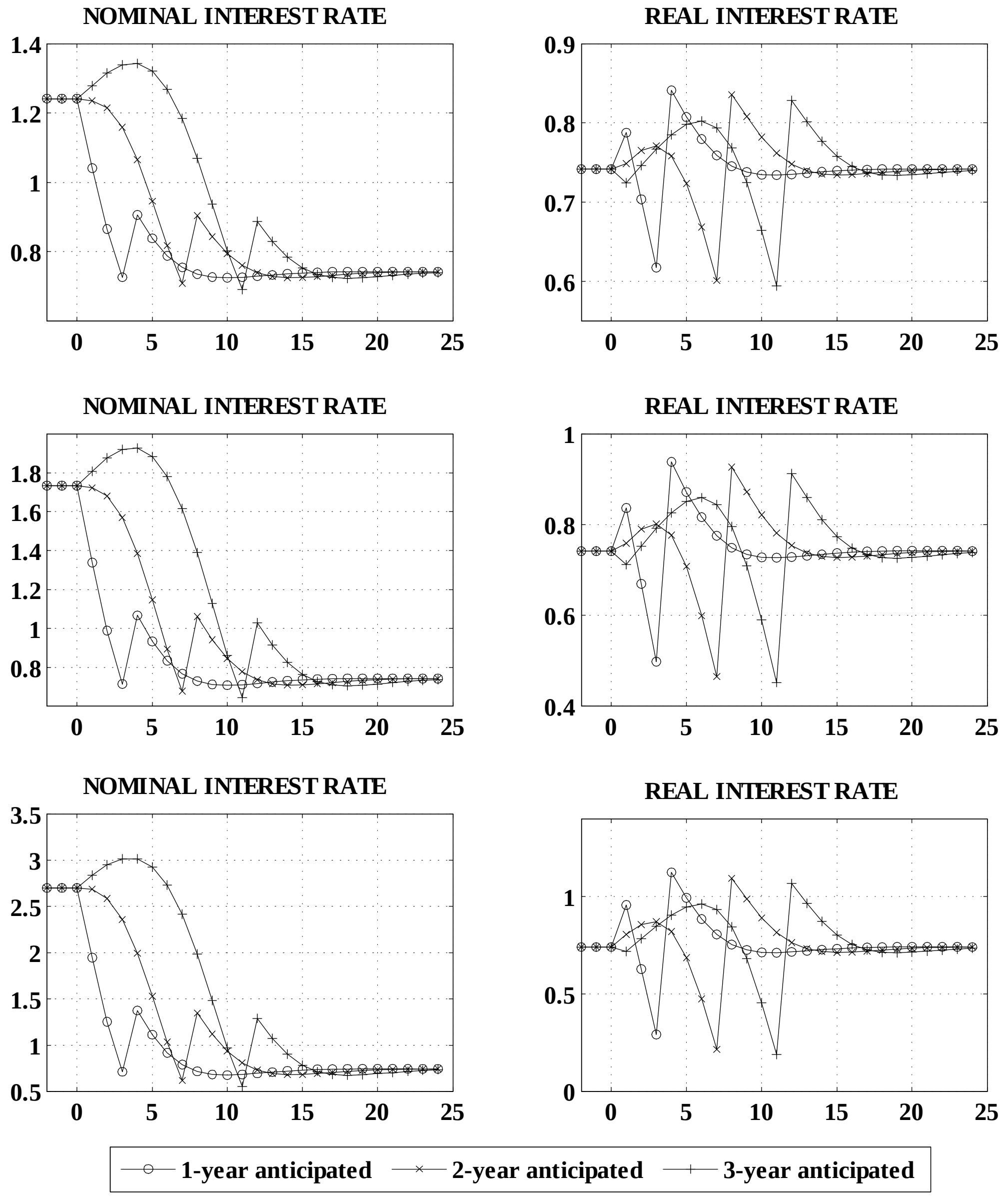

Figure 8: Anticipated disinflation under interest rate rule. First line from 2% to 0; second line: from 4% to 0; third line: from 8% to 0. Transition paths are expressed in percentage deviations from the new steady state.

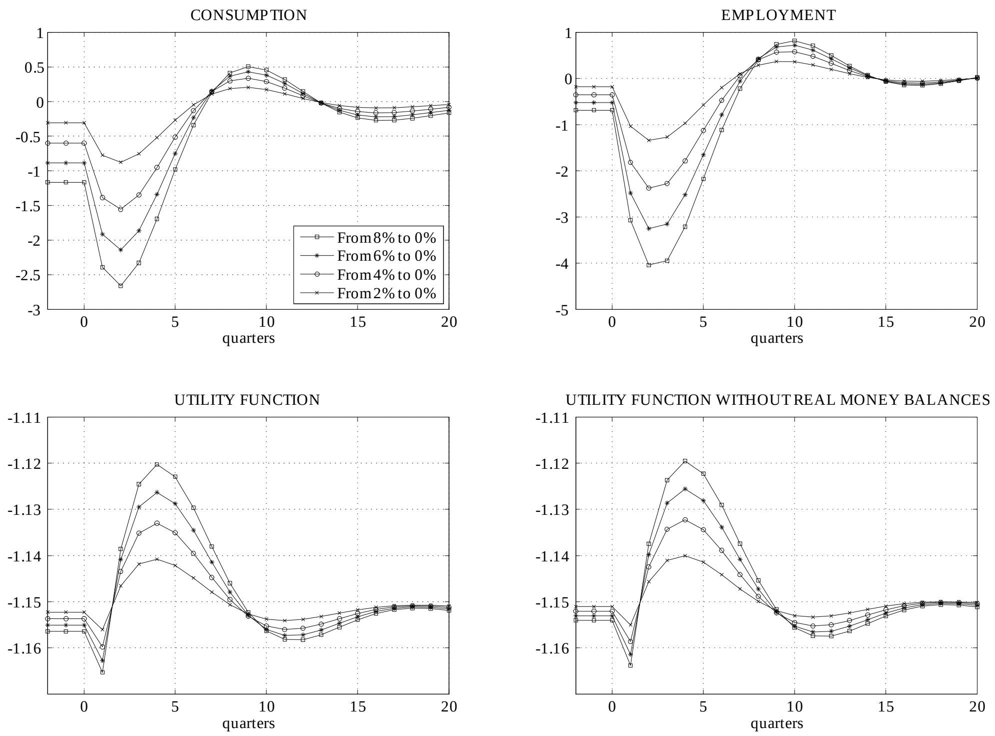

Figure 9: Cold-turkey disinflation under money supply rule. Consumption and employment paths are expressed in percentage deviations from the new steady state. Utilty function are expressed in levels.