House Price Dynamics with Heterogeneous Expectations*

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Abstract

This paper presents a dynamic equilibrium model of the housing market in which agents consume housing services and speculate on future price changes. The model features a fixed supply of housing and a random variation in demand, originating from the fact that agents hold heterogeneous expectations about the future course of prices. The important feature of the model is that heterogeneous expectations generate a non linear demand for housing: agents expecting higher future prices buy in anticipation of capital gains; agents holding pessimistic expectations prefer to rent to avoid capital losses. Because pessimistic agents rent rather than own their expectations are not incorporated in the price for owned houses. As a consequence, the equilibrium price reflects only the expectations of optimistic agents and is thus biased upward. We test the predictions of the model on US state data, using income dispersion as a proxy for information dispersion. Preliminary evidence supports the prediction that house prices are higher in states with more dispersed beliefs about future economic conditions.

1 Introduction

The US housing market has experienced substantial price variations over the last two decades. As shown in Figure 1, the national average index of real house price has risen in the 80s, fallen considerably in the early 90s and moved upward again since 1995. The historical evolution of house prices across US states has been even more drammatic. In the state of New York, for example, real house prices peaked in the late 1980s, fell to a trough in the 1990s and rebounded by 1996. In Wyoming, the house price index followed a "U shaped" pattern, that is, high in the early 1980s, and high again by the end of the sample. In other states such as Oklahoma, real house prices have declined since the 1980s and have not fully recovered since then.[1]

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[1] A similar erratic pattern holds for US cities. See, among others, Himmelberg, Mayer, and Sinai (2005) and Gyourko, Mayer, and Sinai (2006).

In the opinion of many housing-market observers, these large price variations over time and across states are difficult to explain through the lens of standard representative agent model (e.g., Poterba, 1984, 1991) in which houses are treated like any other standard financial asset, with prices depending on current and future expected fundamentals.[2]

The purpose of this paper is to present an alternative to the representative agent model to explain house price dynamics of the type presented in Figure 1. Specifically, we consider a model featuring a fixed supply of housing and a demand that fluctuates stochastically because households hold heterogeneous beliefs about the future course of house prices. Heterogeneous beliefs arise because households are imperfectly informed about the aggregate state of the economy and use their own income together with other signals in the estimation of the underlying fundamentals. Idiosyncratic income variations therefore translate into heterogeneous expectations about future housing demand and *a fortiori*, given the fixed supply, into heterogeneous expectations about future house prices.

The three building blocks of our analysis, i.e., household income, heterogeneous expectations, and fixed housing supply, are motivated by several aspects of the US housing market. First, the available evidence suggests that per capita income remains the main determinant of housing demand, either because richer households tend to demand more (Poterba, 1991, Englund and Ioannides, 1997) or because higher income relaxes credit constraints (Ortalo-Magne' and Rady, 2006, Almeida et al., 2006, Benito, 2005). Second, surveys of households expectations (Case and Shiller, 1988, 2003) have revealed a strong investment motive among home-buyers: the desire to buy houses is driven in part by the expectation of reselling houses in the future at higher prices. Moreover, these surveys document that home buyers' expectations tend to be extrapolative and largely influenced by past and current economic conditions (see also Case, Quigley and Shiller, 2003). Finally, the supply of houses seems, at least in the short-run, considerably inelastic, due to zoning laws, housing market regulations or technological constraints (Glaeser and Gyourko, 2003, Glaeser, Gyourko and Saks, 2005, 2007, Gyourko, Mayer and Sinai, 2006).

Taken together, these observations suggest a specific mechanism through which variations in income generate large swings in house prices: if household income is an important determinant of housing demand *and* shapes expectations of future house prices, a small income shock may initiate a self-reinforcing process that, through heterogeneous expectations, runs from expected prices to house demand and back to house prices. In this paper we formalize this mechanism theoretically and provide some evidence supporting the idea that heterogeneous information influence house prices.

In our model, two groups of households with different expectations about future house prices participate in the housing market to consume housing services and to speculate on future price changes. The important feature of the model is that heterogeneous expectations generate a non linear demand for housing: households who expect higher prices buy in anticipation of future capital gains while those holding pessimistic expectations rent to avoid future capital losses. Thus, renting act as if relatively pessimistic investors face a binding short-sale constraint. That

[2] For example, for US cities Case and Shiller (2003) observe that large variations in house prices cannot be explained by income variations alone. Himmelberg, Mayer and Sinai (2005) find that the price-income ratio and the price-rent ratio cannot explain the bulk of house price indices. Also, Glaeser, Gyourko and Saks (2005b) highlight the lack of a significant relationship between house prices and construction costs, for most US metropolitan areas since the 1980.

is, this group of households would like to short their houses but cannot do so since they have to consume housing services. As a consequence, pessimists prefer to move away from the market of "houses for sale" and use, instead, the rental market to consume housing services. Hence, the equilibrium price of owned houses is biased upward since it reflects only the more optimistic view of the market.

Our paper delivers two main results. First, house prices are higher the larger the difference in expectations between pessimistic and optimistic households. Second, informational shocks have an asymmetric effect on prices: positive shocks bias the equilibrium house price upwards while negative shocks are moot. Both results stem from the assumption that households use their private information to make inference about the unobservable aggregate income. In contrast, if households had homogenous expectations, housing demand would be linear and the equilibrium price would depend only on average income. It turns out that our results survive even when agents use the equilibrium house price — a summary statistics of the disperse information held by households — to update their beliefs about the unknown state of the economy, provided the price is not perfectly revealing due to unobservable demand shifts.

To test the predictions of our model we run panel regressions using US state data. Lacking a direct measurement of imperfect information, we use state-level dispersion of real per capita income to proxy for difference in beliefs. We thus take the model at face value and assume that households use their own income, among other variables, to forecast aggregate economic conditions. In line with the model's prediction, we find that that house prices are higher in states with larger expectations heterogeneity. We also find an asymmetric response of house prices to positive and negative informational shocks: while positive shocks explain significantly house price increases, negative shocks lack statistical predictive power.

In the rest of the paper we proceed as follows. In Section 2 we summarize the related literature. In Section 3 we introduce the baseline model and discuss the determinants of the equilibrium house price. Next, we examine the implications of the information structure about the state of the economy for house prices: imperfect but common information (Section 4), and imperfect and dispersed information (Section 5). In Section 6 we discuss the robustness of our results when households use not only their private source of information but also the endogenous equilibrium price to form beliefs about the state of the economy. Sections 7 and 8, are devoted, respectively, to our empirical methodology and results. We conclude in Section 9. All proofs are in the Appendix.

2 Related Literature

This paper is related to different strands of literature. In the housing literature, Stein (1995) and Ortalo-Magne' and Rady (2006) explain swings in house prices by using borrowing constraints and household leverage. In their models buyers finance the purchase of houses by borrowing and the ability to borrow is directly tied to the value of houses they own. Therefore, a positive income shock that increases the demand for houses and hence their prices, relaxes the borrowing constraint, further increasing the demand for houses and so on. Our paper is related to both studies because changes in household income may have a more than proportional effects on house price. There are, however, three important differences. First, in our story there are no borrowing constraints. Instead, the amplification mechanism operates from expected price, via household income, back to current prices, via changes in speculative demand. Second, households do not need to own houses to consume housing services; they can also use the rental market. Finally, in our set-up not only the level of income but also its dispersion matters for explaining house price dynamics.

In this regard, our paper relates to the recent works of Gyourko, Mayer and Sinai (2006) and Van Nieuwerburgh and Weil (2007). The first paper argues that the interaction between an inelastic supply of houses and the skewing of the income distribution generates a significant price appreciation in superstar cities — cities with unique characteristics preferred by the majority of the population. Wealthy households are willing to pay a significant financial premium to live in these areas, bidding up prices in the face of a relatively inelastic supply of houses. Van Nieuwerburgh and Weil (2006) use a similar mechanism – although households in their model move across cities for productive rather than preference reasons – to explain why the dispersion, and not just the level, of house prices increases with the cross-sectional wage dispersion among US cities. Our paper differs from these contributions because it highlights a different channel through which income dispersion matters. Income affects households perception of local economic conditions, leading to the formation of heterogeneous beliefs about future economic fundamentals. As a consequence, expectation heterogeneity is more pronounced when, ceteris paribus, income is more dispersed. Another important difference is that in our model households are not allowed to move across areas. However, they can use the rental market to consume housing services, an option ruled out in both Gyourko et al., and van Nieuwerburgh and Weil.

Our analysis is also related to a large literature in macroeconomics and finance that studies the role of imperfect information among decision makers. In fact, our story can be seen as an adaptation of the Phelps-Lucas hypothesis to the housing market, in the sense that imperfect information about the nature of disturbances to the economy makes different economic agents react differently to changes in market conditions. Part of our work shares also many features with the literature on the pricing of financial assets in the presence of heterogeneous expectations and short-sale constraints (i.e., Miller, 1977, Harrison and Kreps, 1979, Hong, Scheinkman, and Xiong, 2004 and Sheinkman and Xiong, 2003). In this literature, if agents have heterogeneous beliefs about asset fundamentals and face short sales constraint, the equilibrium asset price reflects the opinion of the more optimistic investors, and it is thus biased upward. We adapt the same idea to the housing market where the short sale constraint arises as a natural constraint when households have the option to consume housing services by either renting or owning.

3 The Model

3.1 Information

The economy is populated by an infinite sequence of overlapping generations of households with constant population. Each generation has unit mass and lives for two periods. In the first period, households supply labor and make saving and housing decisions; in the second period, they consume the return on savings. The wage W_t^j, at which labor is supplied inelastically, is equal to

$$W_t^j = \exp\left(\theta_t + \varepsilon_t^j\right), \tag{1}$$

where θ_t is an economy-wide shock and ε_t^j is an individual-specific shock. We make the assumption that θ_t evolves as a Gaussian random walk,

$$\theta_t = \theta_{t-1} + \eta_t, \tag{2}$$

with η_t independent and normally distributed innovations with zero mean and variance σ_η^2. The individual-specific shocks, ε_t^j, which are the only source of income heterogeneity, are serially independent and have normal distribution with zero mean and variance σ_ε^2. When households cannot observe the realization of θ_t at time t, ε_t^j is also a source of information heterogeneity. In other words, the wage W_t^j is the household $j's$ private signal about the unobservable aggregate shock, θ_t. As usual, in this context, we make the assumption that idiosyncratic shocks cancel in aggregate $\int \varepsilon_t^j dj = 0$, or equivalently the average private signal is an unbiased estimate of the underlying fundamental.

3.2 Preferences

Households have logarithmic preferences over housing services, V_t^j, and second period consumption, C_{t+1}^j,[3]

$$\mathcal{U}_t^j = A_t^j \log V_t^j + E_t^j \left[\log C_{t+1}^j \right], \tag{3}$$

where E_t^j denotes the expectation operator based on household j's information set at time t (to be specified later) and the parameter A_t^j is a preference shock,

$$A_t^j = \exp\left(2\left(a_t + \nu_t^j\right)\right),$$

which consists of two components: an aggregate taste shock, a_t, and an idiosyncratic noise ν_t^j. We assume that a_t follows a random walk

$$a_t = a_{t-1} + \mu_t, \tag{4}$$

where μ_t is a normal random variable with zero mean and variance σ_μ^2, while the idiosyncratic taste shock ν^j is i.i.d., with zero mean and variance σ_ν^2. We also consider the case where the variance of ν_t^j approaches infinity, so that knowing one's own individual taste provides no information about the aggregate taste.[4]

3.3 Budget constraint

In the first period of life, after the realization of the idiosyncratic income, households decide how many housing units to buy, $H_t^j \geq 0$, at unit price, P_t. They also choose the quantity of housing services to consume, V_t^j, and, as a by-product, the units of housing to rent out, $H_t^j - V_t^j$, at the rental price Q_t. For each unit owned, households also incur a cost of owning, equal to a fraction M_t of the nominal value of housing, $P_t H_t^j$.[5] For simplicity we assume that

$$M_t = \exp(m_t),$$

[3] This specification of the utility function subsumes a fixed demand for second-period housing services which, for simplicity, we normalize to zero.

[4] While this assumption implies that a_t is unobservable, the law of motion of a_t is known by all agents.

[5] M can be thought of as including maintenance and depreciation costs, property taxes, interest payments on mortgages, etc.

with m_t following a random walk

$$m_t = m_{t-1} + \zeta_t, \tag{5}$$

where ζ_t is normally distributed with zero mean and variance σ_ζ^2.

At the end of the first period, the residual income is saved at the gross interest rate, R, and at the beginning of the second period the stock of houses owned is sold to the young of the new generation, at the price P_{t+1}. For a typical household j the flow of funds constraint is thus:

$$C_{t+1}^j = R\left(W_t^j - P_t H_t^j + Q_t\left(H_t^j - V_t^j\right) - M_t P_t H_t^j\right) + P_{t+1} H_t^j, \tag{6}$$

with

$$H_t^j \geq 0. \tag{7}$$

3.4 Optimal house demand

Households inter-temporal decisions consist of choosing H_t^j and V_t^j to maximize (3) subject to (6) and (7). It is immediate to establish that the optimal demand for housing service, V_t^j, and housing units, H_t^j, satisfy the following first-order conditions,

$$\frac{A_t^j}{V_t^j} = E_t^j\left[\frac{RQ_t}{C_{t+1}^j}\right], \tag{8}$$

$$E_t^j\left[\frac{R\left(U_t - Q_t\right)}{C_{t+1}^j}\right] \geq 0, \tag{9}$$

where

$$U_t = P_t(1 + M_t) - \frac{P_{t+1}}{R}, \tag{10}$$

denotes the user cost per unit of houses owned, which depends negatively on the future discounted house price P_{t+1}/R, and positively on the costs of buying, P_t and the cost of owning, $P_t M_t$.

Equation (8) establishes that households consume housing services until the current period marginal utility of these services, A_t^j/V_t^j, equal the expected marginal cost in terms of next period consumption utility, $E_t^j\left[RQ_t/C_{t+1}^j\right]$. The optimal quantity of housing units to buy is implicit in equation (9), which relates the the user cost, U_t, to the cost of renting housing services, Q_t. Whether this condition holds with equality depends on households' expectations about future prices. For households holding pessimistic expectations the user cost is perceived higher than the cost of renting. Thus, constraint (7) binds and (9) is satisfied with inequality. The opposite holds for households with relatively optimistic expectations. They demand a strictly positive amount of housing units, $H_t^j > 0$ and (9) holds with equality.

3.5 The linearized optimality conditions

To deliver explicit solutions, we find it convenient to work with a linear approximation of equations (8) and (9) around the "certainty" equilibrium, i.e., the equilibrium prevailing when both aggregate and idiosyncratic shocks are zero. Denoting with lower case letters variables in percentage deviations from the equilibrium with certainty, Appendix I shows that a linear approximation to (9) leads to

$$E_t^j u_t \geq q_t, \tag{11}$$

where

$$u_t = \left(1 + \frac{1}{\kappa}\right) p_t + \left(1 + \frac{1-R}{\kappa}\right) m_t - \frac{1}{\kappa} p_{t+1}, \tag{12}$$

and $\kappa \equiv RU/P > 0$.

Similarly, a linear approximation of (8) leads to

$$v_t^j = w_t^j + a_t^j - q_t, \tag{13}$$

indicating that the consumption of housing services is positively related to the individual income and preferences, and depends negatively on the rental price.

From now on, to make the analysis tractable, we consider only two groups of households, $j = 1$ and $j = 0$, each with equal mass, and adopt the convention that households in the first group receive a relatively more optimistic signal about the current fundamental, i.e., $\varepsilon_t^1 > \varepsilon_t^0$. As a consequence $E_t^1 p_{t+1} > E_t^0 p_{t+1}$, $E_t^0 u_t > E_t^1 u_t$ and (11) can be written as

$$E_t^0 u_t \quad > \quad q_t \quad \text{and} \quad h_t^0 = 0 \tag{14}$$

$$E_t^1 u_t \quad = \quad q_t \quad \text{and} \quad h_t^1 > 0. \tag{15}$$

These equations show that, in equilibrium, pessimistic households decide to own no housing units, $h_t^0 = 0$, because they hold lower expectations about next period prices and thus perceive the cost of ownership to be higher than the cost of renting. On the other hand, optimistic households are just indifferent between owning and renting, or equivalently the user cost is equal to the cost of renting. The implication is that optimistic households choose the units of housing services to consume, v_t^1, out of those owned, h_t^1, and rent out the difference, $h_t^1 - v_t^1 = v_t^0$ to pessimistic households, at the equilibrium rent, q_t.

3.6 The equilibrium house price and rental price

Using (12), the indifference condition (15) can be written as

$$p_t = \frac{\kappa}{1+\kappa} q_t - \left(1 - \frac{R}{1+\kappa}\right) m_t + \frac{1}{1+\kappa} E_t^1 p_{t+1}, \tag{16}$$

suggesting that for a given rental price and maintenance cost, the equilibrium house price reflects only the expectations of optimistic households. With the maintained assumption of fixed housing supply, s, the equilibrium rental price, q_t, is determined by the market clearing condition,

$$s = \frac{v_t^1 + v_t^0}{2},$$

which using (13) yields

$$q_t = \theta_t + a_t - s, \tag{17}$$

where

$$\theta_t = \frac{w_t^1 + w_t^0}{2} \quad \text{and} \quad a_t = \frac{a_t^1 + a_t^2}{2},$$

denote, respectively, the aggregate income and the aggregate preference over housing services. Plugging (17) back into (16), the equilibrium price can be written as

$$p_t = \frac{\kappa}{1+\kappa} f_t + \frac{1}{1+\kappa} \overline{E}_t p_{t+1} + \frac{1}{1+\kappa} \widetilde{E}_t p_{t+1}, \tag{18}$$

where

$$f_t \equiv (\theta_t + a_t - s) - \left(1 + \frac{1-R}{\kappa}\right) m_t, \tag{19}$$

summarizes fundamental variables, and

$$\overline{E}_t p_{t+1} \equiv \frac{E_t^1 p_{t+1} + E_t^0 p_{t+1}}{2}, \quad \tilde{E}_t p_{t+1} \equiv \frac{E_t^1 p_{t+1} - E_t^0 p_{t+1}}{2},$$

denotes respectively the average expectation and the difference in expectations about tomorrow's price.

In equation (18), as in a standard asset pricing equation, the equilibrium price, p_t, depends, on fundamentals, f_t, and on the expected average capital gain from house price appreciation. The extra term, $\widetilde{E}_t p_{t+1}$, is non-standard and arises in our setup because of households' heterogenous expectations. In the next two sections, we will make different assumptions about households' information sets in order to evaluate how $\overline{E}_t p_{t+1}$ and $\widetilde{E}_t p_{t+1}$ influence the relationship between house prices and fundamental variables.

4 Homogenous Information

In order to have a benchmark against which to compare the results, we start with the case where households receive identical information about the underlying unobservable aggregate fundamental, that is $\varepsilon_t^1 = \varepsilon_t^0$. With this assumption individual expectations coincide with average expectations, $E_t^j p_{t+1} = \bar{E}_t p_{t+1}$, and differences in expectation are zero, $\tilde{E}_t p_{t+1} = 0$.

Using equation (18), Appendix II shows that the average expectation of tomorrow's price depends only on current expected fundamentals,

$$\begin{aligned} \overline{E}_t p_{t+1} &= \overline{E}_t \theta_t + \overline{E}_t a_t - s - \left(1 + \frac{1-R}{\kappa}\right) m_t \\ &= \theta_{t-1} + a_{t-1} - s - \left(1 + \frac{1-R}{\kappa}\right) m_t, \end{aligned} \tag{20}$$

where the second equality arises because θ_t and a_t, which are not observable, follow a random walk, so that households' forecast of these variables is just equal to their past realization. Inserting (20) into (18), and recalling that $\tilde{E}_t p_{t+1} = 0$, the equilibrium price under imperfect but homogenous information, p^*, can be written as

$$p_t^* = f_t + \Lambda_t, \tag{21}$$

where f_t is given in (19) and

$$\Lambda_t \equiv \frac{1}{1+\kappa}\left((\theta_{t-1}-\theta_t)+(a_{t-1}-a_t)\right),$$

is an expectation error.

In what follows, we interpret p_t^* as the "fundamental" house price, because it reflects the average opinion in the market that, by assumption, is an unbiased estimate of the unknown fundamental. As we will see, when information is not only imperfect but also heterogeneous among house market participants, the housing market becomes segmented. The upshot is that idiosyncratic shocks are not "washed out" in aggregate and the equilibrium price ends up reflecting only the most optimistic view in the market.

5 Heterogeneous Information

We now consider a setting where households use the current realization of their income, w_t^j, and the exogenous public signal, θ_{t-1}, to make an optimal inference about θ_t. Household j's information set at t is therefore[6]

$$\Omega_t^j = \left\{w_t^j, \theta_{t-1}\right\} \quad j = 0, 1.$$

Under the assumption that households do not share their private information with each other, and since w_t^j is buffeted by idiosyncratic shocks, ε_t^j, households end up holding heterogeneous information about θ_t.

Before proceeding, it is important to notice that the equilibrium price is not included in Ω_t^j. This assumption is made only to simplify the characterization of the channels through which information dispersion affect the equilibrium price. As we will discuss in the following section this assumption is inessential for our results. A way to think about this assumption is to consider the case where the variance of the aggregate unobservable preference shock, μ_t, is arbitrarily large. In such a case the price becomes uninformative about θ_t and house market participants do not learn much after observing the equilibrium price.[7]

With signals w_t^j and θ_{t-1}, the ability of household j to estimate θ_t from available data depends on the relative magnitude of σ_ε^2 and σ_η^2. Because of our assumption of independent and normally distributed errors, the projection theorem implies

$$E_t^j\theta_{t+1} = E_t^j\theta_t = (1-\lambda)\,\theta_{t-1} + \lambda w_t^j, \tag{22}$$

where the weight $\lambda \equiv \sigma_\eta^2/\left(\sigma_\eta^2+\sigma_\varepsilon^2\right)$ reflects the relative precision of the two signals. Thus, with $\lambda > 0$, expectations among households are heterogeneous and both average expectations

[6] Notice that knowing the entire history of aggregate shocks is superfluous, given that θ_t follows a random walk. Similarly, knowing the past realization of household private signals is not relevant, given the *iid* assumption on ε_t^j.

[7] In excluding the equilibrium price from the household information set, me make our analysis akin to a "difference of opinion model", widely used in the finance literature. In such a model investors agree to disagree about the distribution of payoff and signals and therefore do not use the equilibrium prices to infer other investors' beliefs. An alternatively reason households may not condition on the equilibrium price is because they do not know how to use price correctly (e.g. they display bounded rationality, as in Hong and Stein, 1999) or because they exhibit behavioral biases (e.g., they are overconfident, as in Scheinkman and Xiong, 2003).

and expectations differences become important determinants of the equilibrium house prices. Using equations (18) and (22), Appendix III shows that the difference in expectations, and the average expectation of future prices are, respectively,

$$\tilde{E}_t p_{t+1} = \lambda i_t, \tag{23}$$

$$\overline{E}_t p_{t+1} = \theta_{t-1} + a_{t-1} - s - \left(1 + \frac{1-R}{\kappa}\right) m_t + \frac{\lambda}{\kappa} I + \lambda\left(\theta_t - \theta_{t-1}\right), \tag{24}$$

where

$$i_t \equiv \varepsilon_t^1 - \varepsilon_t^0,$$

denotes the informational difference between the two groups of households and

$$I \equiv \int_0^\infty x d\Gamma\left(x\right),$$

measures the average degree of information heterogeneity in the economy, with Γ denoting the distribution of i_t.

Equation (23) arises because households are disparately informed. The difference in expectations reflects uniquely the fact that in estimating θ_t households assign weight λ to their private signal, w_t^j. Differences in expectations, λi_t, are therefore proportional to the difference in private signals, $\varepsilon_t^1 - \varepsilon_t^0$. Equation (24), which measures the average expectation in the market, resembles equation (20), prevailing under imperfect but homogeneous information. It differs from equation (20), however, because heterogenous information introduces two additional terms, each proportional to the positive weight λ that households attach to their private source of information. The first term, $\lambda I/\kappa$ arises because prices are forward looking. It is not only the current degree of information heterogeneity that matters but also the level expected to prevail in the distant future, since future dispersion in information will affect the future course of prices. The second term, $\lambda\left(\theta_t - \theta_{t-1}\right)$, reflects instead the average degree of misperception in the economy and arises because households use only partially the public signal, $\theta_{t-1,}$ to infer the current state of the world. The larger the weight λ assigned to the private signal and the larger the degree of misperception in the economy, $\left(\theta_t - \theta_{t-1}\right)$, the more the expected price deviates from the one prevailing under common information. This partial reaction of households to shifts in the fundamentals has the effect of introducing inertia in the way expectations are formed, which accords well with the idea that in the housing market expectations are formed in an extrapolative manner (see Case and Shiller, 1988, 2003).

Plugging these expressions into (18), the equilibrium house price can be expressed as

$$p_t = p_t^* + \lambda \Upsilon_t. \tag{25}$$

where, p_t^*, is the fundamental price given in (21), and

$$\Upsilon_t \equiv \frac{1}{1+\kappa}\left(\theta_t - \theta_{t-1}\right) + \frac{1}{\kappa(1+\kappa)} I + \frac{1}{1+\kappa} i_t.$$

summarizes the impact of information heterogeneity among households.

Thus, in the presence of heterogenous information, i.e., $\lambda > 0$, p_t differs from p_t^*, due to shifts in Υ_t. In turn, shits in Υ_t are more pronounced the larger is the current and expected degree of information heterogeneity: i_t, and I. The reason is quite intuitive. Households receiving a positive signal expect higher income in the future and thus higher demand and prices. Conversely, households with a negative signal expect lower future prices. However, while households with optimistic expectations demand more houses for speculative reasons the higher the expected price, pessimistic households prefer to consume housing services through the rental market. In other words, renting act as if relatively pessimistic investors were facing a binding short-sale constraint: holding pessimistic expectations about tomorrow's prices these households would like to short their houses. Since they cannot do so, these households drop out of the market of houses for sale and their beliefs are not fully reflected in the equilibrium price. The final result is that the equilibrium price reflects only the expectations of relatively optimistic households, and is thus biased upward. Equation (25) suggests that the larger the divergence in beliefs among households, the more pronounced is this effect.

6 Learning from the equilibrium price

The above result, that differences in beliefs impart an upward drift in price holds under the maintained assumption that the noise in the preference for housing services, μ_t, is arbitrarily large, so that to infer θ_t households rely only on the exogenous public and private signals, but not on the current house price.

We now relax this assumption and allow households to condition on prices. This extension is desirable because house prices, like any other financial price, is a useful summary statistics of the dispersed private information in the economy. Thus, if households use this endogenous public signal to update their beliefs about the underlying fundamental, dispersion in beliefs may vanish, and in the limit the upward bias in the equilibrium house price may disappear.

In extending our analysis to a set-up where households condition on the price, we run into a non-trivial problem. As discussed in the previous section, if households receive symmetrically dispersed information and have the option to consume housing services by either buying or renting, the housing market becomes segmented, in the sense that the noise in the private signals do not cancel out when the demand is aggregated across households. This implies that the opinion of the most optimistic agent, i.e., $\max\left\{\varepsilon_t^i, \varepsilon_t^j\right\}$, or equivalently the difference in opinion between optimists and pessimists, i.e., $i_t = \left|\varepsilon_t^i - \varepsilon_t^j\right|$ ends up affecting the equilibrium price. But, since i_t is not normally distributed p_t will also have a non Gaussian distribution and standard linear filtering methods cannot be applied.[8] To circumvent this problem we proceed by assuming that μ_t is an independent and identically distributed random variable, drawn from a distribution $\mathcal{M}(x)$**,** with zero mean and variance σ_μ^2. Moreover, $\mathcal{M}(x)$ is such that $\mu_t + i_t \equiv \phi_t \sim \mathcal{N}\ (\bar{\imath}, \sigma_\phi^2)$ where $\bar{\imath}$ denotes the unconditional mean of i_t and σ_ϕ^2 the variance of $\mu_t + i_t$.

Although ad-hoc, this assumption enables us to use standard methods to characterize the filtering problem, since it ensures that the equilibrium price is Gaussian. In addition, as typical in a noisy rational expectation model *à la* Grossman and Stiglitz (1976) and Hellwig (1980), Assumption 1 ensures that the average private signals are not fully revealed by the equilibrium price. Specifically, households cannot tell whether prices are high because aggregate economic

[8] See Appendix IV for a derivation of the exact distribution of i_t.

conditions improve or because unobservable taste shocks drive housing demand up. Thus, for a given noise in the exogenous public and private signals, the informative content of the equilibrium price will be decreasing in the preference noise, σ_μ^2.

Under Assumption 1 and using a standard linear solution method, Appendix IV shows that the equilibrium price with learning can be written as,

$$p_t = p_t^* + \frac{\pi_2}{\lambda}\Upsilon_t + \pi_3\Phi_t, \tag{26}$$

where π_2 and π_3 are the weights on the private and the endogenous public signal (the price), respectively, and

$$\Phi_t \equiv \frac{1}{1+\kappa}\left(\theta_t - \theta_{t-1}\right) + \frac{\kappa}{(1+\kappa)(\kappa+\pi_2)}\left(a_t - a_{t-1}\right) + \frac{\pi_2}{(1+\kappa)(\kappa+\pi_2)} i_t,$$

is a term that summarizes the degree of magnification of shocks induced by the process of learning from the price. Intuitively, in the presence of unobservable shocks, households who observe a change in the house price do not understand whether this change is driven by a change in the aggregate income or by changes in preferences and/or private signals. Thus, when $\pi_3 > 0$, each of these shocks will have an amplified effect on the price, since households respond to whatever is the source of movement in the house prices.

A key observation to make in comparing equation (26) with (25) is that i_t – our measure of dispersion in beliefs – continues to shift the equilibrium price away from its fundamental value, p_t^*. More specifically, i_t exerts a direct effect, via Υ_t, for the same reasons discussed in the previous section, and an indirect one, via Φ_t, because of the magnification of shocks induced by the process of learning.

The relative importance of Υ_t and Φ_t depends, however, on π_2 and π_3 which, as shown in Appendix IV, are related to λ as follows,

$$\lambda > \pi_2 \gtreqless \pi_3,$$

with the property that $\pi_2 \to \lambda$, and $\pi_3 \to 0$, as $\sigma_\mu^2 \to \infty$. In words, as the noise in the preference for housing services becomes larger, that is the price gets less and less revealing, the equilibrium price (26) becomes identical to the one prevailing in absence of learning (25). This is illustrated in Figure 2, where we plot the percentage deviation of the equilibrium price from its fundamental level, $p_t - p_t^*$, for different values of the preference noise, σ_μ^2.[9] As the figure shows, regardless of whether households are able to learn, the price misalignments remain of comparable size, for large values of σ_μ^2. Instead, when σ_μ^2 is smaller, p_t is informative about θ_t and households rely less on their private source of information, i.e. $\pi_2 < \lambda$.

Nonetheless, provided preference shocks prevent prices from being fully revealing, i.e., σ_μ^2 is not very small, the private signals continue to provide some information in estimating the underlying fundamental. In such a case Υ_t and Φ_t bias the equilibrium price in a way that depends on the noise in the private signal. This is illustrated in Figure 3, which plots the price differential, $p_t - p_t^*$, for different values of σ_ε^2. As can be seen, the direct effect of Υ_t and Φ_t is

[9] Figure 2 plots the average price differential, $p_t - p_t^*$, of 1000 simulated series using the following parameter values: $\theta_{t-1} = a_{t-1} = s = m_t = 0$ and $\kappa = 1$. These parameter values are chosen without loss of generality given that we consider the deviation of the equilibrium price from its fundamental value.

hump-shaped in the signal noise, regardless of whether households learn from the equilibrium price. The bias is increasing for intermediate values of the private signals' noise, and it is falling for smaller values of σ_ε^2, since in this case households receive similar information. Similarly, the bias will be decreasing for larger values of σ_ε^2 because not only households tend to put less and less weight on their private signals, but also the equilibrium price becomes a more noisy signal of θ_t.

7 Empirical evidence

The model just discussed delivers two main empirical predictions. The first is that higher dispersion beliefs across households raises house prices. The second prediction is that pessimistic expectations do not affect the equilibrium price while optimistic expectations bias the price upward. A perfect test of these predictions would require a direct measure of the degree of information heterogeneity among house market participants, such as the dispersion of real estate analysts' forecasts. For the US, however, this variable is hard to obtain and when available it covers just a few years.[10] To make progress towards a test of our theory we thus need to resort to *indirect* measures of the degree of information heterogeneity. The strategy adopted in this paper is to take the model at face value and use the *dispersion in households' income* as a proxy of dispersed information about local housing market conditions. The assumption underlying this proxy is that housing market trends reflect mainly local economic conditions, with house prices often bid up as a result of higher income and/or better employment opportunities of local residents. This assumption is supported by a large body of empirical work and is grounded on surveys evidence that, for the US housing market, indicates that buyers expectations are strongly influenced by past and current local economic conditions (see e.g., Case and Shiller, 1988, 2003).

7.1 Data and variable definition

To compute an index of income dispersion we interpret our model as describing the dynamic of house prices in a typical US state. Specifically, as in the model, the income process of a representative household in county j, in a given US state, is represented by

$$w_t^j = \theta_t + \varepsilon_t^j,$$

where θ_t is the average state income and ε_t^j the county specific income shock.[11] A proxy for our indicator of information heterogeneity,

$$i_t = \varepsilon_t^j - \varepsilon_t^i \quad i \neq j$$

can then be computed using the dispersion of real per capita income among counties in a given state. For this purpose, we collect income data for 47 US states over the period 1980-2005 from the Regional Economic Accounts of the Bureau of Economic Analysis, and measure the

[10] See for example the housing market monitor of Moody's Economy.com's, which assesses the near-term prospects for single-family homebuildings in the US largest metropolitan areas, starting from 2000.

[11] In light of this interpretation, households in our model are supposed to face non trivial moving costs from one state to another, and thus have to decide whether to buy or rent houses in their state of residence.

distribution of real per capita income with several indexes: 1) the Gini coefficient, 2) the coefficient of variation, 3) the mean log deviation, 4) the standard deviation of logs, and 5) the log ratio of the 99^{th} and the 1^{th} percentiles of the county-state income distribution.[12] For the house prices we use the OFHEO index, deflated with the US CPI less shelter. This index, produced by the Office of Federal Housing Enterprise Oversight, is based on multiple observations of the price of a single-family house and has therefore the advantage of controlling for the changing quality in the mix of houses sold at any time.[13] Our measure of the cost of owning include, as in Himmelberg et al. (2006), the 10-year Treasuries bills rate as the opportunity cost of funds, the 30-year mortgage rate as the cost of borrowing, and the federal marginal income tax and local property tax rates as measures of the fiscal cost of owing versus renting.[14] Finally, in some specifications, we use a proxy of households borrowing capacity to control whether differences in the availability of mortgages finance to households in different states affect the equilibrium house price. Following Lamont and Stein (1999) our proxy for leverage is the state's median loan-to-value-ratio, which is taken from the Federal Housing Finance Board. Summary statistics of our data are given in Table 1.

8 Testing the model with state panel data.

8.1 The basic specification

We start our analysis by examining the empirical relevance of the price equation that prevails under common information. More precisely, we estimate by OLS a variant of equation (21):

$$\Delta p_{kt} = \alpha_0 + \alpha_1 \Delta\theta_{kt} + \alpha_2 \Delta\theta_{kt-1} + \alpha_3 \Delta m_{kt} + \gamma_t + \epsilon_{kt}, \tag{27}$$

where each variables is used in first difference to eliminate state specific factors. Here Δp_{kt} is the log change of the real house price index in state k in year t, $\Delta\theta_{kt}$ the log change in real per capita income, Δm_{kt} the first difference of the owning cost, γ_t a year dummy and ϵ_{kt} the error term. According to the model, α_1 and a_2 are expected positive and α_3 negative. The results reported in the first column of Table 2 (with robust $t-$statistics in parenthesis), indicate that house prices respond positively to contemporaneous and lagged income changes, and negatively to variations in the cost of owning. Both variables are also significant at the one per cent level. In column 2, to control for factors that are common to all states over time, we include year dummies in the regression. The main effect of year dummies is to wipe out

[12] We have data for 50 states and the District of Columbia. We drop, however, the District of Columbia, Delaware, Hawaii and Rhode Island because some measures of income inequality, such as the Gini coefficient, could not be computed with less than four counties.

[13] One major drawback of this index is that it covers only houses whose mortgages have been purchased or securitized by Freddie Mac or Fannie Mae,which are subject to a maximum lending limit (in 2005, the maximum lending limit for such mortgages was $359,650). Thus, high-priced houses are underrepresented. While this is a considerable limitation of our data, there is evidence supporting the hypothesis that markets for high-value houses have their own idiosyncratic dynamics (Mayer 1993). An alternative index of house price is the one produced by the National Association of Realtors (NAR). This index is not subject to the price cap but does not control for the quality of houses sold over time. None of the results presented below depend, however, on the type of house price index chosen.

[14] The interest rate and expected inflation data are from the St. Louis Federal Reserve. Federal marginal tax rates are from the NBER – TAXSIM and property tax rates are from the 2000 Census, reported in Emrath (2002).

the explanatory power for the cost of owning, which is not surprising since this variable varies substantially over time but not across states.[15]

The role played by that our proxies for diversity of opinion is explored in columns 3 to 6, where we estimate the empirical counterpart of (25). Specifically, starting with the benchmark regression (27), we add the first difference of the log of our indicators of income inequality, Δi_{kt}, (as a proxy for households' heterogeneous expectations). We also replace the cost of owning with time dummies, γ_t and estimate by OLS the following regression:

$$\Delta p_{kt} = \beta_0 + \beta_1 \Delta\theta_{kt} + \beta_2 \Delta\theta_{kt-1} + \beta_3 \Delta i_{kt} + \gamma_t + \epsilon_{kt}. \tag{28}$$

In line with the prediction of the model, the results show a strong and significant relationship between dispersion in income and the growth rate of our house price index. Our indicators of information dispersion enter positively and significantly, at the one percent level, in all four regressions. Moreover, the impact is sizeable. For example, in column 3, where income dispersion is measured by the Gini coefficient, the estimated coefficient suggests that relative to the state average, a one-percent increase in the growth rate of the Gini coefficient results in a 5 percentage increase in the growth rate of the house price index. To better gauge the economic effect of this result notice that the standard deviation of the growth rate of the Gini coefficient and of the house price index are 0.07 and 0.05, respectively. Therefore the growth rate of income dispersion explain approximately 8 percent of the variation in the growth rate of the house price, relative to the state average. Similar results are obtained in columns 4 to 7, where we use our alternative measures of income inequality, suggesting that the predictions of our model are entirely consistent with the data.

8.2 Alternative specification

8.2.1 Lamont and Stein's specification

The baseline regressions in Table 2, although based on the equilibrium price equations implied by our model, do not control for some patterns of the house price dynamics that prior work has documented to be important. For example, Lamont and Stein (1999), in their study of the house price dynamics in US cities, have shown that house prices (a) exhibit short run movements, (b) respond to contemporaneous income shocks, and (c) exhibit a long run tendency to fundamental reversion. To take these effects into account, Table 3 presents results based on the preferred specification of Lamont and Stein:

$$\Delta p_{kt} = \gamma_0 + \gamma_1 \Delta p_{kt-1} + \gamma_2 \Delta\theta_{kt} + \gamma_3 \left(p_k/\theta_k\right)_{t-1} + \gamma_k + \gamma_t + \epsilon_{kt}$$

where the annual change in house prices is regressed on its lagged value, Δp_{kt-1}, the contemporaneous change in income, $\Delta\theta_{kt}$, the one period lagged ratio of the house price to per-capita income, $\left(p_k/\theta_k\right)_{t-1}$, as well as state, γ_k, and year dummies, γ_t, to remove unobservable state

[15] The most important source of time variation in the owning cost is the 10 year interest rate and the 30 year mortgage rate. The federal state income tax exhibits instead tiny variations over time. Thus the introduction of year effects, to control for economic factors common to all states, reduce the statistical relevance of the state specific cost of owning.

and year specific supply and demand factors. Column 1 in Table 3 shows that these variables explain significantly a large fraction of house price variations.[16]

To replicate the analysis of Lamont and Stein more closely, column 2 also controls for the possibility that in high leveraged states house prices are more sensitive to income variations. This is done by augmenting the specification of column 1 with the interaction between the lagged measure of leverage (the loan-to-value ratio) with the lagged value of the per capita income growth. We find that this term is positive but, in contract to Lamont and Stein, it is not statistically significant. We thus drop the interaction term and include to the baseline regression in column 1 our proxies of information dispersion, i_{kt}, one after the other. The results, reported in columns 3 to 7 are, by and large, consistent with those presented in Table 2. Our proxies of information dispersion continue to explain significantly variations in house price movements across US states, supporting unambiguously the model's prediction that dispersion in information exerts an upward impact on house prices.[17] The economic effect is again important. For example, the estimated coefficient in column 3 suggests that an exogenous increase in the Gini coefficient from the 25^{th} to the 75^{th} percentile (i.e., from 0.08 to 0.11) accelerates the real house price growth by an additional 1.5 percent per year.

8.2.2 The role of positive and negative shocks

An alternative test of our dispersed information story is to check whether house prices respond more to positive rather than negative "news" shocks. In our model, the speculative motive of households is enhanced when they expect better economic conditions to prevail in the future. On the contrary, households prefer to sit out of the market of house "for sale" after receiving negative income shocks. To check the validity of this prediction, we construct an index for positive shocks using, for each state, the difference between the natural logarithm of real per capita income of those counties at the 99^{th} percentile and those at the 50^{th} percentile. Similarly, negative shocks are measured by the difference between the 50^{th} and the 1^{st} percentile. The results are presented in Table 4, using either the baseline specification of Table 2 or the specification of Table 3. In both cases, our estimates conform with the theory's prediction that positive and negative shocks have asymmetric effects on housing prices. In fact, while positive shocks have a large and significant impact on house prices, the importance of negative shocks is marginal and it is always statistically insignificant.

9 Conclusion

In this paper we have examined the implications of heterogenous expectations within a simple model of the housing market, and observed that the degree of information dispersion among market participants affect the equilibrium house prices. The emphasis on household expectations was motivated by the desire to formalize the popular idea that large house price swings occur when current prices depend upon expectations of future price increases.

[16] In unreported regressions we also found, in line with Lamont and Stein, that longer lags of price and income were insignificant once we included the lagged ratio of price to per-capita income. Thus, the parsimonious specification of Table 2 capture well the effects of longer price and income lags.

[17] The results were essentially the same when the first difference of the log of our proxies of imperfect information, as opposed to its level, were used in the regressions.

Our theoretical model — in which agents have the option to rent and buy houses to consume housing services and/or to speculate on future price change — suggests that the equilibrium housing price is higher, the larger the difference in expectations among house market participants. The intuition is that all households face *de facto* a short position in housing. Therefore, those who hold pessimistic expectations about future prices decide to rent to avoid future capital losses, while those who have optimistic expectations decide to buy to speculate on future price increases. The upshot is that the equilibrium price incorporates only the expectations of the optimists and is thus biased upward.

This theoretical prediction holds empirically in a panel data of US states, if dispersion in households income is used as a proxy for dispersion in beliefs. This proxy was motivated by our model's assumption that households' "window on the world" is given by their individual income, which is perturbed by idiosyncratic shocks and thus leads households to form partial views of the economy. Although in line with the logic of our model, this proxy is undoubtedly a crude one. To the best of our knowledge, however, no direct measures of dispersion of beliefs in the housing markets are available to performs a direct test of our theory.

In keeping our model simple, we have abstracted from a number of issues that might play an important role in the development of a more complete model. For example, we have abstracted from the general equilibrium effects of the interest rate. Changes in R affect, however, the analysis directly since the return on the safe asset influences, for a given level of pessimistic expectations, households' desire to rent rather than owning. We have also prevented households from re-trading. An extension of the model that allows for re-trading, as in Stein (1995) or Ortalo-Magne' and Rady (2006), could, however, provide new light on why heterogenous expectations may induce a positive correlation between house prices and housing transactions.

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Appendix I: Linearization

We linearize equations (8) and (9) around the equilibrium with “certainty”, i.e., when $\varepsilon_t^j = 0$, $\eta_t = 0$, $a_t = 0$ and $\nu_t^j = 0\ \forall t$. In this equilibrium $V = H$, because there is no uncertainty and no heterogeneity among households. Denoting with X any variable X_t in the “certainty” equilibrium, the first order conditions (8) and (9), with interior solutions, can be written as

$$V^j = V > 0 \Longrightarrow V = \frac{C}{R}\frac{1}{Q}, \tag{29}$$

$$H^j = H > 0 \Longrightarrow Q = U. \tag{30}$$

Moreover, using equations (6), (10) and the fact that $V = H$,

$$\begin{aligned}\frac{C}{R} &= W - HP\left((1+M) + \frac{1}{R}\right) \\ &= W - VQ. \end{aligned} \tag{31}$$

Thus combining (31) and (29) one obtains

$$V = \frac{W}{2Q}.$$

Under the assumption of fixed housing supply, S, the market clearing condition is,

$$V = S,$$

which implies that the following relationships must hold in a certainty equilibrium,

$$U = Q, \quad Q = \frac{W}{2S}, \quad C = \frac{RW}{2}.$$

Denoting with lower-case letters variables in percent deviation from the equilibrium with certainty, and recalling our definition of user cost,

$$U_t = P_t(1 + M_t) - \frac{P_{t+1}}{R} \tag{32}$$

a linearization of (9) around the certainty equilibrium yields,

$$E_t^j \left[\begin{array}{c} \frac{RP}{C}\left(1+p_t-c_{t+1}^j\right)+\frac{RPM}{C}\left(1+p_t+m_t-c_{t+1}^j\right) \\ -\frac{RQ}{C}\left(1+q_t-c_{t+1}^j\right)-\frac{P}{C}\left(1+p_{t+1}-c_{t+1}^j\right) \end{array} \right] \geq 0.$$

Rearranging,

$$\begin{aligned} E_t^j \left[\begin{array}{c} \frac{RP}{C}p_t+\frac{RPM}{C}\left(p_t+m_t\right)-\frac{RQ}{C}q_t-\frac{P}{C}p_{t+1} \\ -c_{t+1}^j\left(\frac{RP}{C}+\frac{RPM}{C}-\frac{RQ}{C}-\frac{P}{C}\right) \end{array} \right] &\geq 0 \Rightarrow \\ E_t^j\left[RPp_t+RPM\left(p_t+m_t\right)-RQq_t-Pp_{t+1}\right] &\geq 0 \Rightarrow \\ E_t^j\left[p_t+Mp_t+Mm_t-\frac{Q}{P}q_t-\frac{1}{R}p_{t+1}\right] &\geq 0, \end{aligned}$$

we obtain

$$p_t \geq \frac{\kappa}{1+\kappa}q_t-\left(1-\frac{R}{1+\kappa}\right)m_t+\frac{1}{1+\kappa}E_t^j p_{t+1}, \tag{33}$$

where

$$\kappa = RU/P > 0 \quad \text{and} \quad 1-\frac{R}{1+\kappa} > 0 \quad \text{iff } M > 0.$$

Notice also that a linearization of (32) gives

$$\begin{aligned} u_t &= \frac{P}{U}p_t+\frac{PM}{U}\left(p_t+m_t\right)-\frac{P}{RU}p_{t+1} \\ &= \left(1+\frac{1}{\kappa}\right)p_t+\left(1+\frac{1-R}{\kappa}\right)m_t-\frac{1}{\kappa}p_{t+1}. \end{aligned}$$

Therefore, (33) can be rewritten as

$$E_t^j u_t \geq q_t. \tag{34}$$

Since, by assumption, $E_t^1 p_{t+1} > E_t^0 p_{t+1}$, it follows that $E_t^0 u_t > E_t^1 u_t$. Thus, in equilibrium, equation (34) can be written as,

$$E_t^1 u_t = q_t \quad \text{and} \quad h_t^1 > 0, \tag{35}$$

$$E_t^0 u_t > q_t \quad \text{and} \quad h_t^0 = 0. \tag{36}$$

Proceeding as above, a linearization of equation (8), around the certainty equilibrium, gives

$$\begin{aligned} E_t^j \frac{RQ}{C}\left(q_t-c_{t+1}^j\right) &= \frac{A}{V}(2a_t^j-v_t^j) \\ E_t^j \frac{1}{S}\left(q_t-c_{t+1}^j\right) &= \frac{1}{V}(2a_t^j-v_t^j) \end{aligned}$$

which defines the optimal demand of housing services

$$v_t^j = 2a_t^j - q_t + E_t^j c_{t+1}^j. \tag{37}$$

The term $E_t^j c_{t+1}^j$ in (37) is obtained by linearizing the flow of budget constraint (6), that for the two groups of households reads as follows,

$$\begin{aligned} C_{t+1}^1 &= R\left(W_t^1-P_tH_t^1+Q_t\left(H_t^1-V_t^1\right)-M_tP_tH_t^1\right)+P_{t+1}H_t^1, \qquad (38) \\ C_{t+1}^0 &= R\left(W_t^0-Q_tV_t^0\right). \end{aligned}$$

A bit of algebra establishes[18]

$$
\begin{aligned}
E_t^1 c_{t+1}^1 &= 2w_t^1 - v_t^1 - \left(1+\frac{1}{\kappa}\right)p_t + \frac{1}{\kappa}E_t^1 p_{t+1} - \left(1+\frac{1-R}{\kappa}\right)m_t && (39)\\
&= 2w_t^1 - v_t^1 - E_t^1 u_t,\\
E_t^0 c_{t+1}^0 &= 2w_t^0 - v_t^0 - q_t. && (40)
\end{aligned}
$$

Plugging these expressions in (37) and using equation (35) it follows that

$$
\begin{aligned}
v_t^1 &= w_t^1 + a_t^1 - \frac{1}{2}\left(q_t + E_t^1 u_t\right)\\
&= w_t^1 + a_t^1 - q_t,\\
v_t^0 &= w_t^0 + a_t^0 - q_t.
\end{aligned}
$$

Using the market clearing condition,

$$s = \frac{1}{2}h_t^1 \tag{41}$$

and the fact that

$$\frac{1}{2}\left(h_t^1 - v_t^1\right) = \frac{1}{2}v_t^0$$

equation (41) can be written as

$$\frac{1}{2}v_t^0 + \frac{1}{2}v_t^1 = s,$$

from which is it is immediate to pin down the equilibrium rental price,

$$q_t = \theta_t + a_t - s, \tag{42}$$

where

$$\theta_t = \frac{w_t^1 + w_t^0}{2} \quad \text{and} \quad a_t = \frac{a_t^1 + a_t^0}{2}.$$

Finally, inserting (42) into (35) gives,

$$
\begin{aligned}
p_t &= \frac{\kappa}{1+\kappa}\left(\theta_t + a_t - s\right) - \left(1 - \frac{R}{1+\kappa}\right)m_t + \frac{1}{1+\kappa}E_t^1 p_{t+1}\\
&= \frac{\kappa}{1+\kappa}\left(\theta_t + a_t - s\right) - \left(1 - \frac{R}{1+\kappa}\right)m_t + \frac{1}{1+\kappa}\overline{E}_t p_{t+1} + \frac{1}{1+\kappa}\widetilde{E}_t p_{t+1}, && (43)
\end{aligned}
$$

where

$$\overline{E}_t p_{t+1} = \frac{E_t^1 p_{t+1} + E_t^0 p_{t+1}}{2} \quad \text{and} \quad \widetilde{E}_t p_{t+1} = \frac{E_t^1 p_{t+1} - E_t^0 p_{t+1}}{2}.$$

[18] Linearizing (38) yields

$$
\begin{aligned}
E_t^1 c_{t+1}^1 &= \frac{RW}{C}w_t^1 - \frac{RPH}{C}(p_t + h_t^1) + \frac{RQH}{C}(q_t + h_t^1) - \frac{RQV}{C}(q_t + v_t^1)\\
&\quad - \frac{RPHM}{C}(p_t + m_t + h_t^1) + \frac{PH}{C}(E_t^1 p_{t+1} + h_t^1)\\
&= 2w_t^1 - \frac{P}{U}(p_t + h_t^1) + (q_t + h_t^1) - (q_t + v_t^1)\\
&\quad - \frac{MP}{U}(p_t + m_t + h_t^1) + \frac{P}{RU}(E_t^1 p_{t+1} + h_t^1).
\end{aligned}
$$

Rearranging this equation gives (39). Proceeding in a similar way one obtains (40).

Appendix II: Common Information

In the case of common expectations, $E_t^j p_{t+1} = \bar{E}_t p_{t+1}$ and $\tilde{E}_t p_{t+1} = 0$. Therefore, equation (43), shifted one period forward, gives

$$p_{t+1} = \frac{\kappa}{1+\kappa}\left(\theta_{t+1} + a_{t+1} - s\right) - \left(1 - \frac{R}{1+\kappa}\right) m_{t+1} + \frac{1}{1+\kappa}\bar{E}_{t+1} p_{t+2}.$$

Taking expectations on both sides, conditional on time t information, and excluding explosive price paths, a forward iteration of the expression above gives

$$\overline{E}_t p_{t+1} = \frac{\kappa}{1+\kappa}\sum_{\tau=0}^{\infty}\left(\frac{1}{1+\kappa}\right)^{\tau}\overline{E}_t\left(\theta_{t+1+\tau} + a_{t+1+\tau} - s\right) - \left(1 - \frac{R}{1+\kappa}\right)\sum_{\tau=0}^{\infty}\left(\frac{1}{1+\kappa}\right)^{\tau}\overline{E}_t m_{t+1+\tau},$$

Using equations (2), (4) and (5) and the fact that θ_t and a_t are not observable at time t, we have

$$\overline{E}_t\left[\theta_{t+1+\tau} + a_{t+1+\tau} - s\right] = \theta_{t-1} + a_{t-1} - s$$

and

$$\overline{E}_t\left[m_{t+\tau}\right] = m_t.$$

It is therefore immediate to obtain

$$\overline{E}_t p_{t+1} = \left(\theta_{t-1} + a_{t-1} - s\right) - \left(1 + \frac{1-R}{\kappa}\right) m_t. \tag{44}$$

Plugging (44) back into (43) and recalling that $\tilde{E}_t p_{t+1} = 0$, the equilibrium price under common information can then be written as

$$p_t^* = \left(\theta_t + a_t - s\right) - \left(1 + \frac{1-R}{\kappa}\right) m_t + \frac{1}{1+\kappa}\left(\left(\theta_{t-1} - \theta_t\right) + \left(a_{t-1} - a_t\right)\right).$$

Appendix III: Heterogeneous Information

In the presence of heterogeneous expectations, $E_t^j p_{t+1} \neq \bar{E}_t p_{t+1}$ and $\tilde{E}_t p_{t+1} \neq 0$. Shifting equation (43) one period forward

$$p_{t+1} = \frac{\kappa}{1+\kappa}(\theta_{t+1} + a_{t+1} - s) - \left(1 - \frac{R}{1+\kappa}\right) m_{t+1} + \frac{1}{1+\kappa}\overline{E}_{t+1} p_{t+2} + \frac{1}{1+\kappa}\widetilde{E}_{t+1} p_{t+2}$$

and guessing that $\tilde{E}_t\left[p_{t+1}\right] = \lambda i_t$, yields

$$\begin{aligned}
E_t^j p_{t+1} &= \frac{\kappa}{1+\kappa} E_t^j(\theta_{t+1} + a_{t+1} - s) - \left(1 - \frac{R}{1+\kappa}\right) E_t^j m_{t+1} + \frac{1}{1+\kappa} E_t^j \bar{E}_{t+1} p_{t+2} + \frac{\lambda}{1+\kappa} I \\
\overline{E}_t p_{t+1} &= \frac{\kappa}{1+\kappa}\bar{E}_t(\theta_{t+1} + a_{t+1} - s) - \left(1 - \frac{R}{1+\kappa}\right)\bar{E}_t m_{t+1} + \frac{1}{1+\kappa}\bar{E}_t \bar{E}_{t+1} p_{t+2} + \frac{\lambda}{1+\kappa} I, \\
\widetilde{E}_t p_{t+1} &= \frac{\kappa}{1+\kappa}\tilde{E}_t \theta_{t+1} + \frac{1}{1+\kappa}\widetilde{E}_t \bar{E}_{t+1} p_{t+2}
\end{aligned}$$

where the last equality holds because households hold heterogeneous expectations with respect to θ_{t+1} but not with respect to a_{t+1} and m_{t+1}. In the expressions above,

$$I \equiv \int_0^{\infty} x d\Gamma\left(x\right),$$

denotes the average degree of information heterogeneity where Γ is the density of

$$i_t = \left|\varepsilon_t^j - \varepsilon_t^i\right| \quad \text{for} \quad i \neq j.$$

Iterating the expressions above forward and excluding explosive price paths, we obtain:

$$\begin{aligned}
E_t^j p_{t+1} &= E_t^j(\theta_{t+1}+a_{t+1}-s)-\left(1+\frac{1-R}{\kappa}\right)E_t^j m_{t+1}+\frac{\lambda}{\kappa}I, \\
\bar{E}_t p_{t+1} &= \bar{E}_t(\theta_{t+1}+a_{t+1}-s)-\left(1+\frac{1-R}{\kappa}\right)\bar{E}_t m_{t+1}+\frac{\lambda}{\kappa}I, \\
\tilde{E}_t p_{t+1} &= \tilde{E}_t\theta_{t+1}.
\end{aligned}$$

Moreover, using equation (22)

$$E_t^j\theta_{t+1} = E_t^j\theta_t = (1-\lambda)\theta_{t-1} + \lambda w_t^j$$

and equations (4) and (5), it is to see that

$$\begin{aligned}
\bar{E}_t p_{t+1} &= ((1-\lambda)\theta_{t-1}+\lambda\theta_t)+a_{t-1}-s-\left(1+\frac{1-R}{\kappa}\right)m_t+\frac{\lambda}{\kappa}I, \\
&= (\theta_{t-1}+a_{t-1}-s)-\left(1+\frac{1-R}{\kappa}\right)m_t+\lambda\left(\theta_t-\theta_{t-1}\right)+\frac{\lambda}{\kappa}I, \\
\tilde{E}_t p_{t+1} &= \lambda i_t.
\end{aligned}$$

so that $\tilde{E}_t\left[p_{t+1}\right] = \lambda i_t$ as claimed. Plugging $\bar{E}_t p_{t+1}$ and $\tilde{E}_t p_{t+1}$ in (43), the equilibrium house prices can be written as

$$\begin{aligned}
p_t &= (\theta_t+a_t-s)-\left(1+\frac{1-R}{\kappa}\right)m_t+\frac{1}{1+\kappa}\left((\theta_{t-1}-\theta_t)+(a_{t-1}-a_t)\right) \\
&\quad +\frac{\lambda}{1+\kappa}\left(\theta_t-\theta_{t-1}\right)+\frac{\lambda}{\kappa(1+\kappa)}I+\frac{\lambda}{1+\kappa}i_t. \\
&= p_t^*+\frac{\lambda}{1+\kappa}\left(\theta_t-\theta_{t-1}\right)+\frac{\lambda}{\kappa(1+\kappa)}I+\frac{\lambda}{1+\kappa}i_t.
\end{aligned}$$

Appendix IV: Learning from the equilibrium price

In this appendix we provide a solution to the signal extraction problem when households condition on the price to learn the unknown fundamental, θ_t. As explained in Section 6, the inference problem is involved since the equilibrium price in the presence of heterogenous information is not normally distributed. To characterize this non-standard signal extraction problem we assume that the distribution of the preference shock μ_t, is such that sum of i_t and μ_t follows a normal distribution. This assumption enables us to recover a Gaussian distribution for the equilibrium price and allows us to apply standard linear filtering techniques.

We proceed in three steps. First, we define the exact distribution for i_t. Next, we determine the form of the distribution of μ_t that makes the equilibrium price normally distributed. Finally, using a method of undetermined coefficients we characterize the inference problem for θ_t and the resulting equilibrium price.

The distribution of $i = |\varepsilon^i - \varepsilon^j|$ for $i \neq j$

Consider two independent random variables, ε^i and ε^j, distributed normally with zero mean and equal variance σ_ε^2. Define,

$$\tilde{\varepsilon} = \varepsilon^j - \varepsilon^i \sim \mathcal{N}(0, 2\sigma_\varepsilon^2).$$

The cumulative distribution function of $i = |\tilde{\varepsilon}|$ is

$$F_i(y) = \Pr\left(i = |\tilde{\varepsilon}| \leq y\right) = 2\int_0^y \frac{1}{\sqrt{2\pi}\sqrt{2}\sigma_\varepsilon} \exp\left(-\frac{1}{2}\frac{z^2}{2\sigma_\varepsilon^2}\right) dz,$$

and the associated density,

$$f_i\left(y\right) = \begin{cases} \frac{\partial F_i(y)}{\partial y} = \frac{2}{\sqrt{2\pi}\sqrt{2}\sigma_\varepsilon} \exp\left(-\frac{1}{2}\frac{y^2}{2\sigma_\varepsilon^2}\right) & \text{if } y \geq 0 \\ 0 & \text{otherwise} \end{cases}. \tag{45}$$

Denote with $\bar{\imath}$, the mean of i,

$$\bar{\imath} = \int_0^\infty y f_i\left(y\right) dy.$$

The distribution of the aggregate preference shock, μ.

We wish to find the distribution of a random variable, μ, with zero mean and variance σ_μ^2, such that

$$\mu + i \sim \mathcal{N}(\bar{\imath}, \;\; \sigma_\mu^2 + \sigma_i^2).$$

The cumulative function of $\mu + i$ is

$$F_{\mu+i}\left(y\right) = \Pr\left(\mu + i \leq y\right) = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{y-\mu} f_i\left(i\right) di\right) f_\mu\left(\mu\right) d\mu,$$

where f_μ is the density of μ and f_i is defined in (45). Differentiating $F_{\mu+i}\left(y\right)$ w.r.t. y yields the probability density of $\mu + i$,

$$f_{\mu+i}\left(y\right) = \int_{-\infty}^{\infty} f_i\left(y - a\right) f_\mu\left(\mu\right) d\mu.$$

Since by assumption $\mu + i$ follows a normal distribution, it must be

$$f_{\mu+i}\left(y\right) = \frac{1}{\sqrt{2\pi}\sqrt{\sigma_\mu^2 + \sigma_i^2}} \exp\left(-\frac{1}{2}\frac{\left(y - \bar{\imath}\right)^2}{\sigma_\mu^2 + \sigma_i^2}\right).$$

Therefore the density $f_\mu\left(\mu\right)$ is recovered by solving the following integral,

$$\int_{-\infty}^{\infty} f_i\left(y - \mu\right) f_\mu\left(\mu\right) d\mu = \frac{1}{\sqrt{2\pi}\sqrt{\sigma_\mu^2 + \sigma_i^2}} \exp\left(-\frac{1}{2}\frac{\left(y - \bar{\imath}\right)^2}{\sigma_\mu^2 + \sigma_i^2}\right).$$

Lemma 1

The correlation coefficient between ε^j and $i \equiv \left|\varepsilon^j - \varepsilon^i\right|$ is zero.
Proof.

$$\begin{aligned} Cov\left(\varepsilon^j, \left|\varepsilon^j - \varepsilon^i\right|\right) &= Cov\left(\varepsilon^j, \varepsilon^j - \varepsilon^i\right) \Pr\left(\varepsilon^j > \varepsilon^i\right) + Cov\left(\varepsilon^j, -(\varepsilon^j - \varepsilon^i)\right) \Pr\left(\varepsilon^j < \varepsilon^i\right) \\ &= Cov\left(\varepsilon^j, \tilde{\varepsilon}\right) \Pr\left(\varepsilon^j > \varepsilon^i\right) - Cov\left(\varepsilon^j, \tilde{\varepsilon}\right) \Pr\left(\varepsilon^j < \varepsilon^i\right) \\ &= Cov\left(\varepsilon^j, \tilde{\varepsilon}\right) \left[\Pr\left(\varepsilon^j > \varepsilon^i\right) - \Pr\left(\varepsilon^j < \varepsilon^i\right)\right] = 0 \end{aligned}$$

The last equation holds because ε^j and ε^i are independent and identically distributed normal random variable with zero mean and equal variance, so that $\Pr\left(\varepsilon^j > \varepsilon^i\right) - \Pr\left(\varepsilon^j < \varepsilon^i\right) = 0.$ ■

The method of undetermined coefficients

Starting from equation (25), we guess that the equilibrium price is a linear function of the current unobservable fundamental θ_t and preference shock, a_t, the current observable maintenance cost, m_t, and the difference in households private signals, i_t,

$$p_t = b_0 + b_\theta \theta_{t-1} + b_\eta \eta_t + b_a a_{t-1} + b_\mu \mu_t + b_m m_t + b_i i_t. \tag{46}$$

where b_0, b_θ, b_η, b_a, b_μ, b_m and b_i are undetermined coefficients. It is convenient to rewrite equation (46) as

$$p_t = b_\eta \eta_t + b_\mu \mu_t + b_i i_t + X_t, \tag{47}$$

where

$$X_t \equiv b_0 + b_\theta \theta_{t-1} + b_a a_{t-1} + b_m m_t,$$

is non-stochastic. Defining,

$$\widehat{p}_t \equiv \frac{p_t - X_t}{b_\eta},$$

equation (47) can be written as

$$\widehat{p}_t = \eta_t + \phi_t,$$

where,

$$\phi_t = \frac{b_\mu}{b_\eta}\mu_t + \frac{b_i}{b_\eta} i_t. \tag{48}$$

Under the assumption made on the distribution of μ_t, ϕ_t is normally distributed,

$$\phi_t \sim \mathcal{N}\left(\frac{b_i}{b_\eta}\bar{\imath},\quad \left(\frac{b_\mu}{b_\eta}\right)^2 \sigma_\mu^2 + \left(\frac{b_i}{b_\eta}\right)^2 \sigma_i^2\right)$$

and, as a consequence $\widehat{p}_t$ is also normally distributed,

$$\widehat{p}_t \sim \mathcal{N}\left(\frac{b_i}{b_\eta}\bar{\imath},\quad \sigma_\eta^2 + \frac{b_\mu^2 \sigma_\mu^2 + b_i^2 \sigma_i^2}{b_\eta^2}\right). \tag{49}$$

The inference problem

Household j estimates the unknown fundamental θ_t by solving a standard filtering problem, based on the normally distributed (a) private signal, w_t^j, (b) exogenous public signal, θ_{t-1}, and (c) endogenous public signal, $\widehat{p}_t$. Recalling that

$$\begin{aligned} \theta_t &= \theta_{t-1} + \eta_t, \\ w_t^j &= \theta_t + \varepsilon_t^j, \\ \widehat{p}_t &= \eta_t + \phi_t, \end{aligned}$$

and using (49) and Lemma 1, the log-likelihood function can be written as

$$L = -\frac{1}{2\sigma_\eta^2}\left(\theta_{t-1} - E_t^j \theta_t\right)^2 - \frac{1}{2\sigma_\varepsilon^2}\left(w_t^j - E_t^j \theta_t\right)^2 - \frac{1}{2\sigma_\phi^2}\left(\widehat{p}_t - E_t^j \eta_t\right)^2.$$

Thus, the optimal filtering solves the following first order condition,

$$-\frac{1}{\sigma_\eta^2}\left(-E_t^j\eta_t\right)+\frac{1}{\sigma_\varepsilon^2}\left(w_t^j-\theta_{t-1}-E_t^j\eta_t\right)+\frac{1}{\sigma_\phi^2}\left(\widehat{p}_t-E_t^j\eta_t\right)=0,$$

or,

$$E_t^j\eta_t=\frac{\sigma_\eta^2\sigma_\phi^2\left(w_t^j-\theta_{t-1}\right)+\sigma_\eta^2\sigma_\varepsilon^2\widehat{p}_t}{\sigma_\varepsilon^2\sigma_\phi^2+\sigma_\eta^2\sigma_\phi^2+\sigma_\eta^2\sigma_\varepsilon^2}.$$

The best linear estimate of θ_t is therefore,

$$E_t^j\theta_t=(\pi_1+\pi_3)\,\theta_{t-1}+\pi_2 w_t^j+\pi_3\widehat{p}_t, \tag{50}$$

where

$$\pi_1 = \frac{\sigma_\varepsilon^2\sigma_\phi^2}{\sigma_\varepsilon^2\sigma_\phi^2+\sigma_\eta^2\sigma_\phi^2+\sigma_\eta^2\sigma_\varepsilon^2} \tag{51}$$

$$\pi_2 = \frac{\sigma_\eta^2\sigma_\phi^2}{\sigma_\varepsilon^2\sigma_\phi^2+\sigma_\eta^2\sigma_\phi^2+\sigma_\eta^2\sigma_\varepsilon^2}, \tag{52}$$

$$\pi_3 = \frac{\sigma_\eta^2\sigma_\varepsilon^2}{\sigma_\varepsilon^2\sigma_\phi^2+\sigma_\eta^2\sigma_\phi^2+\sigma_\eta^2\sigma_\varepsilon^2}. \tag{53}$$

Notice that if $\sigma_\phi^2 \to \infty$ (for example, because $\sigma_\mu^2 \to \infty$, i.e., the preference shock has a very large variance) then

$$\pi_1 \to \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_\eta^2}=1-\lambda, \quad \pi_2 \to \frac{\sigma_\eta^2}{\sigma_\varepsilon^2+\sigma_\eta^2}=\lambda \quad \text{and} \quad \pi_3 \to 0.$$

In other words, households have nothing to learn from the equilibrium price and the weights used for inferring the unobservable aggregate fundamental are the same as in Section 5. Notice also that away from this limiting case the following is always true

$$\lambda > \pi_2 > \pi_3.$$

The equilibrium price

To solve for the equilibrium price we follow the same steps as in Appendix III. By guessing that $\tilde{E}_t p_{t+1} = \pi_2 i_t$, we have

$$\begin{aligned}
E_t^j p_{t+1} &= E_t^j(\theta_{t+1}+a_{t+1}-s)-\left(1+\frac{1-R}{\kappa}\right)E_t^j m_{t+1}+\frac{\pi_2}{\kappa}I,\\
\bar{E}_t p_{t+1} &= \bar{E}_t(\theta_{t+1}+a_{t+1}-s)-\left(1+\frac{1-R}{\kappa}\right)\bar{E}_t m_{t+1}+\frac{\pi_2 I}{\kappa},\\
\tilde{E}_t p_{t+1} &= \tilde{E}_t\theta_{t+1}.
\end{aligned}$$

Moreover, using (50) the last two equations can be written as :

$$\begin{aligned}
\bar{E}_t p_{t+1} &= (\theta_{t-1}+a_{t-1}-s)-\left(1+\frac{1-R}{\kappa}\right)m_t+\pi_2\eta_t+\pi_3\widehat{p}_t+\frac{\pi_2}{\kappa}I,\\
\tilde{E}_t p_{t+1} &= \pi_2 i_t,
\end{aligned}$$

confirming the claim that $\tilde{E}_t p_{t+1} = \pi_2 i_t$. Inserting now $\overline{E}_t p_{t+1}$ and $\widetilde{E}_t p_{t+1}$ in (18), the equilibrium price becomes

$$\begin{aligned} p_t &= \frac{\kappa}{1+\kappa}\left((\theta_t + a_t - s) - \left(1+\frac{1-R}{\kappa}\right) m_t\right) \\ &\quad + \frac{1}{1+\kappa}\left(\begin{array}{c} (\theta_{t-1} + a_{t-1} - s) - \left(1+\frac{1-R}{\kappa}\right) m_t + \pi_2 \eta_t + \frac{\pi_2 I}{\kappa} \\ +\pi_3\left(\frac{p_t - b_0 - b_\theta \theta_{t-1} - b_a a_{t-1} - b_m m_t}{b_\eta}\right) \end{array}\right) + \frac{\pi_2}{1+\kappa} i_t, \end{aligned}$$

from which it follows,

$$\begin{aligned} p_t &= \left(1 - \frac{\pi_3}{1+\kappa}\frac{b_\theta}{b_\eta}\right)\theta_{t-1} + \left(1 - \frac{\pi_3}{1+\kappa}\frac{b_a}{b_\eta}\right) a_{t-1} - s - \left(\left(1 + \frac{1-R}{\kappa}\right) + \frac{\pi_3}{1+\kappa}\frac{b_m}{b_\eta}\right) m_t \\ &\quad + \frac{\kappa+\pi_2}{1+\kappa}\eta_t + \frac{\kappa}{1+\kappa}\mu_t + \frac{1}{1+\kappa}\left(-\pi_3 \frac{b_0}{b_\eta} + \frac{\pi_2 I}{\kappa}\right) + \frac{\pi_3}{1+\kappa}\frac{1}{b_\eta} p_t + \frac{\pi_2}{1+\kappa} i_t, \end{aligned}$$

or,

$$\begin{aligned} p_t &= \frac{\frac{1}{1+\kappa}\left(-\pi_3 \frac{b_0}{b_\eta} + \frac{\pi_2 I}{\kappa}\right) - s}{1 - \frac{\pi_3}{1+\kappa}\frac{1}{b_\eta}} + \frac{1 - \frac{\pi_3}{1+\kappa}\frac{b_\theta}{b_\eta}}{1 - \frac{\pi_3}{1+\kappa}\frac{1}{b_\eta}}\theta_{t-1} + \frac{1 - \frac{\pi_3}{1+\kappa}\frac{b_a}{b_\eta}}{1 - \frac{\pi_3}{1+\kappa}\frac{1}{b_\eta}} a_{t-1} \\ &\quad - \frac{\left(1 + \frac{1-R}{\kappa}\right) + \frac{\pi_3}{1+\kappa}\frac{b_m}{b_\eta}}{1 - \frac{\pi_3}{1+\kappa}\frac{1}{b_\eta}} m_t + \frac{\frac{\kappa+\pi_2}{1+\kappa}}{1 - \frac{\pi_3}{1+\kappa}\frac{1}{b_\eta}}\eta_t + \frac{\frac{\kappa}{1+\kappa}}{1 - \frac{\pi_3}{1+\kappa}\frac{1}{b_\eta}}\mu_t + \frac{\frac{\pi_2}{1+\kappa}}{1 - \frac{\pi_3}{1+\kappa}\frac{1}{b_\eta}} i_t. \end{aligned}$$

The undetermined coefficients can therefore be written as,

$$\begin{array}{ll} b_0 = \frac{\frac{1}{1+\kappa}\left(-\pi_3 \frac{b_0}{b_\eta} + \frac{\pi_2 I}{\kappa}\right) - s}{1 - \frac{\pi_3}{1+\kappa}\frac{1}{b_\eta}} & \Longrightarrow b_0 = \frac{\pi_2}{\kappa(1+\kappa)} I - s \\ b_\theta = \frac{1 - \frac{\pi_3}{1+\kappa}\frac{b_\theta}{b_\eta}}{1 - \frac{\pi_3}{1+\kappa}\frac{1}{b_\eta}} & \Longrightarrow b_\theta = 1 \\ b_a = \frac{1 - \frac{\pi_3}{1+\kappa}\frac{b_a}{b_\eta}}{1 - \frac{\pi_3}{1+\kappa}\frac{1}{b_\eta}} & \Longrightarrow b_a = 1 \\ b_m = -\frac{\left(1 + \frac{1-R}{\kappa}\right) + \frac{\pi_3}{1+\kappa}\frac{b_m}{b_\eta}}{1 - \frac{\pi_3}{1+\kappa}\frac{1}{b_\eta}} & \Longrightarrow b_m = -\left(1 + \frac{1-R}{\kappa}\right) \\ b_\eta = \frac{\frac{\kappa+\pi_2}{1+\kappa}}{1 - \frac{\pi_3}{1+\kappa}\frac{1}{b_\eta}} & \Longrightarrow b_\eta = \frac{\kappa}{1+\kappa}\left(1 + \frac{\pi_2+\pi_3}{\kappa}\right) \\ b_\mu = \frac{\frac{\kappa}{1+\kappa}}{1 - \frac{\pi_3}{1+\kappa}\frac{1}{b_\eta}} & \Longrightarrow b_\mu = \frac{\kappa}{1+\kappa}\left(1 + \frac{\pi_3}{\kappa+\pi_2}\right) \\ b_i = \frac{\frac{\pi_2}{1+\kappa}}{1 - \frac{\pi_3}{1+\kappa}\frac{1}{b_\eta}} & \Longrightarrow b_i = \frac{\pi_2}{1+\kappa}\left(1 + \frac{\pi_3}{\kappa+\pi_2}\right), \end{array}$$

and the equilibrium price as,

$$\begin{aligned} p_t &= (\theta_{t-1} + a_{t-1} - s) - \left(1 - \frac{1-R}{\kappa}\right) m_t \\ &\quad + \frac{\kappa}{1+\kappa}\left(1 + \frac{\pi_2+\pi_3}{\kappa}\right)\eta_t + \frac{\kappa}{1+\kappa}\left(1 + \frac{\pi_3}{\kappa+\pi_2}\right)\mu_t + \frac{\pi_2}{1+\kappa}\left(1 + \frac{\pi_3}{\kappa+\pi_2}\right) i_t \\ &\quad + \frac{\pi_2}{\kappa(1+\kappa)} I \end{aligned}$$

or, after some manipulation:

$$p_t = p_t^* + \frac{\pi_2}{\lambda}\Upsilon_t + \pi_3 \Phi_t.$$

As in section 4 and 5,

$$p_t^* = (\theta_t + a_t - s) - \left(1 - \frac{1-R}{\kappa}\right) m_t + \frac{1}{1+\kappa}\left((\theta_{t-1} - \theta_t) + (a_{t-1} - a_t)\right)$$

denotes the fundamental price, and

$$\Upsilon_t \equiv \frac{\lambda}{1+\kappa}\left(\theta_t - \theta_{t-1}\right) + \frac{\lambda}{1+\kappa} i_t + \frac{\lambda}{\kappa(1+\kappa)} I$$

measures the degree of dispersion in beliefs. The new term,

$$\Phi_t \equiv \frac{1}{1+\kappa}\left(\theta_t - \theta_{t-1}\right) + \frac{\kappa}{(1+\kappa)(\kappa+\pi_2)}\left(a_t - a_{t-1}\right) + \frac{\pi_2}{(1+\kappa)(\kappa+\pi_2)} i_t,$$

captures instead the degree of magnification of shocks induced by the the process of learning from price.

Solving for b_η, b_μ and b_i.

A closer look at the variance of ϕ_t,

$$\sigma_\phi^2 = \left(\frac{b_\mu}{b_\eta}\right)^2 \sigma_\mu^2 + \left(\frac{b_i}{b_\eta}\right)^2 \sigma_i^2 \tag{54}$$

suggests that π_1, π_2 and π_3, which are functions of σ_ϕ^2, depend in turn on b_η, b_μ and b_i. To pin down these undetermined coefficients it is thus necessary to use equations (52), (53) and (54). This leads to

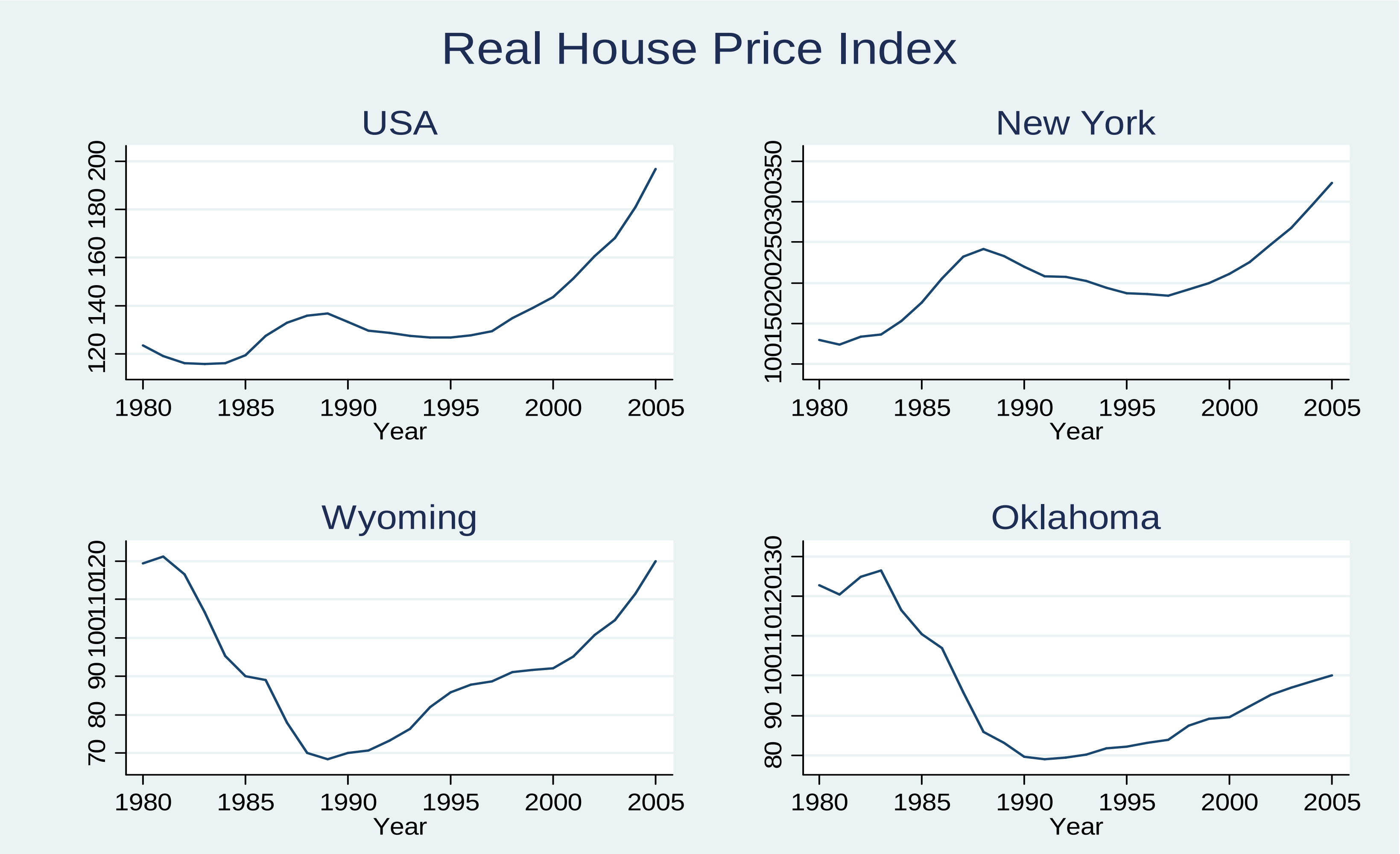
$$\begin{aligned} b_\eta &= \frac{\kappa}{1+\kappa}\left(1 + \frac{\pi_2 + \pi_3}{\kappa}\right) \\ &= \frac{\kappa}{1+\kappa} + \frac{1}{1+\kappa}\left(\frac{\sigma_\eta^2\left(\frac{b_\mu^2\sigma_\mu^2 + b_i^2\sigma_i^2}{b_\eta^2}\right) + \sigma_\eta^2\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2 + \sigma_\eta^2\right)\left(\frac{b_\mu^2\sigma_\mu^2 + b_i^2\sigma_i^2}{b_\eta^2}\right) + \sigma_\eta^2\sigma_\varepsilon^2}\right), \end{aligned}$$

$$\frac{b_i}{b_\mu} = \frac{\pi_2}{\kappa} = \frac{1}{\kappa}\left(\frac{\sigma_\eta^2\left(\frac{b_\mu^2\sigma_\mu^2 + b_i^2\sigma_i^2}{b_\eta^2}\right)}{\left(\sigma_\varepsilon^2 + \sigma_\eta^2\right)\left(\frac{b_\mu^2\sigma_\mu^2 + b_i^2\sigma_i^2}{b_\eta^2}\right) + \sigma_\eta^2\sigma_\varepsilon^2}\right),$$

and

$$b_\eta = b_\mu + b_i,$$

which define a system of three equations in the three unknowns, b_η, b_μ and b_i. Unfortunately, this system of equations does not admit closed-form solutions. However, numerical values can be easily computed.



Source: Office of Federal Housing Enterprise Oversight and Bureau of Labor Statistics

Figure 2: Equilibrium price and the preference noise σ_μ

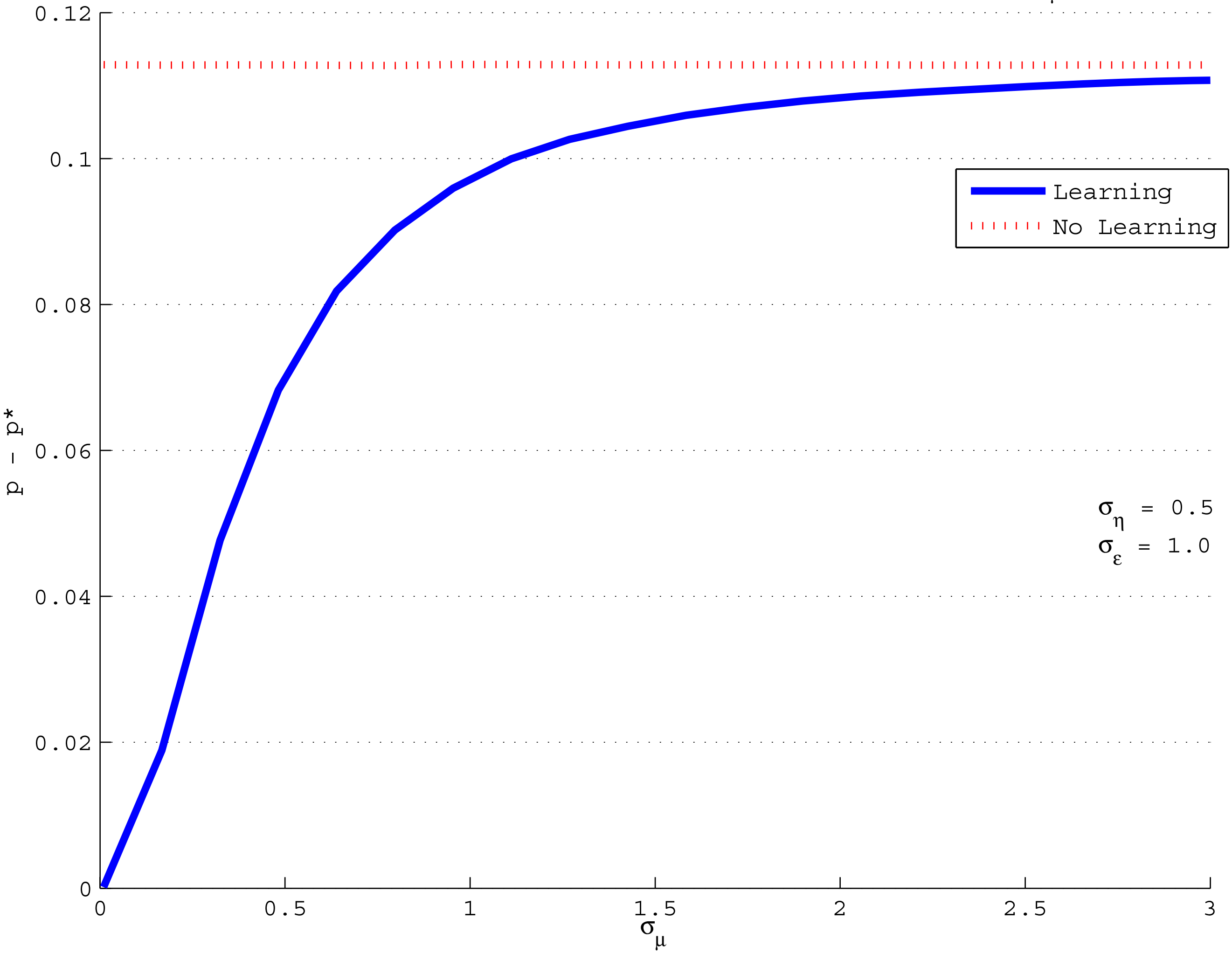
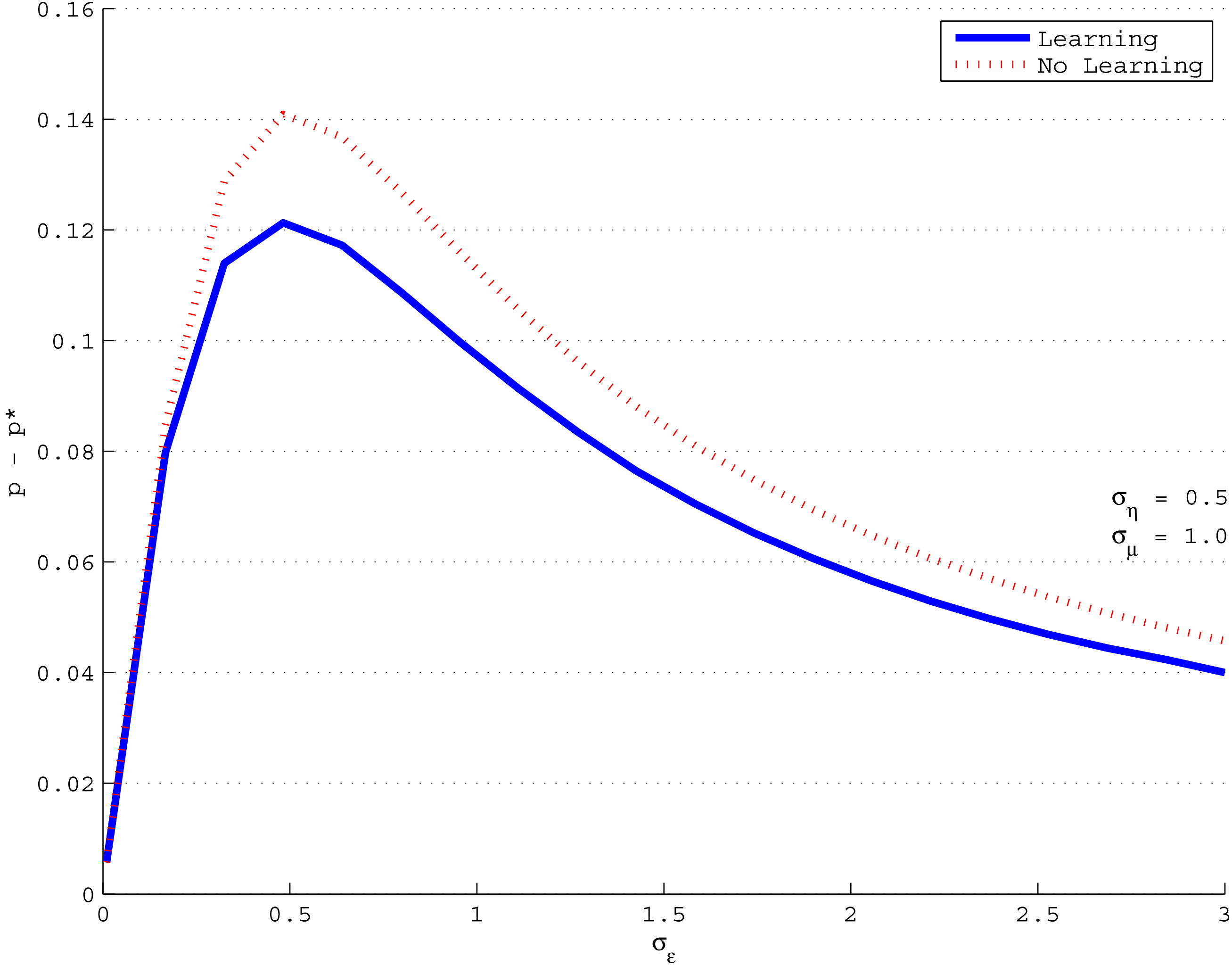


Figure 3: Equilibrium price and the idiosyncratic shock σ_ε



Tab 1. Summary Statistics

Variable	Obs	Mean	Standard Deviation	Minimum	Maximum
Dp(t)	1175	.0129	.0493	-.2717	.2248
Dθ(t)	1175	.0148	.0221	-.1270	.1701
m(t)	1222	.1421	.0392	.0647	.2487
lvr	1127	.7677	.0460	.2570	.9669
gini(t)	1222	.0962	.0229	.0434	.1848
cov(t)	1222	.1870	.0508	.0824	.3771
sdl(t)	1222	.1766	.0420	.0812	.3282
mld(t)	1222	.0165	.0082	.002	.0548
$(99\text{-}1)^{th}$	1221	.8494	.2569	.2567	1.953
$(99\text{-}50)^{th}$	1221	.4885	.1893	.1037	1.271
$(1\text{-}50)^{th}$	1221	-.3608	.1671	-1.627	-.086

Dp(t) is the log dif of the real house price index, Dθ(t) the log dif of the real per capita income, m(t) the user cost, lvr(t) the loan to value ratio, gini(t) the gini coefficient cov(t) the coefficient of variation, sdl(t) the standard deviation of logs, mld(t) the mean log deviation, (99-1)th the difference between the 99th and 1st percentile of the log of per capita income distribution. Statistics shown are for annual observations pooled across 47 US states for the 1980-2005 period.

Tab 2. The role of dispersion in information

Dependent variable Dp(t)

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Dθ(t)	.5326	.7116	.7188	.7169	.7247	.7222	.7238
	(7.49)	(8.52)	(8.86)	(8.87)	(9.11)	(9.04)	(9.14)
Dθ(t-1)	.4201	.6796	.6617	.6656	.6621	.6625	.6782
	(3.56)	(3.57)	(3.46)	(3.48)	(3.46)	(3.46)	(2.37)
Dm(t)	-1.123	.0188					
	(5.80)	(0.01)					
Dgini(t)			.0549				
			(2.54)				
Dcov(t)				.0481			
				(2.40)			
Dsdl(t)					.050		
					(3.06)		
Dmld(t)						.025	
						(2.88)	
$D(99\text{-}1)^{th}$							.0283
							(2.37)
Fixed Effects	no	no	no	no	no	no	no
Year Effects	no	yes	yes	yes	yes	yes	yes
Obs	1175	1128	1128	1128	1128	1128	1128
R2	0.14	0.41	0.41	0.41	0.41	0.41	0.41

The dependent variable, Dp(t), is the log change of the real house price index at time t. Dθ(t) is the log difference of the real per capita income, Dm(t) the first difference in the cost of owning, Dgini(t) the first difference of the log gini coefficient, Dcov(t) the first difference of the log coefficient of variation, Dsdl(t) the first difference of the log standard deviation of log income, Dmld(t) the first difference of the log mean log deviation. D(99-1)th the first difference of the 99^{th} minus the 1^{st} percentile of the log of per capita income distribution. Estimation is by OLS for the period 1980-2005. Robust t-statistics are in parenthesis.

Tab 3. The role of dispersion in information
using the specification of Lamont and Stein (1999)

Dependent variable Dp(t)

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Dp(t-1)	.6263	.6023	.6245	.6248	.6231	.6236	.6552
	(12.08)	(11.17)	(12.25)	(12.30)	(12.14)	(12.14)	(12.25)
$D\theta(t)$	.3399	.3539	.3384	.3338	.3473	.3423	.3510
	(5.93)	(5.94)	(6.15)	(6.07)	(6.26)	(6.26)	(6.13)
$(p/\theta)(t-1)$	-.1134	-.1251	-.1278	-.1237	-.1246	-.1248	-.1174
	(9.01)	(10.19)	(9.82)	(9.71)	(9.71)	(9.71)	(9.33)
$D\theta(t-1)$*lvr(t-1)		.7170					
		(0.22)					
gini(t)			.0499				
			(3.33)				
cov(t)				.0393			
				(2.79)			
sdl(t)					.0431		
					(3.18)		
mld(t)						.0291	
						(3.12)	
$(99-1)^{th}$							.0255
							(1.84)
Fixed Effects	yes	yes	yes	yes	yes	yes	yes
Year Effects	yes	yes	yes	yes	yes	yes	yes
Obs	1128	1128	1128	1128	1128	1128	1128
R2	0.65	0.64	0.66	0.66	0.66	0.66	0.65

The dependent variable, Dp(t), is the log change of the real house price index at time t. $D\theta(t)$ is the log diff. of the real per capita income, $(p/\theta)(t-1)$ the lagged ratio of real price and real per capita income. $D\theta(t-1)$*lvr(t-1) is the interaction of the once lagged loan-to-value-ratio and the lagged value of real per capita income growth, gini(t) is the log gini coefficient, cov(t) the log coefficient of variation, sdl(t) the log standard deviation of log income, mld(t) the log mean log deviation, (99-1)th the first difference of the 99th and the 1st percentile of the log of per capita income distribution. Estimation is by fixed effects for the period 1980-2005. Robust t-statistics are in parenthesis

Tab 4. The role of dispersion in information

Dependent variable Dp(t)

Variable	(1)	(2)
Dp(t-1)		.6272
		(12.73)
Dθ(t)	.7136	.3380
	(8.61)	(6.06)
(p/θ)(t-1)		-.1186
		(9.44)
Dθ(t-1)	.6842	
	(3.66)	
D(99-50)th	.0471	
	(1.68)	
D(1-50)th	-.030	
	(0.63)	
(99-50)th		.0421
		(1.70)
(1-50)th		-.0146
		(1.21)
Fixed Effects	no	yes
Year Effects	yes	yes
Obs	1127	1128
R2	0.41	0.65

See notes to Table 1, 2 and 3 for the definition of variables. In column (1) estimation is by OLS. In column (2) estimation is by fixed effects. The sample period is 1980-2005. Robust t-statistics are in parenthesis