Loan Securitization

and the

Monetary Transmission Mechanism

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PRELIMINARY

Abstract

We examine the monetary transmission mechanism in a DSGE model with asymmetric information across financial intermediaries. Adverse selection results in a time-varying market share of individually risk-priced loans held by banks, and of securitized loans held by the secondary market. This financial market structure generates a discrete interest rate spread across the prime and subprime segment of the market, and captures some essential features of the US mortgage market. Monetary policy affects the subprime-prime loan rate spread, the distribution of interest rates across loans with different default risk, and the degree of loan securitization. We analyze the impact of conventional monetary policies and credit market policies in response to an increase in default risk, and discuss its welfare implications. In our model the impact of default risk can be interpreted as a price distortion, which is smaller the larger the degree of securitization.

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1 Introduction

In most models used in monetary policy analysis the financial intermediation sector plays no role. There is a well-established literature of DSGE models deviating from the assumption of frictionless financial markets. These approaches allow for a spread between the interest paid by borrowers and received by the household sector, and build on models of lending embedding an agency problem where firms have complete information but lack funds to finance projects, and lenders lack information but have an elastic supply of funds.[1]

In this paper we study the implications for the business cycle and monetary policy of loan securitization in a credit market with imperfect information. Information asymmetries lead to adverse selection between financial intermediaries, resulting in endogenous volatility of the share of loans which are securitized and priced as a loan pool, and the share of loans which are risk-priced. The model generates an equilibrium risk-profile of interest rates, and a prime and subprime segment of the financial market with a discrete interest rate spread across the two segments. We use our model to discuss the amplification mechanism generated by financial market imperfections over the business cycle, the behaviour of spreads and interest rates across different levels of risk, the impact of securitization, and its policy and welfare implications. Because default risk plays an explicit role, we can analyze the impact of disturbances originating in the financial market, and the options available to policymaker in response.

Our setup is necessarily stylized, having to describe an economy with heterogeneous default rate across loans, but has the advantage of providing a useful interpretation of the distortion generated by defaults and (lack of) securitization as a price distortion, which we can measure both for the aggregate economy and for any given pool of loans.

Our model of financial intermediation is closely related to partial equilibrium models of mortgage markets in Heuson, Passmore, and Sparks (2001) and Crews Cutts and Van Order (2005),

[1] A short selection of recent work includes Carlstrom, Fuerst and Paustian (2009), Christiano, Motto and Rostagno (2009), Curdia and Woodford (2008), De Fiore and Tristani, (2007), Gertler and Karadi (2009). Bernanke and Gertler (1989) and Bernanke, Gertler and Gilchrist (1999) model the so-called financial accelerator, where borrowers are not rationed but spreads depend on the leverage ratio. Borrowing constraints play instead a central role in another strand of literature, including the work of Kiyotaki and Moore (1997) and Iacoviello (2005). Stiglitz and Weiss (1981) discuss adverse selection in the credit market, a mechanism central to our analysis.

and to new Keynesian models with nominal price rigidity. It can be interpreted as follows. Uncollateralized loans are issued to borrowers who have to finance purchases in advance, with loans being heterogeneous with respect to the default probability. Underwriters sell loans to the highest bidder in the financial sector. Loans are either risk-priced, if held on the balance sheet of banks, or priced as part of an heterogeneous pool, if held on the balance sheet of secondary market investors. Banks incur higher cost of issuing credit relative to the secondary market, but have better information on the borrowers. The secondary market is selected against by the underwriters, and always ends up with a portfolio including the riskiest loans, all identically priced. The model can also be interpreted as describing an economy where banks have an informational advantage over the secondary market, underwrite all loans and then either hold on to the investment, or sell its payoff as a pass-through security to secondary market investors. Securitization happens because some financial intermediaries have a cost-advantage in underwriting loans; thus it is welfare-enhancing. The cost advantage allows overcoming the adverse selection problem for the financial intermediaries with an information disadvantage.

This setup is consistent with the working of the US residential mortgage market, where over the last twenty years the fraction of mortgages sold into the secondary market has increased substantially, reaching about 50% in the 1990s, and has been high but fluctuating ever since. Indeed the originate-to-distribute model of lending has been criticized by some economists as contributing to the severity of the recent credit-market crisis in the US (see Basel committee on Banking Supervision, 2008, and Gorton, 2008). A related development is the increasing share of subprime mortgages over the total flow of new originations. US subprime originations amounted to $35 billion in 1993, or 5% of the overall mortgage market. In 2000, 2003 and 2006 the subprime share in total residential mortgage originations was respectively 13.2%, 8.7% and 20.1%. Of the subprime originations in 2006, 80.5% were securitized. As in our model, US data show that the loan pricing differential is much higher between similar loans in different market segments than between slightly different loans in the same segment, a feature which in our model we label the *subprime-jump.*

Because securitization in our model is the result of a relative screening cost advantage of certain financial intermediaries, it reduces average equilibrium interest rates. For the particular parameterization that we consider, it lowers the average interest rate paid by borrowers by more than 100 basis points. Moreover, this decline in steady-state interest rates is bigger for high-risk loans than for low-risk ones. That is, subprime borrowers are the ones that gain the most from securitization.

Interest rate spreads in our model act as a distortionary tax on the goods the purchase of which is funded by loans in the financial market. We call this the *default wedge*. Since these spreads are determined by the endogenous segmentation of the financial market into a non-securitized prime loans segment, a securitized prime segment, a non-securitized subprime segment, and a securitized subprime segment, the degree of securitization in the financial market affects the size of this default wedge. The volatility in the default wedge over the business cycle is correlated with the volatility in the share of securitized loans, which in turn depends on the degree of adverse selection in the loan market.

Monetary policy affects the extent of securitization in the financial market. As a result, monetary policy does not only affect the commonly analyzed distortion caused by nominal rigidities but also the financial market distortion. The latter causes an amplification of the monetary transmission mechanism in our model compared to that in a standard new Keynesian framework with nominal price rigidity. A negative productivity shock increases equilibrium interest rate spreads for two reasons. First it, raises default rates and second it reduces loan securitization.

A financial disturbance, in the form of an exogenous increase in average default rates in the economy, has much more dramatic effects on the risk-profile of interest rates and the demand for goods funded through the financial market than has a productivity shock. Moreover, a financial disturbance mainly affects the default wedge distortion rather than the distortion caused by nominal rigidities. As a result, a monetary policy rule that puts more weight on output does a better job at alleviating the allocative effects of such a disturbance.

However, the nominal interest rate is not necessarily the only monetary policy instrument in this economy. We also consider an alternative monetary policy intervention in response to a financial disturbance that lowers the screening cost in the financial market. Such an intervention, which is very much in the spirit of the funding facilities set up by the Federal Reserve in response to the financial crisis in the Fall of 2008, substantially reduces the effect of the financial disturbance compared to a conventional interest rate policy. This intervention turns out to have a particularly big effect on the prime market segment. A traditional expansionary monetary policy supporting aggregate demand is instead much less effective, and highly inflationary.

The paper is organized as follows. In the next section we introduce the model. We pay particular attention to the imperfect credit market and describe the standard new Keynesian core of the model more briefly. In Section 3, we derive the optimal behavior of households and firms in the economy, prove the existence and properties of the equilibrium in the financial market, and present the market clearing conditions. In Section 4 we present the results of several numerical simulations that illustrate the impact of defaults and securitization on the monetary transmission mechanism. Finally, we conclude with Section 5. Most of the mathematical details are left for the technical Appendix.

2 Preferences, Technology, and Policy

We set up our model to resemble a new Keynesian model of monetary policy transmission with the addition of a market for credit-goods. These are consumption goods the purchase of which has to be financed through a debt market where lenders have imperfect information about the quality of the borrowers. We first introduce the household's problem to explain how this second class of consumption goods affects the households' decisions. We then discuss how lenders can obtain an imperfect signal about the quality of potential borrowers. After that, we discuss the production technologies of both types of consumption goods as well as the process that drives nominal rigidities in this economy. Finally, we describe the deviations of the model from the efficient equilibrium.

2.1 Household's problem

The unit measure of households in this economy obtain utility from the consumption of three things: (*i*) standard consumption goods, which we denote by C_t; (*ii*) consumption goods that need to be financed through an imperfect credit market, which we denote by B_{it} because their purchase requires *borrowing* in this market; (*iii*) leisure. There is a continuum of imperfect credit goods, which we index by $i \in [0,1]$. Each household is expected to maximize the expected present discounted value of the flow of utility from these three sources. This flow is given by

$$U = \ln Y_t + \nu \left(1 - L_t\right), \text{ where } Y_t = C_t^{1-\gamma} \exp\left\{\gamma \int_0^1 \ln B_{it} di\right\} \tag{1}$$

and the preference parameter $1 > \gamma \geq 0$ determines the relative importance of the imperfect-credit good. We use a perfectly elastic labor supply like Hansen(1985) and assume that labor is allocated using the lottery mechanism described in Rogerson (1988).

The resulting objective that the households maximize is given by

$$E_t\left[\sum_{j=0}^{\infty}\beta^j\left\{(1-\gamma)\ln C_{t+j}+\gamma\int_0^1\ln B_{it+j}di+\nu\left(1-L_{t+j}\right)\right\}\right], \quad (2)$$

where E_t denotes the expectation based on all information available to the household at time t and β is the discount factor.

For each type of imperfect credit good, i, that households purchase, they can either be honest and repay the principal and interest on the loan that they get to finance the purchase, or they can can be dishonest, not repay, and default on the loan.

For simplicity, in our analysis we do not model explicitly the optimal defaulting decision of households. Each household defaults on a fraction $P_t[d]$ of the loans for imperfect credit goods it buys in period t. When it purchases each of the goods, it knows on which ones it will default and which one not.

Lenders in the credit market do not observe whether individual borrowers are honest or dishonest, i.e. whether borrowers will pay off their loan. The best that they can do is to obtain information about the borrower's credit-score, which we denote by $\sigma\in[0,1]$, for the particular good i. This credit score is borrower–good-specific, exogenous, and known to the household. It provides imperfect information about the borrower's likelihood to default on the loan underwritten for the purchases of a good i. The credit score is uniformly distributed over all household-good combinations.

Throughout, we assume that the higher the score σ assigned to the loan for the purchase of good i, the more likely the borrower is to default. That is, σ is strictly increasing in the riskiness of a borrower. The result is that the interest rate charged to a household for the purchases of B_t is a function of the household's credit score. We denote the gross interest rate charged to consumers with credit score σ by $R_t^B(\sigma)$. It is the gross interest rate set at time t that the household pays in period $t+1$ for the purchases of B_t. Because the cost of credit is different across different credit-scores, the level of consumption of B_t also varies with σ. We denote this consumption profile by $B_t(\sigma)$.

Because each of the imperfect-credit goods makes up an infinitesimally small part of the household's expenditures, the household does not face any uncertainty over its overall budget constraint.[2]

[2] We assume a two-dimension continuum of B_i goods, so that the defaulted share of borrowing for each i good

In particular, if we denote the household's nominal wealth level at the end of period t by A_t, then we can write

$$A_t = R_{t-1}A_{t-1} + W_tL_t + \Pi_t + \Gamma_t - P_t^C C_t - \int_0^1 (1 - P_{t-1}[d|\sigma]) R_{t-1}^B(\sigma) P_{t-1}^B B_{t-1}(\sigma)d\sigma \qquad (3)$$

Here P_t^C is the price of the first consumption good C_t, and P_t^B is the price of the imperfect-credit good, B_t, at time t, W_t is the nominal hourly wage, while Π_t denotes the potential flow of profits from the business sector to the households, and Γ_t is the income share from renting the fixed amount of capital available in the economy to firms.

In addition, $P_t[d|\sigma]$ is the chance that the households will default on the loans used for purchases of goods for which they have credit score σ. This function is assumed to be continuous and strictly increasing in σ to reflect that a higher score σ is associated with more risky borrowers. Moreover, it depends on time t because we assume that default rates depend on overall economic conditions.

Allowing households to default only on a fraction of the loans results in a model with a representative agent. While the idea that each loan i gets assigned a credit score σ_i for the same household may appear unrealistic, it is simply a device to allow easily aggregation. We could alternatively rewrite the specification of the household sector, assuming each household is either honest or dishonest, is assigned a single credit score and purchases an amount $B_t(\sigma)$. While a system of lump-sum transfers could ensure that honest and dishonest households have the same level of income and savings, and identical consumption expenditure shares, each household would consume a different amount of credit good. None of the aggregate equilibrium conditions would change, since by construction $\int_0^1 B_t(\sigma)d\sigma = \int_0^1 B_{it}di$.

2.2 Screening technology

For each borrower, a potential lender can incur a cost of $\Lambda(\sigma)$ hours of labor, this is the time that the lender needs to screen a borrower and to determine whether or not they have a score of σ *or lower*. This screening cost is assumed to be non-increasing in σ. This reflects that it is cheaper to determine that a potential borrower is at least of a low quality than it is to assure that they are of high quality.

Throughout the rest of this paper we assume a particular form of this screening cost. Namely,

purchased with a loan of type σ is equal to the probability $P_t[d|\sigma]$ of each loan being defaulted.

a step-function of the form:

$$\Lambda(\sigma) = \begin{cases} \overline{\Lambda} & \text{for} \quad \sigma \in [0, \sigma_m) \\ \underline{\Lambda} & \text{for} \quad \sigma \in [\sigma_m, 1) \\ \Lambda_1 & \text{when} \quad \sigma = 1 \end{cases} \tag{4}$$

where $0 \leq \Lambda_1 \leq \underline{\Lambda} \leq \overline{\Lambda}$, and $0 \leq \sigma_m \leq 1$. An example of this profile is depicted in Figure 1. We interpret Λ_1 as the fixed underwriting cost per loan and the additional cost $\Lambda(\sigma) - \Lambda_1$ as the actual screening costs. Hence, since all borrowers are at least of quality $\sigma = 1$, there are only underwriting costs and no screening costs at $\sigma = 1$.

In addition to the credit score, the lender also observes, B_t, for which it underwrites the loan. If a household applies for a particular loan with a minimum credit score requirement $s \in [0, 1]$ and, after screening, turns out not to satisfy the criteria, it is charged the screening cost, $\Lambda(s)$.

All potential entrants in the lending market have access to this same screening technology and there is free entry in the market for lending in the imperfect-credit market.

2.3 Production Technologies

There are two types of goods in this economy: (i) regular consumption goods, C_t; and (ii) imperfect-credit goods, B_t. We assume that each of these two goods are produced using similar technologies that are subject to the same productivity shocks. In order for our analysis to nest a simple model with nominal rigidities, we assume that retail producers of both of these goods use the same intermediate goods. Firms in the wholesale sector are constrained in the price setting, which is modeled following the Calvo (1983) adjustment mechanism.

Final goods production

The production of both types of goods involves the use of a common set of intermediate goods, $X_t(j)$ where $j \in [0, 1]$. These goods are sold at prices $P_t^X(j)$. Regular consumption goods and imperfect-credit goods are competitively produced using the production functions

$$C_t = X_t^C \text{ and } B_t = X_t^B, \tag{5}$$

where

$$X_t^i = \left(\int_0^1 X_t^{\frac{\epsilon-1}{\epsilon}}(j) dj \right)^{\frac{\epsilon}{\epsilon-1}} \text{ for } i \in \{C, B\} \text{ and } \epsilon > 1. \tag{6}$$

There is free entry in both regular final goods as well as imperfect-credit goods production.

Intermediates production and nominal rigidities

Each intermediate good is produced using the CRS technology

$$X_t^s(j) \leq Z_t L_t^X(j)^{1-\alpha} \overline{K}_t(j)^{\alpha}., \text{ where } 0 \leq \alpha < 1, \tag{7}$$

where the productivity level, Z_t, evolves according to

$$\ln Z_t = \rho_z \ln Z_{t-1} + \varepsilon_t^Z. \tag{8}$$

Here, ε_t^Z is the productivity shock. The aggregate capital stock is fixed at $\overline{K}$, and is perfectly mobile across firms. Households own capital, and receive the proceeds Γ_t from renting it out to firms.[3]

The intermediate goods producers are monopolistic competitors that take the wage rate, W_t, overall demand for intermediates, prices charged by their competitors, and the demand function they face, as given. There is no entry into or exit out of the production of intermediate goods. We use Calvo's (1983) price stickiness assumption that in each period a random fraction is allowed to re-optimize their price $P_t^X(j)$. The remaining fraction, $0 \leq \xi < 1$, of intermediate goods suppliers have to satisfy demand at the posted price. Hence, ξ represents the degree of price stickiness in the economy.

3 Utility and Profit Maximization and Market Clearing

3.1 Households: Utility Maximization

The consumption decision of a households who does not default on the loan i indexed by σ is guided by the following two intertemporal optimality conditions

$$\frac{1}{P_t^C C_t} = \beta R_t E_t \left[\frac{1}{P_{t+1}^C C_{t+1}} \right], \tag{9}$$

and

$$\frac{\gamma}{P_t^B B_t(\sigma)} = \beta R_t^B(\sigma) E_t \left[\frac{1-\gamma}{P_{t+1}^C C_{t+1}} \right], \tag{10}$$

[3] The introduction of a fixed capital stock allows production to have a CRS technology, and a downward sloping demand function for labor. The latter is essential for equilibrium, since labor supply is linear in the wage,

which equate the current marginal utility of each of the two consumption goods to the expected future marginal utility loss due to the decrease in savings caused by additional purchases.

The above two conditions combine to the intratemporal condition that determines the relative demand for the imperfect-credit good, B_t. That is,

$$(1-\gamma)\left(\frac{R_t^B(\sigma)}{R_t}\right)P_t^B B_t(\sigma)=\gamma P_t^C C_t \tag{11}$$

Note that a defaulting household, in principle, face no cost of buying $B_t(\sigma)$, and would thus like to buy as much as possible. However, if, at the score σ, the household indicates the intention to buy more than the amount demanded if not defaulting, its type would be revealed, and access to purchase of any of the imperfect-credit goods would be denied . Hence, the best defaulting households can do is to buy as much of the imperfect-credit good, $B_t(\sigma)$, as their honest counterparts. As a consequence, the demand for $B_t(\sigma)$ by defaulting households is guided by (11) as well. The optimal savings condition (9) is identical across all households.

The aggregate demand for imperfect-credit goods equals

$$B_t=\int_0^1 B_t(\sigma)\,d\sigma \tag{12}$$

This aggregate representation of the decisions made by the household sector will be useful for the derivation of the equilibrium allocation in this economy. Moreover, the aggregate fraction loan losses equals

$$\int P_t[d\,|\sigma]\,d\sigma=P_t[d]\,. \tag{13}$$

Hence, the loan-loss ratio on loans underwritten for the purchase of B_t is equal to the fraction of loans that are defaulted on.

In order for households to be on the perfectly elastic part of their labor supply curve, the wage rate has to satisfy the labor supply condition

$$\frac{W_t}{P_t^C}=\frac{\nu}{1-\gamma}C_t. \tag{14}$$

Throughout the rest of our analysis, we assume that this is the case.

3.2 Lenders: Debt Market Segmentation

The main contribution of this paper is the addition of a particular imperfect credit market in a new Keynesian framework. The way we model this market is closely related to the model of subprime and prime mortgage markets laid out in Crews Cutts and Van Order (2005). In this subsection, we consider the partial equilibrium in this market. That is, we consider how the market structure and optimality conditions result in an equilibrium interest rate schedule $R_t^B(\sigma)$, for a given demand for loans and cost of funding equal to the riskless (gross) nominal interest rate R_t.

A lender in this market chooses to screen potential borrowers for a critical credit score, which we denote by s. The lender can not discriminate between the households that pass this screening and it does not supply loans to households that fail.

Given that the lender does not supply loans to households with a credit score higher than s and that he faces competition from other lenders in the market, in equilibrium the lender ends up supplying a portfolio of loans of, potentially different, quality. Let this set be denoted by $\mathcal{S}(s) \subseteq [0,1]$ and let the gross nominal interest rate that the lender charges be given by $R_t^B(\sigma)$ for all $\sigma \in \mathcal{S}(s)$.

Note that the household's problem implies that households that get charged the same interest rate will buy the same amount of B_t. Since the lender can not discriminate between households that it lends to and thus charges them all the same interest rate, all these households will also buy the same amount $B_t(\sigma)$ for all $\sigma \in \mathcal{S}(s)$. The resulting principal of the loans to all customers of the lender is $P_t^B B_t(\sigma)$ for all $\sigma \in \mathcal{S}(s)$.

The expected revenue that the lender gets from the loan is the sum of the interest and principal paid on the loan corrected for the probability that the household to which the lender lends will default. This means that the expected revenue for the lender equals

$$\int_{\mathcal{S}(s)} \{1 - P_t[d|\sigma]\} R_t^B(\sigma) P_t^B B_t(\sigma) dF(\sigma). \tag{15}$$

The costs for the lender of underwriting and financing this loan consist of two parts. The first is the screening cost which consists of the labor cost of $\Lambda(s)$ hours of work per loan. Since the lender pays the labor costs for this screening using borrowed funds the nominal costs equal $R_t \Lambda(s) W_t$ per loan. The second is the financing cost for the principal, which equals $R_t P_t^B B_t(\sigma)$ for all $\sigma \in \mathcal{S}(s)$.

In equilibrium, we prove that there are two types of possible loan portfolios. Portfolios that are of measure zero, such that $S(s) = \{s\}$. These are not really loan portfolios but instead individual loans of quality s. Throughout the rest of this paper, we refer to these types of loans as *bank loans*. Note that while the contract offered by the financial intermediary is conditional only on the applicant having a score $\sigma \leq s$, in equilibrium the issuer of a bank loan knows the signal of all the loans in its portfolio, where $\sigma = s$ and each loan is priced based on its default risk. The second type of loans are those that are part of a portfolio of loans of heterogenous quality, i.e. those for which $\sigma \in \mathcal{S}(s) \supset \sigma$. Because these loans are packaged in a portfolio with other loans of different quality, we refer to them as *securitized loans*.

By equating the expected revenue (15) to the costs of the loan, we obtain that the zero expected profit condition for the lender that supplies loans in the portfolio $\mathcal{S}(s)$ reads

$$\int_{\mathcal{S}(s)} \{1 - P_t[d|\sigma]\} R_t^B(\sigma) P_t^B B_t(\sigma) dF(\sigma) = \int_{\mathcal{S}(s)} R_t(P_t^B B_t(\sigma) + \Lambda(s) W_t) dF(\sigma) \tag{16}$$

Since this lender would offer the same interest rate to all borrowers with $\sigma \in \mathcal{S}(s)$, all loans will be for identical amount and we obtain

$$R_t^B(S) P_t^B B_t(S) \int_{\mathcal{S}(s)} (1 - P_t[d|\sigma]) dF(\sigma) = R_t[P_t^B B_t(S) + \Lambda(s) W_t] \int_{\mathcal{S}(s)} dF(\sigma) \tag{17}$$

We assume the marginal distribution of σ is uniform. This allows writing the zero profit condition for a lender offering loans in the market segment $\mathcal{S}(s)$ as

$$\{1 - P_t[d|\sigma \in \mathcal{S}(s)]\} R_t^B(S) P_t^B B_t(S) = R_t[P_t^B B_t(S) + \Lambda(s) W_t] \tag{18}$$

Combining the zero profit condition with (11), this can be written in terms of the break-even interest rate

$$R_t^B(S) = \frac{R_t}{\{1 - P_t[d|\sigma \in \mathcal{S}(s)]\} - \frac{1-\gamma}{\gamma}\frac{\Lambda(s)W_t}{P_t^C C_t}}. \tag{19}$$

Equilibrium in this market is a set of loan portfolios such that: (i) The lender that supplies each of these portfolios of loans makes zero expected profits, and (ii) there does not exist a portfolio of loans over which a potential entrant can make strictly positive expected profits.

As we show in Appendix A, equilibrium in this imperfect-credit market leads to an endogenous segmentation of the market into, at maximum, four possible segments. The first segment, which we call the *securitized subprime* segment consists of securitized portfolios of loans of the lowest possible qualities that are not screened at all. The lenders in this market do not check the credit score of any of the loan applicants and provide all of them with a loan at a constant, but high, interest rate which reflects the high expected default probability among their customers. The second segment, which we call *subprime bank loans*, consists of bank loans of such a quality that the borrowers are better off applying for a loan at the bank than having their loan priced as part of the riskiest loan-pool. However, they are not of high enough quality to qualify for the next segment of loans. This, the third, is the second securitized segment of the market and consists of loans of relatively high quality that are screened relative to the threshold quality level, σ_m. We refer to these as *prime securitized* loans. The final possible market segment consists of loans underwritten to such reliable borrowers that lenders are willing to cover the high screening cost because it is offset by the reduction in their interest rate due to their low default probability. We call these very high quality bank loans the *prime bank loans* market segment.

Bank loans belong to measure-zero portfolios, implying in equilibrium $R_t^B(S) = R_t^B(\sigma)$. Since we have four potential endogenous market segments and one, exogenously given cut-off between them, we need two more additional cutoffs to formally define equilibrium. We show in Appendix A, that equilibrium can be written in terms of two endogenous cut-off credit quality levels, which we denote by $\sigma_{h,t}$ and $\sigma_{l,t}$. The cut-off quality $\sigma_{l,t}$ is the maximum loan quality for which not-securitizing the loan would yield a lower or equal zero profit interest rate than when it is part of a securitized portfolio that includes all loans of quality $\sigma_{l,t}$ through σ_m. It is the cut-off between the *prime bank loans* and *prime securitized* segments of the market. Formally,

$$\sigma_{l,t} = \max_{\sigma^* \in [0,\sigma_m]} \left\{ \sigma^* \ s.t. \ \left(P_t\left[d\left|\sigma \in [\sigma^*, \sigma_m]\right.\right] - P_t\left[d\left|\sigma^*\right.\right]\right) \leq \frac{1-\gamma}{\gamma} \frac{\left(\overline{\Lambda} - \underline{\Lambda}\right) W_t}{P_t^C C_t} \right\} \tag{20}$$

If, in equilibrium, there is any securitization in the prime market, then $\sigma_{l,t}$ is the highest quality loan (i.e. the lowest credit-score) that is securitized in this market.

Define the hypothetical equilibrium interest rate schedule $\widetilde{R}_t^B(\sigma)$ in case there is no securitized sub-prime market as

$$\widetilde{R}_t^B(\sigma) = \begin{cases} \frac{R_t}{\{1-P_t[d|\sigma]\} - \frac{1-\gamma}{\gamma}\frac{\overline{\Lambda} W_t}{P_t^C C_t}} & \text{for} \quad \sigma \in [0, \sigma_{l,t}] \\ \frac{R_t}{\left\{1-P_t\left[d\middle|\sigma \in \left(\sigma_{l,t}, \sigma_m\right]\right]\right\} - \frac{1-\gamma}{\gamma}\frac{\underline{\Lambda} W_t}{P_t^C C_t}} & \text{for} \quad \sigma \in (\sigma_{l,t}, \sigma_m] \\ \frac{R_t}{\{1-P_t[d|\sigma]\} - \frac{1-\gamma}{\gamma}\frac{\underline{\Lambda} W_t}{P_t^C C_t}} & \text{for} \quad \sigma \in (\sigma_m, 1] \end{cases} \tag{21}$$

then, given this interest rate profile, the maximum quality of loan that is, in equilibrium, securitized in the prime market, which we denote by $\sigma_{h,t}$, equals

$$\sigma_{h,t} = \max_{\sigma^* \in [0,1]} \left\{ \sigma^* \ s.t. \ \frac{R_t}{\left\{1 - P_t\left[d\left|\sigma \in [\sigma^*, 1]\right.\right]\right\} - \frac{1-\gamma}{\gamma}\frac{\Lambda_1 W_t}{P_t^C C_t}} \geq \widetilde{R}_t^B(\sigma^*) \right\}. \tag{22}$$

This is the endogenous cut-off level between the *bank loans* and *securitized prime* market segments. The cut-off between *prime* and *subprime* is given exogenously and equals σ_m.

Given these definitions, the equilibrium interest rate profile in this market, as well as the endogenous debt market segmentation into the four segments described above are as given in Table 1. The min operators in the market segment definitions are there to allow for cases in which some of the segments do not exist in equilibrium. Figure 2 depicts a stylized example of the case where all four market segments exist.

The endogenous segmentation of the imperfect-credit market involves the following labor input into the screening process

$$L_t^{\Lambda} = \underline{\Lambda}\sigma_{h,t} + \left(\overline{\Lambda} - \underline{\Lambda}\right) \min\left\{\sigma_{h,t}, \sigma_{l,t}\right\} + \Lambda_1 \left[1 - \sigma_{h,t}\right]. \tag{23}$$

It is the overhead cost, in terms of labor, that the lenders incur to screen all the borrowers in the market.

In the description of the equilibrium we assumed borrowers can apply directly to financial intermediaries, who offer the same conditions for all borrowers with signal $\sigma \in \mathcal{S}(s)$. In this sense our use of the terms 'bank loans' and 'securitized loans', in a model without a financial sector balance sheet and explicit secondary markets for asset-backed securities, might be considered somewhat of a misnomer. However, we would obtain the same equilibrium if we had underwriters in a competitive market offer to buy loan contracts, which are then resold to banks or to the secondary market. Since the underwriter makes zero profit in equilibrium, the loan contracts' resale price would imply the same risk-profile of loan rates as in our model. The secondary market can earn zero-profit by buying pools of loan contracts with different σ, and pay the underwriter a premium relative to the risk-based price that would be offered by competitive banks for each individual σ-risk class of loan contracts in the pool. Whether the underwriters are independent brokers, or banks who resell the loans to the secondary market is irrelevant for the equilibrium. In this setup there exists an explicit market for pass-through securities delivering the loan payoff they are linked to, but banks and the secondary market are still ex-ante identical financial intermediaries, which optimally choose to acquire loan pools with different characteristics, either homogeneous or heterogeneous with respect to repayment risk. The appendix discusses in detail how to obtain the same equilibrium in a setup where banks have by assumption an informational advantage over the secondary market, and resell a portion of their loan portfolio as pass-through security.

3.2.1 Determinants of interest rate spread profile

Equilibrium interest rates in both the prime and subprime markets are basically determined by two mechanisms. The first, which we refer to as the *asymmetric information* problem, is simply the consequence of lenders being unable to distinguish between honest and dishonest borrowers and thus loans that are subject to default. The second, which we refer to as the *adverse selection* mechanism, is that the cost advantage of some financial intermediaries leads to segments of the market in which the lowest quality loans endogenously get pooled and securitized. This can be interpreted as a classic adverse selection outcome, in the sense of Akerlof (1970), because the securitizers of loans screen for a minimal loan quality but know they only get customers with loan qualities that are the worst of the set of borrowers that would pass their screening, since the better customers would select to get a bank loan instead. This type of adverse selection in prime and subprime securitized markets for mortgages has been considered and documented before.[4]

The result of this equilibrium is a profile of interest rates, $R_t^B(\sigma)$, that maps into a continuum of interest rate spreads. Though one could potentially follow the whole profile, for much of our analysis we focus on the following four spreads. The first spread we consider is that between the riskiest and safest loan, given by $R_t^B(1) - R_t^B(0)$. We then build the weighted average interest rate over all loans. This is the gross rate that if applied to the aggregate loan amount B_t would generate the aggregate claim of the financial sector $\int_0^1 R_t^B(\sigma)\, B_t(\sigma) d\sigma$:

$$\widetilde{R}_t^{all} = \frac{\int_0^1 R_t^B(\sigma)\, B_t(\sigma) d\sigma}{B_t}. \tag{24}$$

We report the spread between the average risky rate and the risky rate for safest borrower $\widetilde{R}_t^{all} - R_t^B(0)$. Since the safest borrower never defaults, changes in this spread are also equal to changes in the average risky to riskless spread. Our steady state parameterization also depends on the *safest-to-riskless spread*, given by $R_t^B(0) - R_t$.

If there is a prime securitized market, then there is a jump in rates around σ_m. This is a discrete jump, which we call the *subprime jump*, between the worst prime loan and the best subprime loan

[4] Heuson, Passmore, and Sparks (2001) contains an interpretation of this mechanism that distinguishes between loan origination and securitization. Keys et. al. (2008) document the difference in the risk profile of securitized and non-securitized loans. They argue that some of this difference can be explained by reduced screening incentives of securitizers of loans.

and equals

$$\lim_{\sigma \downarrow \sigma_{m,t}} R_t^B(\sigma) - R_t^B(\sigma_{m,t}). \tag{25}$$

Rather than focusing on this spread, which is difficult to track in the data, we report a *prime-subprime* spread for the two market segments. It is defined as the difference between the weighted average rate per loan charged in the prime market and the subprime market, i.e.

$$\widetilde{R}_t^{subprime} - \widetilde{R}_t^{prime}, \text{ where } \widetilde{R}_t^{subprime} = \frac{\int_{\sigma_{m,t}}^1 R_t^B(\sigma) B_t(\sigma) d\sigma}{\int_{\sigma_{m,t}}^1 B_t(\sigma) d\sigma}, \text{ and } \widetilde{R}_t^{prime} = \frac{\int_0^{\sigma_{m,t}} R_t^B(\sigma) B_t(\sigma) d\sigma}{\int_0^{\sigma_{m,t}} B_t(\sigma) d\sigma}. \tag{26}$$

Aggregate demand for imperfect-credit goods

The above imperfect-credit market equilibrium together with the household's problem described in the previous subsection yields that the aggregate demand for imperfect-credit goods can be written as

$$B_t = \frac{\gamma}{1-\gamma} \frac{P_t^C}{P_t^B} C_t \left\{ [1 - P_t[d]] - \frac{1-\gamma}{\gamma} \frac{W_t L_t^\Lambda}{P_t^C C_t} \right\} = \frac{\gamma}{1-\gamma} \frac{P_t^C}{\widetilde{P}_t^B} C_t, \tag{27}$$

where

$$\widetilde{P}_t^B = \Omega_t^{-1} P_t^B \text{ and } \Omega_t = \left\{ [1 - P_t[d]] - \frac{1-\gamma}{\gamma} \frac{W_t L_t^\Lambda}{P_t^C C_t} \right\}. \tag{28}$$

Equation (27) has a fairly straightforward interpretation. Without the imperfect credit market, the expenditure share of the imperfect credit good would be equal to γ. Hence the first term of (27) reflects the demand for B_t in case there are no credit frictions. The last term reflects the distortion caused by the credit frictions.

First of all, demand for the imperfect-credit good is reduced in equilibrium because the risk of default, represented by the fraction of loans which are not repaid, $P_t[d]$, raises the equilibrium interest rate charged. The second factor that increases the equilibrium interest rates charged, and thus reduces demand for B_t, is the screening cost that is incurred per loan.

The screening costs are increasing in the amount of hours of labor needed for the screening, i.e. L_t^Λ, as well as in the wage paid for these hours, W_t. Because the screening cost is a fixed overhead cost per loan, the cost per dollar of loans issued is decreasing in the average size per loan. This average size is what is related to $P_t^C C_t$. Hence, the higher the average size per loan, the lower the distortion due to screening.

One way to interpret this equation is that the existence of dishonest households and defaults in each market results in an increase in equilibrium spreads that acts as a distortionary tax on the cost of credit for loans that will be repaid, and on the cost of B_t. The aggregate distortion caused by the existence of defaults is given by the *default wedge* Ω_t.

3.3 Producers: Profit maximization

Final good producers

The intermediate goods demand functions that result from the final goods producers' factor input choices are

$$X_t^i(j) = \left(\frac{P_t^X(j)}{P_t^X}\right)^{-\epsilon} X_t^i, \text{ where } i \in \{C, B\}, \tag{29}$$

and

$$P_t^X = \left(\int_0^1 \left(\frac{1}{P_t^X(j)}\right)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}} \tag{30}$$

The resulting average production costs for both types of consumption goods, and thus their prices, are the same. We denote this common price by P_t. It is defined as

$$P_t = P_t^C = P_t^B = P_t^X. \tag{31}$$

Intermediate goods producers

The optimal price setting decision of those intermediate goods suppliers that can adjust their price in period t, is to set their price equal to gross markup factor times a weighted sum of current and expected future marginal cost levels. The firm's optimality condition can be written as:

$$P_t^X(j) E_t \sum_{k=0}^{\infty} (\xi\beta)^k \Lambda_{t,t+k} \left[\frac{P_t^X(j)}{P_{t+k}^X}\right]^{1-\varepsilon} X_{t+k}^d = \frac{\varepsilon}{\varepsilon - 1} E_t \sum_{k=0}^{\infty} (\xi\beta)^k \Lambda_{t,t+k} MC_{t+k}^n \left[\frac{P_t^X(j)}{P_{t+k}^X}\right]^{1-\varepsilon} X_{t+k}^d \tag{32}$$

where $P_t^X(j)$ denotes the price level that the j firm is choosing. Because this is a relatively standard problem, the full derivation of the optimality conditions for the intermediate firms is omitted from the main text and derived in Appendix A.

3.4 Monetary policy

We assume that the monetary authority in this economy implements monetary policy through a simple Taylor rule of the form

$$R_t = f(\pi_t^X, X_t). \tag{33}$$

Hence, it sets the one-period riskless rate as a function of observed levels of output and retail price inflation. The inflation measure π_t^X that the monetary authority uses for the implementation of its policy is

$$\pi_t^X = (1-\gamma)\,\pi_t^C + \gamma\pi_t^B, \tag{34}$$

where π_t^C and π_t^B are the percentage increases in P_t^C and P_t^B. This is the correct consumer price inflation measure in case there are only nominal rigidities, but there is no imperfect credit market, in this economy. As we will show later on, with credit market imperfections, it is possible to build a distortion-adjusted inflation measure that includes a correction for the average interest rate spread in this imperfect credit market relative to the riskless rate.

3.5 Market clearing

Equilibrium in the labor market implies

$$L_t = L_t^X + L_t^\Lambda, \text{ where } 0 \leq L_t \leq 1, \tag{35}$$

and we will assume throughout that we are on the perfectly elastic portion of the labor supply curve, such that

$$\frac{W_t}{P_t} = \frac{\nu}{1-\gamma} C_t. \tag{36}$$

Aggregating over the budget constraint of all households, we obtain

$$B_t + C_t = \frac{W_t}{P_t} L_t^X + \Pi_t^r + \Gamma_t^r = \int X_t^s(j)dj = X_t \tag{37}$$

where Π_t^r, Γ_t^r are real profits an real rental income from the intermediate firms sector, and we used the market clearing condition $X_t^s(j) = X_t(j)$ where $X_t(j)$ denotes demand for good j and $X_t = \int X_t(j)dj$. Finally,

$$Z_t \left(L_t^X\right)^{1-\alpha} K_t^\alpha = \int_0^1 \left(\frac{P_t^X(j)}{P_t^X}\right)^{-\varepsilon} dj X_t = f_t X_t \tag{38}$$

The latter equation can be rewritten to obtain the intermediate goods sectors labor demand

$$L_t^X = \int_0^1 L_t^X(j)dj = \left[f_t X_t \frac{1}{Z_t K_t^\alpha}\right]^{\frac{1}{1-\alpha}} \tag{39}$$

where $f_t < 1$ reflects the distortion in the labor allocation due to suppliers having to satisfy demand at suboptimal price levels. Appendix A provides the complete derivation of the aggregate equilibrium conditions.

3.6 Distortions and Monetary Policy

We do not conduct a full-fledged welfare analysis but instead limit ourselves to studying the effect of monetary policy through simple policy rules. However, in this model equilibrium distortions are easily characterized. There are two distortions in this economy.

First, sticky prices move markups and create price dispersion, reflected by f_t in (39). This is the distortion that is the source of the monetary transmission mechanism in the basic new Keynesian model to which our model simplifies if defaulting is not allowed

Second, defaults cause the *default wedge*, Ω_t, which eq. (27) shows acting as a price distortion for the imperfect-credit good. As a result, it raises the effective relative price of B_t. In the aggregate, this reduces the quantity of the credit good purchased, even though the proceeds from defaulting accrue entirely to the household, and in the aggregate there is no destruction of resources resulting from loan defaults.

The default wedge is given by the risk-profile of interest rates relative to the riskless rate,

$$\Omega_t = \int_0^1 \left(\frac{R_t}{R_t^B(\sigma)}\right) d\sigma$$

which is affected both by defaults and by the extent of securitization. In equilibrium this distortion is also equal to

$$\Omega_t = \left\{[1 - P_t[d]] - \frac{1-\gamma}{\gamma}\frac{W_t L_t^\Lambda}{P_t^C C_t}\right\}$$

which provides a convenient measure independent of σ. The first term in brackets reflects the cost of aggregate defaults, that is, the cost of the distortion in interest rates caused by the *asymmetric information* between borrowers and lenders. If we had an endogenous optimal defaulting decision - even if limited to the fraction of loans where the households chooses to act dishonestly - this term would be related to the agency cost. The second term in brackets is the underwriting cost.

Securitization affects the default wedge by changing this cost over the business cycle. This cost depends on the degree of *adverse selection* in the loan market. An increase in defaults impacts Ω_t through two channels. The worsening of the asymmetric information cost lowers the first term in brackets, while the reduction of the market share that the more efficient lenders can cover increases the second term. The two effects reinforce each other. Because the default probabilities are assumed to depend on the aggregate state of the economy, the central bank can actively affect the equilibrium in the imperfect credit market. This provides a second transmission channel for monetary policy, when nominal price rigidities are present.

The default wedge effectively reduces the aggregate amount of B_t available to the households, for each unit of C_t that the household would give up. Note that Ω_t can be computed for any pool of loans $[\sigma_a, \sigma_b]$. In an economy with heterogeneous borrowers, rather than heterogeneous loans and identical borrowers, the difference in the distortion across pools of loans would translate into a difference in the distortion across households.

It is possible to build a distortion-adjusted price index that accounts for the default wedge. The inflation measure

$$\overline{\pi}_t = \frac{\overline{P}_t - \overline{P}_{t-1}}{\overline{P}_{t-1}}. \tag{40}$$

is the percentage change in the price for an aggregate unit of the consumption aggregate Y_t purchased at retail prices P_t^B and P_t^C. We call $\overline{\pi}_t$ *retail output inflation.* Consider an economy with consumption C_t, B_t identical to the equilibrium levels in our model, but with no default. In this economy the household would buy identical quantity of B_{it} for any i, so that we can write $Y_t = C_t^{1-\gamma} B_t^{\gamma}$. Optimal consumption would still be dictated by eq. (11), which now would read:

$$B_t = \frac{\gamma}{1-\gamma} \frac{P_t^C}{\widetilde{P}_t^B} C_t$$

Since B_t,C_t and P_t^C are the same as in the economy with default, it must be the case that

$$\widetilde{P}_t^B = P_t^B \left\{ [1 - P_t[d]] - \frac{1-\gamma}{\gamma} \frac{W_t L_t^{\Lambda}}{P_t^C C_t} \right\}^{-1}$$

As we show in Appendix A, a unit of aggregate consumption would then have a price of $\widetilde{P}_t$:

$$\widetilde{P}_t \sim \left(\int_0^1 \left(\frac{R_t}{R_t^B(\sigma)} \right) d\sigma \right)^{-\gamma} \overline{P}_t = \Omega_t^{-\gamma} \overline{P}_t \tag{41}$$

Hence, fluctuations in the interest rate spread paid in the imperfect credit market affect the output-inflation trade-off in this economy. We denote the inflation rate associated with this price level by

$$\widetilde{\pi}_t = \frac{\widetilde{P}_t - \widetilde{P}_{t-1}}{\widetilde{P}_{t-1}}. \tag{42}$$

We refer to it as *consumer price inflation*, although this would be the CPI only in an economy with the same amount of aggregate consumption but no dispersion across the B_{it} quantities. It is important to note that while this price index contains a measure of interest rate spreads, such spreads are currently in neither the Consumer Price Index nor the Personal Consumption Expenditures deflator, which are the two price indices most closely followed for monetary policy purposes in the U.S.[5]

4 Results

We illustrate the effect of loan securitization using a detailed numerical simulation in this section. We do so in three steps. First we consider how securitization affects the steady-state risk-profile of interest rates. Second, we show how the existence of loan securitization affects the propagation of a productivity shock in our model economy and how it influences the cyclical behavior of the risk profile of interest rates. Finally, and most importantly, we consider how different monetary policy responses can dampen the effect of financial disturbances that affect the overall default rate in the economy. Before we present this analysis, however, we first discuss the parameterization and calibration of the model for which we do our simulations.

4.1 Parameterization and Calibration

Cyclical properties and risk profile of default rates

So far, we have derived the model for a very general specification of the default rate process that determines $P_t [d \,|\sigma]$. The two main properties that we have discussed are that: (i) aggregate defaults are countercyclical; and (ii) default rates are strictly increasing in the signal σ. Throughout the rest of our results, we capture these two properties with the functional form

$$P_t [d \,|\sigma] = \Xi_t \Gamma_t \sigma^2, \tag{43}$$

[5] They are also not part of the Euro Area's Harmonized Index of Consumer Prices.

where Ξ_t is an exogenous random shock and Γ_t is a function of aggregate economic conditions. We assume Γ_t is countercyclical and depends on output X_t. Up to first order, Γ_t can be approximated by

$$\widehat{\Gamma}_t = -\xi \widehat{X}_t$$

where ξ is the elasticity with respect to output and $\widehat{\Gamma}_t,\ \widehat{X}_t$ are log-deviations from the steady state value.

We let Ξ_t vary to reflect potential disturbances to the financial sector. In particular, we choose

$$\left(\ln \Xi_t - \ln \overline{\Xi}\right) = \rho_{\Xi} \left(\ln \Xi_t - \ln \overline{\Xi}\right) + \varepsilon_{\Xi,t}, \tag{44}$$

where $\varepsilon_{\Xi,t}$ is the aggregate default risk shock, and $\frac{1}{3}\overline{\Xi\Gamma}$ is the perfect-foresight steady-state aggregate default rate $\overline{P}[d]$.[6]

We choose the quadratic form in the signal σ to reflect that the marginal probability of default, i.e.

$$\frac{\partial}{\partial \sigma} P_t \left[d \,|\sigma \right], \tag{45}$$

is strictly increasing in the credit score σ. This is important for the equilibrium in the securitization market because it implies that the marginal benefit of screening potential borrowers is much higher for low quality loans than for high quality ones.

This also is consistent with the data, as can be seen from Figure 3. It depicts the risk-profile of delinquency rates in terms of FICO scores. The horizontal axis is the percentile of the credit score, from best-to-worst, which can be considered an empirical proxy for the signal σ, while the vertical axis depicts the fraction of loans issued to persons with such scores that become delinquent.[7]

Calibration

Because our model is a standard new Keynesian model, as in Woodford (2003, Chapter 2), with an added imperfect credit market, we divide the model parameters in two separate sets for calibration

[6] Up to first order, different specifications for Γ_t can be calibrated to deliver the desired elasticity of default probability with respect to economic conditions. We assume $\Gamma_t = 3 \exp \left\{ -\frac{\xi}{2} \left(\left(\frac{X_t}{\overline{X}} \right)^2 - 1 \right) \right\}$. Then $\overline{P}[d] = \frac{1}{3}\overline{\Xi\Gamma} = \overline{\Xi}$.

[7] Unfortunately, we do not have similar data on default rates. However, if the same fraction of delinquent loans default for all FICO scores, or if a smaller fraction of delinquent loans default for higher FICO scores, Figure 3 is consistent with our functional form assumption that the marginal default probability is increasing in σ.

purposes. The first set consists of preference, technology, and monetary policy parameters that are part of the nested new Keynesian model. We use commonly applied values for these parameters in our calculations.

Table 2 lists the preference and technology parameters and their values. For the monetary policy rule, (33), we use a standard Taylor Rule of the form

$$\ln R_t = -\ln \beta + \chi \ln R_{t-1} + (1-\chi)\left[\phi_\pi \pi_t + \phi_y \left(\ln X_t - \ln \overline{X}\right)\right]. \tag{46}$$

As a baseline policy we assume $\phi_\pi = 1.5$, $\phi_y = 0$ and $\chi = 0.5$. We also consider the impact of alternative policies. Steady state inflation is set equal to zero.

The calibration of the second set of parameters, those that guide the equilibrium in the imperfect credit market, is more involved. We calibrate them to match evidence on aggregate mortgage defaults and interest rate spreads, using data on US foreclosure rates and interest rates reported in Chomsisengphet and Pennington-Cross (2006). The average quarterly default rate we use is 0.75% and the interest rate spreads that we match are listed in Table 3. The elasticity, ξ, and persistence parameter, ρ_Ξ, are chosen to capture some of quantitative countercyclical properties of default rates. With our parameterization, we obtain a that the share of prime and subprime borrowing are respectively 75% and 25%, and the share of securitized loans is equal to 44.2%.

4.2 Steady-State impact of securitization

We assess the impact of securitization by comparing the baseline economy with an economy where perfect risk-pricing is optimal. This outcome would obtain in equilibrium if no financial intermediary had any cost advantage, and all paid the screening cost $\overline{\Lambda}$. Figure 4 shows the steady-state risk profile of interest rates for the two economies.

When no financial intermediary has any cost advantage, the risk profile of interest rates is completely defined by the asymmetric information problem, i.e. by the risk profile of default rates $P_t[d|\sigma]$. Interest rates (and prices for the credit good) are distorted since lenders need to cover the cost of defaults across agents in each class of risk σ. When we allow for the screening cost to decrease as the quality of the borrower worsens (in other words, it is more costly to ensure the bank has a low cost of default, rather than a high cost of default incurred by accepting loans which are more likely to be defaulted on), securitization emerges endogenously. Free entry in the market ensures that some lenders will buy pools of loans, up to a given default risk signal σ, guaranteeing a safe return to the investor. This lowers the interest rate for two segments of the market, where bank loans are competed out of the market. That is, loans issued in these segments obtain a *securitization discount*. Because the screening cost is a step function, we endogenously obtain a prime-subprime spread.

Securitization lowers $\widetilde{R}_t^{all}$ by over 100 basis points (annualized) and reduces the default wedge, Ω_t. Table 5 summarizes the steady state impact of securitization. In the aggregate, without securitization the economy would have to give up an additional 0.24% of the imperfect-credit good consumption that would be obtained if there were no defaults. This loss is unevenly distributed across markets for loans of different qualities. Prime loans would lose only an additional 0.10%, while subprime loans would lose 0.62%. This is because the the benefits from securitization accrue mainly to high-risk loans, who get the most substantial securitization discount in equilibrium.[8]

The degree of securitization observed in the economy depends on the relative weight of the cost advantage and the degree of adverse selection in each segment of the market. Figure 5 shows the steady state risk profile of interest rates with a constant marginal default rate profile, which is

$$\overline{P}\left[d\left|\sigma\right.\right] = G\overline{\Xi\Gamma}\sigma. \tag{47}$$

where $G = \frac{2}{3}$ ensures that the unconditional default probability is identical across the two conditional distributions used to generate the interest rate risk-profiles in the figure. Relative to the quadratic case, the linear conditional default probability increases adverse selection for low σ, and lowers it for high σ, compared to the baseline case (43). Given our parameterization, the change in the risk profile is enough to allow the most efficient lender to take over all of the subprime as well as the securitized prime market. Relative to the baseline case, interest rates are lower for agents who used to belong to the subprime segment, and higher for agents in the prime segment. The higher degree of securitization ensures a lower level of distortion Ω, since now the more efficient lender has gained market share.

[8] In welfare terms, this gains should be evaluated keeping constant the disutility from labor supplied to the financial sector, which is exogenously reduced by the cost advantage in the securitized economy. The extra labor required in a world without securitization is equal to 0.21% of total steady state labor, which is nearly identical across the two economies.

4.2.1 Productivity Shocks: Response of Real Variables

The propagation mechanism is modified by the existence of the asymmetric information problem (time-varying defaults) and of the adverse selection problem (time-varying securitization). Both affect utility by changing the degree of interest rate distortion, and the premium paid over the riskless rate R_t. Because the default rate is countercyclical, as defaults increase in a recession, the value of $\Omega_t = \int_0^1 \left(\frac{R_t}{R_t^B(\sigma)} \right) d\sigma$ decreases. Defaults increase the severity of the asymmetric information problem. This in turn reduces the degree of adverse selection that the cost advantage of the most efficient financial intermediaries can compensate for, and the two effects cumulate in reducing Ω_t.

Figure 6 shows the impulse response function to a 1% negative technology shock in three economies: a new Keynesian model without defaults, a model with default risk but no securitization, and our baseline model. Both defaults and securitization make the average risky rate more sensitive to recessionary shock. This results in an increase in the effective price of the imperfect-credit good, reducing its consumption. Consumption of the consumption good is not directly affected, since credit is used neither in the purchase nor in the production of C_t. The general equilibrium impact on C_t is modest. The additional amplification generated by the existence of endogenous securitization appears small. But consider that the model is calibrated to a very low steady state default rate. Given the elasticity of defaults to output, the probability of default increases on impact by only 0.15 percentage points. For an increase of one percentage point in defaults, the response of B_t increases by 120% relative to the new Keynesian model, and by 96% in an economy with default risk but no securitization. Defaults also double the impact of the shock on CPI: the consumption basket is effectively 0.8% more expensive, even if retail output inflation increases only by 0.4%.

While we do not explicitly discuss optimal policy, it is easy to see that technology shocks present a trade-off to the policymaker. With our policy rule, markups decrease while the default rate increases. Reducing markup volatility by stabilizing inflation results in a larger swing in output. This in turn generates more defaults and a higher interest rate distortion, increasing the difference between CPI and retail output inflation. To reduce the interest rate distortion, monetary policy would need to be more output-stabilizing. For example, a policy which would systematically react to output with a positive feedback coefficient of $\phi_y = 1$ would halve the peak recessionary impact of the productivity shock. At the same time, while reducing defaults and the interest rate distortion Ω_t, stimulating demand would lead to an annualized CPI inflation of about 5% on impact. This results in a higher nominal level of the riskless interest rate, but in a lower risky-riskless spread.

4.2.2 Productivity Shocks: Response of Financial Variables

Our model provides a theory of the behaviour of spreads over the business cycle. The impact of defaults and securitization on the dynamics of real quantities can be traced through the changes in the risk structure of interest rates, shown in figure 7.

We define the prime and subprime segments of the loans market in an economy without securitization using the same marginal signal σ_m, so that the amount of default risk in each segment of the market is identical to what would obtain in the economy with securitization. While there is no subprime jump in an economy without securitization, we can still define a subprime premium - the spread between the weighted average interest rate for the prime and subprime segments of the market. In response to a negative productivity shock, the subprime premium increases about 25% more in the economy with securitization. The share of securitized debt falls on impact from 44% to 33%, as the limit risk-signals for the securitized pools of loans, that we defined as σ_l and σ_h, endogenously respond to the shock. Since securitization lowers the interest rate paid by agents, the subprime premium elasticity to output is larger relative to the model without securitization. This translates into a larger increase of the average risky rate relative to the risky rate paid by the safest borrower. The rate spread paid by the riskiest borrower is unaffected by securitization (though the level is different), since the riskiest borrower will in equilibrium always get a loan from the most efficient financial intermediary.

The impact of an endogenous change in securitization is unevenly spread across loans: there are indeed winners and losers. The most affected are loans that were priced on the secondary market, but now can belong to the market segment where loans are risk-priced. Figure 7 shows two of such classes of risk, with $\sigma = 0.6$ and $\sigma = 0.87$. The increase in the spread for an agent within the risk class $\sigma = 0.87$ is larger than the increase in spread for the worst possible borrower, since the agent is being charged the risk-based price for the loan, and is losing the securitization discount. We discuss more in depth the welfare implications of endogenous securitization in the next section.

4.2.3 Financial Shocks

We use our economy to gain insight into how financial shocks propagate through the economy, affect welfare, and to investigate the options available to policymakers. We model a financial disturbance as the combination of two shocks. First, the economy is hit by an exogenous raise in defaults. We assume the default rate increases from 0.75% to 1.125%. In the period between 2007 and 2008, foreclosure rates increased in the order of $3-400\%$ among prime loans, so the magnitude of the shock we consider is conservative relative to periods of extreme financial stress. At the same time, we assume the screening technology changes, and σ_m temporarily increases by 15%. This implies it becomes more costly to screen loans for levels of risk between the steady state level of σ_m and its new higher level. Intuitively, it has become more difficult to ascertain the level of risk associated with some loans, and lenders need to expend more resources towards this task. The change in σ_m has an interesting equilibrium outcome, which is useful to model a financial disturbance changing the risk contained in loan pools: as σ_m increases, any securitized prime loan pool necessarily includes more risky loans. We believe this captures an important aspect of disturbances in loan markets.

While σ_m represents a change in the screening technology, the default rate increase is a pure financial shock. In our model, there is no destruction of resources associated with default: all that changes is the distortion in the pricing of the imperfect-credit good. Any change in the price that financial intermediary assign to a given level of risk - even if the risk profile of agents were unchanged - would deliver the same response of the economy.

Fig. 8 and fig. 9 illustrate the dynamic impact of the financial disturbance. The change in the risk profile of lending leads to a substantial fall in the demand for the imperfect-credit good, and some substitution of the imperfect-credit good for the consumption good in the consumption basket. Correspondingly, while retail inflation hardly changes, CPI inflation increases by over 1%. Securitization greatly amplifies the impact of the shock, and the change in the overall measure of distortion Ω_t that falls by nearly 0.6%. The change in the risk structure of interest rates (shown at period $t = 1$ in figure 10) explains this outcome. Securitization is reduced. Therefore the spread between the average and the safest risky rate is much more responsive to the financial disturbance, generating a higher aggregate distortion relative to the economy without securitization.

The impact of the disturbance is unevenly distributed across loans.[9] First, there is a curvature

[9] The default wedge for the prime market is equal to $\Omega_t^p = \int_0^{\sigma_m} \left\{ [1 - P_t[d|\sigma]] - \frac{1-\gamma}{\gamma} \frac{W_t \Lambda(\sigma)}{P_t^C C_t} \right\} \sigma_m^{-1}$. The wedge for

effect. A proportional increase in default rates across all loans affects more the riskiest segment of the market, since the risky interest rate depends inversely on the probability of repaying the debt. This leads to an increase in the interest rate distortion Ω_t more than twice as large among subprime loans relative to the overall distortion. The risky rate for prime loans increases by far less than the subprime risky rate. Second, securitization has a stronger impact in the prime market relative to the subprime market - as can be seen by observing both the corresponding interest rates and distortion measures. Adverse selection is smaller in the prime market, given the information contained in σ for the risk profile across loans. An increase in defaults has a relatively larger impact on the share of securitized loans when adverse selection is lower. Lenders in the secondary market must offer a rate competitive with banks. A small change in the rate offered, to compensate for the increase in default risk, translates into a much higher marginal risk-signal σ_l for the loan that the household will want to borrow at the rate offered by the secondary market lender, and a much smaller pool of securitized loans.

If we interpreted our setup as an economy with heterogenous borrowers, rather than as an economy with heterogenous loans but identical borrowers, we would obtain that the distortionary impact Ω_t of financial disturbances falls disproportionately on subprime borrowers, but securitization makes the distortionary impact on prime borrowers, and on the economy as a whole, relatively more sensitive to the shock relative to an economy without securitization.

4.2.4 Monetary Policy and Financial Shocks

Expansionary Policy to Lower Default Rate In a financial crisis the first line of defense is an expansionary policy that lowers the cost of credit, supports demand and contains the increase in defaults. This can be achieved with direct government intervention (support to borrowers) or with a policy aimed at lowering the interest rate on loans. In either case, the objective is to lessen the impact of defaults. Figure 11 compares two policy rule: our baseline policy where the monetary authority reacts only to inflation, and one aimed also at stabilizing output, where we set $\phi_y = 0.25$. Therefore in a recession the private sector expects that the monetary authority will systematically lower interest rates, for given inflation level.

The interest rate policy dampens the severity of the fall in consumption of the imperfect-

the subprime market is similarly defined.

credit good, and more than doubles the increase in consumption of C_t. This comes at a cost of a considerable increase in retail inflation. The financial disturbance acts like a negative technology cost in the financial sector. The labor devoted to borrowers' screening increases by 20%, increasing the CPI and the overall distortion in interest rates. An expansionary policy increases consumption of imperfect-credit goods mainly through a general equilibrium effect by lowering markups, and the spot price of X_t in terms of labor hours, since the elasticity of default rates to output is not high enough. This translates in a large increase in inflation, posing a dilemma to the monetary authority. In equilibrium, the risky rate increases - although the risky-riskless rate spread decreases - since the increase in inflation calls for a an equilibrium increase in nominal interest rates.

Allowing More Risk in Government-Sponsored Secondary Market Agencies' Portfolio The Housing and Economic Recovery Act of 2008 changed Fannie Mae's charter to expand the definition of a "conforming" loan - loans that could be purchased by the government-sponsored loan securitizer. Relaxing the standards for a loan to be purchased by financial intermediaries acting as securitizers allows implicitly more risk to enter into the securitized loan pools. In the case of a government-sponsored loan securitizer this would only be true to the extent that the added securitization is not substituting for lower supply of securitized loans by private market labels. Note that in and of itself this policy - which corresponds to an increase in σ_m, the maximum level of risk accepted by the buyer of loan pool - will increase the average risky rate and reduce securitization, since the same absolute cost advantage now has to compensate the lender for a higher degree of adverse selection. This policy is effective only inasmuch relaxing the rules for a loan to be purchased by a government-sponsored securitizer also lowers the cost of the loan, since these agencies issue liabilities implicitly backed by a government guarantee, and are effectively subsidized relative to the private market.[10] The correct way of thinking about this policy is as an attempt to lower the overall cost of lending. We take up this policy next.

Reducing the Cost of Underwriting Loans Both the change in the rules for conforming loans, and the Federal Reserve policy of taking risk on its balance sheet and providing assets that can be used as collateral by private lenders effectively subsidize the operating cost of the financial

[10] If the loan purchased by the government-sponsored agency were to stay on its balance sheet, this would obviously increase the total amount of credit available.

intermediation market. Figure 12 shows the impact in our model of a 20% reduction in the screening cost across all lenders, in response to the financial disturbance. Policymakers cannot directly reduce the degree of adverse selection in the market, but can offer to lenders more leeway to overcome it by making the cost of issuing credit lower. The policy is very effective in fighting the recessionary impact of the financial disturbance, and manages to nearly compensate for the overall increase in the financial sector screening labor seen under the baseline policy.

Securitization works *against* the effectiveness of this policy. Because it depends on the absolute, rather than the percentage, cost advantage of the efficient loan-pooling financial intermediaries, compensating for the increase in default risk by lowering overall cost of issuing credit results in less securitization. Additionally, the share of subprime market that is securitized is in equilibrium much larger than the securitized share of the prime market, relative to the respective market segment. Under the credit market policy, the risky-riskless interest rate spread for the subprime securitized segment is virtually unchanged relative to the baseline policy.[11] In the prime segment instead, were securitization is lower, risk-priced loans accrue the full benefit of the reduction in underwriting cost. This leads to the benefits of this policy falling disproportionately on the prime market, and especially on its unsecuritized segment. Thus the overall risk profile of interest rates falls, but the spread between the safest and average risky rate, and the subprime premium, increase relative to the baseline policy response.

A more effective policy, aimed at reducing the increase in the subprime market distortion after the financial disturbance, would reduce *by the same absolute amount*, rather than proportionally, the cost of issuing credit, since it would leave the relative cost advantage of efficient lenders unchanged. Thus it would lead to a large fall in the risky-riskless interest rate spread in the subprime market as well as in the prime market, and contain the fall in securitization.

5 Conclusion

In this paper we added an imperfect credit market with endogenous loan securitization to a standard new Keynesian model of monetary policy. The equilibrium in this market consists of an

[11] This is a consequence of the increase in σ_h with the credit market policy, and of the very high degree of adverse selection in the subprime poll of securitized loans. As σ_h increases, the default risk in the loan pool rises fast, wiping out the reduction in underwriting cost and resulting in a virtually unchanged interest rate $R_t^B(\sigma_h)$.

endogenously determined risk-profile of interest rates, as well as a prime and subprime segment of the financial market. We use our model to illustrate how the endogenous securitization of loans amplifies the propagation mechanism and adds a new channel for monetary policy to influence the economy.

Interest rate spreads cause a default wedge that acts as a distortionary tax. These spreads are determined by the endogenous segmentation of the financial market. In steady state, securitization reduces this wedge and leads to a decline in interest rate spreads that especially benefits subprime borrowers.

Since monetary policy affects the equilibrium financial market segmentation, it does not only affect the commonly analyzed distortion caused by nominal rigidities but also the financial market distortion, represented by the default wedge. This distortion is composed of two terms, one reflecting the severity of the information asymmetry between borrowers and lenders, the other the extent to which lenders can overcome the adverse selection problem and securitize pools of loans. Volatility in the default wedge amplifies the propagation mechanism compared to the one in a standard new Keynesian framework. It also affects the degree of countercyclicality of spreads between interest rates paid in the subprime and prime market segments as well as procyclicality of the market shares of securitized loans. Moreover, we show that changes in securitization can have a very different effect across different groups of loans (or borrowers), and different segments of the loan market.

The addition of financial market frictions allows us to consider the effects of different policies in case of a financial disturbance, i.e. an exogenous increase in average default rates in the economy. Such a disturbance directly affects the risk-profile of interest rates and the demand for goods funded through the financial market. Moreover, a financial disturbance mainly affects the default wedge distortion rather than the distortion caused by nominal rigidities. Consequently, a monetary policy rule that puts more weight on output does a better job at alleviating the allocative effects of such a disturbance, at the cost of generating a much higher inflation.

We find that, in case of such a financial disturbance, the effectiveness of monetary policy is greatly enhanced by not only using the nominal interest rate as a policy instrument but by directly affecting the cost of issuing credit for lenders. This is very much in line with some of the unconventional measures that have been enacted during and after the 2008 financial crisis by many central banks. Our model shows that these policies predominantly help the prime segment of the market, if the subprime market is extensively securitized.

Though our analysis is the first to formally include a market for securitized loans in a model of the monetary transmission mechanism, it does so in a framework with some major limitations. Most importantly, because the stock of debt is equal to the flow, financial disturbances have no effect on the balance sheets of the household and financial sectors. Moreover, financial intermediaries do not face any uncertainty about the default probabilities when they make the loans nor is there an agency problem that requires borrowers to be leveraged and results in endogenous defaults, as in the financial accelerator framework of Bernanke, Gertler and Gilchrist (1999) and the analysis of unconventional monetary policy by Gertler and Karadi (2009). Dealing with some of these limitations is the focus of our future research.

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A Mathematical details

This appendix contains the derivations with the mathematical details underlying the results in the main text. The results are derived in the order that they appear in the text.

Derivation of (13):

The total amount of loans outstanding for the purchase of the imperfect-credit good and interest to be (re)paid at time t, D_t^B, is

$$D_t^B = \int R_t^B(\sigma) P_{t-1}^B B_{t-1}(\sigma) d\sigma. \tag{48}$$

When we substitute the demand equation (11) into this equation, we obtain that

$$D_t^B = \frac{\gamma}{1-\gamma} R_t P_{t-1}^C C_{t-1}. \tag{49}$$

The total amount of loans outstanding for the purchase of B_{t-1} that is actually repaid with interest in period t, equals

$$\int (1 - P_t[d|\sigma]) R_t^B(\sigma) P_{t-1}^B B_{t-1}(\sigma) d\sigma. \tag{50}$$

Again substituting in the relative demand equation (11), we can write this as

$$\left(\int (1 - P_t[d|\sigma]) d\sigma\right) \frac{\gamma}{1-\gamma} R_t P_{t-1}^C C_{t-1} = (1 - P_t[d]) \frac{\gamma}{1-\gamma} R_t P_{t-1}^C C_{t-1}. \tag{51}$$

Here, the unconditional probability follows from the assumption that the credit-score is assumed to be uniformly distributed across households. Hence, the fraction of the total amount of loans and interest owed in the imperfect credit market that does not get repaid is equal to the fraction of households that are dishonest.

Derivation of (18):

For this derivation, we used the result:

$$\int_{\mathcal{S}(s)} \{1 - P[d|\sigma]\} d\sigma = \int_{\mathcal{S}(s)} d\sigma - \left[\frac{\frac{1}{b-a}\int_{\mathcal{S}(s)} P[d|\sigma] d\sigma}{\frac{1}{b-a}\int_{\mathcal{S}(s)} d\sigma}\right] \int_{\mathcal{S}(s)} d\sigma = \int_{\mathcal{S}(s)} d\sigma - \left[\frac{\int_{\mathcal{S}(s)} f(d,\sigma) d\sigma}{\int_{\mathcal{S}(s)} f_\sigma d\sigma}\right] \int_{\mathcal{S}(s)} d\sigma$$

for $\sigma \sim U(a, b)$.

Derivation of (19):

The zero expected profit condition implies that for all $\sigma \in \mathcal{S}(s)$ it must be the case that

$$\{1 - P[d|\sigma' \in \mathcal{S}(s)]\} R_t^B(\sigma) P_t^B B_t(\sigma) = R_t[P_t^B B_t(\sigma) + \Lambda(s) W_t]. \tag{52}$$

Note that, from (11), we know that

$$(1-\gamma) R_t^B(\sigma) P_t^B B_t = \gamma R_t P_t^C C_t. \tag{53}$$

This allows us to rewrite the above zero expected profits condition as

$$\begin{aligned} \{1 - P[d|\sigma' \in \mathcal{S}(s)]\} \frac{\gamma}{1-\gamma} R_t P_t^C C_t &= \frac{R_t}{R_t^B(\sigma)} R_t^B(\sigma) P_t^B B_t(\sigma) + R_t \Lambda(s) W_t & (54) \\ &= \left(\frac{R_t}{R_t^B(\sigma)}\right) \frac{\gamma}{1-\gamma} R_t P_t^C C_t + R_t \Lambda(s) W_t. & (55) \end{aligned}$$

Rearranging terms in this equation allows us to write this as (19).

Derivation of imperfect credit market equilibrium:

We derive the equilibrium in the imperfect credit market in five steps. The first two steps show that securitized segments in the market can only exist with upper cutoffs of 1 and σ_m. In the third step we show that the loan shark segment of the market always exists if $\underline{\Lambda} > \Lambda_1$. In the fourth step we show that, if $\overline{\Lambda} > \underline{\Lambda}$, then there is always a set of loans below σ_m that is securitized. In the final step, we combine these four results to define equilibrium and provide some more detail than is given in the main text.

Step 1: In equilibrium, there are no loan portfolios supplied with range

$$\mathcal{S}(s) = [s - \varepsilon, s], \text{ with } \varepsilon > 0 \text{ and } s < \sigma_m. \tag{56}$$

Proof: Suppose such a portfolio would be supplied in equilibrium, then, under free entry in the lending market, it must be the case that if we define

$$\widetilde{B}_t = B_t(\sigma) \text{ and } \widetilde{R}_t^B = R_t^B(\sigma) \text{ for all } \sigma \in \mathcal{S}(s) \tag{57}$$

then

$$\{1 - P[d|\sigma \in \mathcal{S}(s)]\} \widetilde{R}_t^B P_t^B \widetilde{B}_t = R_t P_t^B \widetilde{B}_t + R_t \overline{\Lambda} W_t. \tag{58}$$

However, since for any $\widetilde{s} \in (s - \varepsilon, s)$ it is the case that

$$P[d|\sigma \in [s - \varepsilon, \widetilde{s}]] < P[d|\sigma \in [s - \varepsilon, s]], \tag{59}$$

potential entrants could offer a portfolio of loans of the form $[s - \varepsilon, \widetilde{s}]$ and make expected profits equal to

$$\begin{aligned} \{1 - P[d|\sigma \in [s - \varepsilon, \widetilde{s}]]\} \widetilde{R}_t^B P_t^B \widetilde{B}_t - R_t P_t^B \widetilde{B}_t - R_t \overline{\Lambda} W_t &> \\ \{1 - P[d|\sigma \in \mathcal{S}(s)]\} \widetilde{R}_t^B P_t^B \widetilde{B}_t - R_t P_t^B \widetilde{B}_t - R_t \overline{\Lambda} W_t &= 0. \end{aligned} \tag{60}$$

This thus implies an entry opportunity with strictly positive expected profits. Hence, in this case entry will occur and the equilibrium interest rate will be undercut. As a result, this can not be the equilibrium interest rate schedule in the first place.

An implication of this result is that for each $\sigma \in \mathcal{S}(s)$ with $s < \sigma_m$, it must be the case that $\mathcal{S}(s) = \{\sigma\}$. Hence, households with such credit scores get bank loans in equilibrium.

Step 2: In equilibrium, there do not exist any loan portfolios of the form

$$\mathcal{S}(s) = [s - \varepsilon, s], \text{ with } \varepsilon > 0 \text{ and } \sigma_m < s < 1. \tag{61}$$

Proof: Suppose that, in equilibrium, there was such a loan portfolio, then if we define $\widetilde{B}_t$ and $\widetilde{R}_t^B$ in a similar way to (57), we know that the supplier must make zero expected profits. That is

$$\{1 - P[d|\sigma \in \mathcal{S}(s)]\} \widetilde{R}_t^B P_t^B \widetilde{B}_t = R_t P_t^B \widetilde{B}_t + R_t \underline{\Lambda} W_t. \tag{62}$$

However, since for any $\widetilde{s} \in (s-\varepsilon, s)$ it is the case that

$$P\left[d\left|\sigma \in [s-\varepsilon, \widetilde{s}]\right.\right] < P\left[d\left|\sigma \in [s-\varepsilon, s]\right.\right], \tag{63}$$

potential entrants could offer a portfolio of loans of the form $[s-\varepsilon, \widetilde{s}]$ and make expected profits equal to

$$\begin{aligned} \{1 - P[d|\sigma \in [s-\varepsilon, \widetilde{s}]]\} \widetilde{R}_t^B P_t^B \widetilde{B}_t - R_t P_t^B \widetilde{B}_t - R_t \underline{\Lambda} W_t &> \\ \{1 - P[d|\sigma \in \mathcal{S}(s)]\} \widetilde{R}_t^B P_t^B \widetilde{B}_t - R_t P_t^B \widetilde{B}_t - R_t \underline{\Lambda} W_t &= 0. \end{aligned} \tag{64}$$

This thus implies an entry opportunity with strictly positive expected profits. Hence, in this case entry will occur and the equilibrium interest rate will be undercut. As a result, this can not be the equilibrium interest rate schedule in the first place.

An implication of this result is that for each $\sigma \in \mathcal{S}(s)$ with $\sigma_m < s < 1$, it must be the case that $\mathcal{S}(s) = \{\sigma\}$. Hence, households with such credit scores get bank loans in equilibrium.

Step 3: If $\underline{\Lambda} > \Lambda_1$ then the loan shark market segment exists in equilibrium.

Proof: Suppose not, then $\exists\, \varepsilon > 0$ such that for all $\sigma \in (1-\varepsilon, 1)$ it is the case that if $\sigma \in \mathcal{S}(s)$, then $\mathcal{S}(s) = \{s\}$. In that case, for all $\sigma \in (1-\varepsilon, 1)$ zero profits are made on bank loans provided to borrowers with these credit scores, such that

$$\{1 - P[d|\sigma]\} R_t^B(\sigma) P_t^B B_t(\sigma) = R_t^B(\sigma) P_t^B B_t(\sigma) + R_t \underline{\Lambda} W_t. \tag{65}$$

However, since $P[d|\sigma]$ is continuous and strictly increasing in σ, it is also the case that $P[d|\sigma \in (1-\varepsilon, 1)]$ is continuous and strictly increasing in ε. Consider the break-even interest rate for $\sigma \in (1-\varepsilon, 1)$

$$R_t^B(\sigma) = \frac{R_t}{\{1 - P[d|\sigma]\} - \frac{1-\gamma}{\gamma} \frac{\underline{\Lambda} W_t}{P_t^C C_t}} \tag{66}$$

Note that over this interval $R_t^B(\sigma)$ is strictly increasing in σ. The break-even interest rate on a securitized subprime loan portfolio when this market segment covered $\sigma \in (1-\delta, 1)$ with $\varepsilon' > 0$ would be

$$R_{t+1}^{*B} = \frac{R_t}{\{1 - P[d|\sigma \in (1-\varepsilon', 1)]\} - \frac{1-\gamma}{\gamma} \frac{\Lambda_1 W_t}{P_t^C C_t}} \tag{67}$$

Equilibrium would require that for all $0 < \varepsilon' \leq \varepsilon$ the above interest rate is higher than that charged for bank loans. This means that for all $0 < \varepsilon' \leq \varepsilon$

$$P\left[d\left|\sigma \in \left(1-\varepsilon', 1\right)\right.\right] - P\left[d\left|1-\varepsilon'\right.\right] > \frac{1-\gamma}{\gamma} \frac{\left(\underline{\Lambda} - \Lambda_1\right) W_t}{P_t^C C_t} > 0. \tag{68}$$

However, this is not the case because that would mean a violation of the continuity property of $P[d|\sigma]$ in σ. Hence, this can not be an equilibrium.

Step 4: If $\overline{\Lambda} > \underline{\Lambda}$ then $\exists\, \varepsilon > 0$ such that $\forall \sigma \in (\sigma_m - \varepsilon, \sigma_m]$ it is the case that if $\sigma \in \mathcal{S}(s)$ then $\sigma_m \in \mathcal{S}(s)$.

Proof: Suppose not, then either one of two things have to be true, according to the results from the above three steps. Either the loans of quality σ_m are bank loans. That is

$$(i) \text{ If } \sigma_m \in \mathcal{S}(s) \text{ then } \mathcal{S}(s) = \{\sigma_m\} \text{ and } R_t^B(\sigma_m) = \frac{R_t}{\{1 - P[d|\sigma_m]\} - \frac{1-\gamma}{\gamma} \frac{\underline{\Lambda} W_t}{P_t^C C_t}}. \tag{69}$$

Or the loans of quality σ_m are part of securitized subprime segment and are the best loans in this portfolio, such that

$$(ii) \text{ If } \sigma_m \in \mathcal{S}(s) \text{ then } s = 1 \text{ and } R_t^B(\sigma_m) = \frac{R_t}{\{1 - P[d|\sigma \in [\sigma_m, 1]]\} - \frac{1-\gamma}{\gamma}\frac{\Lambda_1 W_t}{P_t^C C_t}}. \tag{70}$$

If the latter is the case then

$$\frac{R_t}{\{1 - P[d|\sigma_m]\} - \frac{1-\gamma}{\gamma}\frac{\underline{\Lambda} W_t}{R_t P_t^C C_t}} > \frac{R_t}{\{1 - P[d|\sigma \in [\sigma_m, 1]]\} - \frac{1-\gamma}{\gamma}\frac{\Lambda_1 W_t}{P_t^C C_t}}. \tag{71}$$

Moreover, the third thing that needs to be the case is that for all $\sigma \in (\sigma_m - \varepsilon, \sigma_m)$, if $\sigma \in \mathcal{S}(s)$ then $\sigma_m \in \mathcal{S}(s)$ then $\mathcal{S}(s) = \{\sigma_m\}$. That is, borrowers with credit scores in this interval all get bank loans.

The rest of this prove involves showing that there are always some bank loans provided to borrowers with credit scores in the interval $\sigma \in (\sigma_m - \varepsilon, \sigma_m)$ for which it must be the case that the break even interest rate strictly exceeds the left-hand size of (71). That is, there will always be an interest rates on particular bank loans that can be undercut by the expanding the portfolio of loans that includes σ_m to include the borrowers that get these bank loans. This is very similar to the proof of step 3.

The break-even interest rate a lender in the securitized prime market would charge when his market segment covered $\sigma \in (1 - \varepsilon', 1)$ with $\varepsilon' > 0$ would be

$$R_{t+1}^{*B} \leq \frac{R_t}{\{1 - P[d|\sigma \in (\sigma_m - \varepsilon', \sigma_m)]\} - \frac{1-\gamma}{\gamma}\frac{\underline{\Lambda} W_t}{P_t^C C_t}} \tag{72}$$

Equilibrium would require that for all $0 < \varepsilon' \leq \varepsilon$ the prime securitized loan provider's interest rate is higher than that charged for bank loans. This means that for all $0 < \varepsilon' \leq \varepsilon$

$$P\left[d\left|\sigma \in \left(\sigma_m - \varepsilon', \sigma_m\right)\right.\right] - P\left[d\left|\sigma_m - \varepsilon'\right.\right] > \frac{1-\gamma}{\gamma}\frac{\left(\overline{\Lambda} - \underline{\Lambda}\right)W_t}{P_t^C C_t} \tag{73}$$

Again, this is not the case because that would mean a violation of the continuity property of $P[d|\sigma]$ in σ. Hence, this can not be an equilibrium.

Step 5: Definition of equilibrium.

The above four steps imply that there are, at maximum, four possible market segments. The middle cut-off between these segments is always σ_m. The other cutoffs are as defined by $\sigma_{h,t}$ and $\sigma_{l,t}$ in the main text. This results in the following four market segments: (i) A set of securitized subprime loans covering $\sigma \in [\sigma_{h,t}, 1]$; (ii) a segment of subprime bank loans $\sigma \in [\min\{\sigma_m, \sigma_{h,t}\}, \sigma_{h,t})$; (iii) a set of securitized prime loans $\sigma \in [\min\{\sigma_{l,t}, \sigma_{h,t}\}, \min\{\sigma_m, \sigma_{h,t}\}]$; and (iv) a set of prime bank loans that cover $\sigma \in [0, \min\{\sigma_{l,t}, \sigma_{h,t}\})$.

Financial market setup with pass-through security issue and asymmetric information across lenders:

In this setup some of the variables presented in the paper get re-labeled and take a different interpretation. The working of the financial market hinges though on the same key assumptions: in equilibrium, some financial intermediary have a cost-advantage, and there exists adverse selection.

Assume banks can screen for σ at constant cost, which we set equal to zero without loss of generality. The cost of funding is equal to R_t per dollar of loan, plus $R_t\Lambda(s)W_t$ per loan, where $\Lambda(s) = \overline{\Lambda}$ for banks and $\Lambda(s) = \Lambda_1 < \overline{\Lambda}$

for the secondary market.[12]

We first discuss an equilibrium with only two segments in the loans market, where effectively $\sigma_m = 1$ and there is no subrprime jump in the interest rate profile. Banks underwrite all loans, and can either fund the loan by issuing deposits, or by selling the loan payoff to the secondary market. In equilibrium, free entry implies that the interest rate $R_t^B(\sigma)$ charged by banks for loans held in portfolio with signal σ satisfies:

$$R_t^B(\sigma)\, P_t^B B_t(\sigma)\,(1 - P_t[d|\sigma]) = R_t[P_t^B B_t(\sigma) + \overline{\Lambda} W_t] \tag{74}$$

Note that, for the arguments discussed in the previous section, the same equilibrium risk-profile would obtain regardless of whether the screening technology allows banks to determine whether a loan has a score of σ *or lower,* or whether the loan has a score exactly equal to σ.

The secondary market cannot screen for the loans to which the payoff of the security is linked. Intermediaries buy a nominal amount $P_t^B \widetilde{B}_t(S)/\widetilde{v}_t(S)$ at price $\widetilde{v}_t(S)$ of a pass-through security that pays P_t^B units of currency for every unit of loan, measured in terms of the imperfect-credit good, which is not defaulted. The zero profit condition for a financial intermediary in the competitive secondary market can be written as:

$$D_t(S) R_t \int_{\mathcal{S}(s)} dF(\sigma) = P_t^B \widetilde{B}_t(S) \int_{\mathcal{S}(s)} (1 - P_t[d|\sigma]) dF(\sigma) \tag{75}$$

where $D_t(S)\int_{\mathcal{S}(s)} dF(\sigma)$ is the amount of funding from the household's savings necessary to finance the purchase of the pass-through security. Since the secondary market is selected against by the seller of the pass-through security, it knows that it will always get the worse possible portfolio of loans in the security, implying $\mathcal{S}(s) = [\sigma_h, 1]$. The financial intermediary cash flow must obey the constraint:

$$\left[\Lambda_1 W_t + P_t^B \widetilde{B}_t(S)\widetilde{v}_t(S)\right] \int_{\mathcal{S}(s)} dF(\sigma) = D_t(S) \int_{\mathcal{S}(s)} dF(\sigma) \tag{76}$$

The bank selling loans to be funded by the proceeds from the pass-through securities' sales offers to purchase loan contracts at price v_t per dollar, to be repaid at face value. The bank maximizes profits:

$$\int_{\mathcal{S}(s)} \left[P_t^B \widetilde{B}_t(S)\widetilde{v}_t(S) - P_t^B \widetilde{B}_t(S) v_t(S)\right] dF(\sigma) \tag{77}$$

subject to the demand constraint

$$\widetilde{B}_t(S) v_t(S) \int_{\mathcal{S}(s)} dF(\sigma) = B_t(S) \int_{\mathcal{S}(s)} dF(\sigma) \tag{78}$$

and where we define $v_t(S) = \frac{1}{R_t^B(S)}$, $\widetilde{v}_t(S) = \frac{1}{\widetilde{R}_t^B(S)}$, $R_t^B(S)$ is the implied interest rate offered by the bank on the loan market, $\widetilde{R}_t^B(S)$ is the implied interest rate earned by the secondary market on each loan that does not default within the loan pool in the pass-through security.

Free entry in the competitive banking sector results in $\widetilde{v}_t(S) = v_t(S)$ and $\widetilde{R}_t^B(S) = R_t^B(S)$. Then eqs. (75) through (78) imply:

$$R_t^B(S)\, P_t^B B_t(S) \int_{\mathcal{S}(s)} (1 - P_t[d|\sigma]) dF(\sigma) = R_t[P_t^B B_t(S) + \Lambda_1 W_t] \int_{\mathcal{S}(s)} dF(\sigma)$$

[12] Assuming the cost advantage of the secondary market is in the funding of the loan allows for the total cost of credit to be paid by the financial institution holding the loan on the balance sheet at maturity, even if banks underwrite and screen all loans.

This condition is met for more than one portfolio $\mathcal{S}(s)$: as loans of decreasing riskiness are added to the security $\widetilde{B}_t(S)$, the implied zero-profit interest rate $R_t^B(S)$ gets lower and lower, since each additional loan is of a lower risk class then the ones already in the security. In this setup, $\sigma_{h,t}$ is still defined by the first order condition (20). If the least risky loan in the pass-through security is of quality $\sigma > \sigma_{h,t}$, then a bank could always add a contract of higher quality $\sigma - \varepsilon$ and sell the security at the same price, at a profit. This can only occur until $\sigma = \sigma_{h,t}$. A bank can then still add an even safer loan to the security, but would not be able to buy such a loan contract from any borrower of equivalent higher quality, since the borrower would be served by a bank offering a loan at $R_t^B(\sigma) < R_t^B(S)$.

To obtain an equilibrium with four market segments, and a subprime jump in the interest rate profile, we need incentives for the banking system to deliver prime loans to the secondary market. This can be achieved by assuming that banks have three choices to fund a loan: they can fund it with saving deposits at cost $R_t\overline{\Lambda}W_t$; sell the loan on the secondary market, where the per-loan cost of issuing credit is $R_t\Lambda_1 W_t$; or pay an amount $R_t(\underline{\Lambda} - \Lambda_1)W_t$ to get the pass-through security rated by a third party, certifying that the riskiest loan in the security is of quality no worse than σ_m, and then sell the security. The total per-loan cost of issuing credit to the borrowers funded through this security is then $R_t\underline{\Lambda}W_t$, and the same equilibrium described in the paper obtains.

Derivation of (23):

We can write

$$\begin{aligned} L_t^{\Lambda} &= [1-\sigma_{h,t}]\,\Lambda_1 + [\sigma_{h,t} - \min\{\sigma_m, \sigma_{h,t}\}]\,\underline{\Lambda} + \\ &\quad [\min\{\sigma_m, \sigma_{h,t}\} - \min\{\sigma_{l,t}, \sigma_{h,t}\}]\,\underline{\Lambda} + \min\{\sigma_{l,t}, \sigma_{h,t}\}\,\overline{\Lambda} \\ &= \sigma_{h,t}\underline{\Lambda} + \min\{\sigma_{l,t}, \sigma_{h,t}\}\left(\overline{\Lambda} - \underline{\Lambda}\right) + [1-\sigma_{h,t}]\,\Lambda_1 \end{aligned} \tag{79}$$

which corresponds to (23) in the text.

Derivation of (27):

Define the lending standard function

$$s^*(\sigma) = \{s\,|\sigma \in \mathcal{S}(s)\}. \tag{80}$$

The zero expected profit condition in equilibrium implies that for all $\sigma' \in [0,1]$ it must be the case that

$$R_t^B(\sigma')\,P_t^B B_t(\sigma') = \frac{R_t P_t^B B_t(\sigma') + R_t\Lambda(s^*(\sigma'))\,W_t}{1 - P\left[d\,|\sigma \in \mathcal{S}(s^*(\sigma'))\right]}. \tag{81}$$

Given (11), this can be written as

$$\frac{\gamma}{1-\gamma}R_t P_t^C C_t = \frac{R_t P_t^B B_t(\sigma') + R_t\Lambda(s^*(\sigma'))\,W_t}{1 - P\left[d\,|\sigma \in \mathcal{S}(s^*(\sigma'))\right]}. \tag{82}$$

This allows us to solve for $B_t(\sigma')$ to get

$$B_t(\sigma') = \frac{\gamma}{1-\gamma}\frac{P_t^C C_t}{P_t^B}\left\{\left[1 - P\left[d\,|\sigma \in \mathcal{S}\left(s^*(\sigma')\right)\right]\right] - \frac{1-\gamma}{\gamma}\frac{\Lambda(s^*(\sigma'))\,W_t}{P_t^C C_t}\right\}. \tag{83}$$

Aggregate demand for the imperfect-credit good can now be derived by integrating over σ'. That is,

$$\begin{aligned} B_t &= \int_0^1 B_t(\sigma')\,d\sigma' \\ &= \frac{\gamma}{1-\gamma}\frac{P_t^C C_t}{P_t^B}\left\{\left[1 - \int_0^1 P\left[d\,|\sigma \in \mathcal{S}\left(s^*(\sigma')\right)\right]d\sigma'\right] - \frac{1-\gamma}{\gamma}\frac{W_t}{P_t^C C_t}\left[\int_0^1 \Lambda\left(s^*(\sigma')\right)d\sigma'\right]\right\}. \end{aligned} \tag{84}$$

The final two things to realize for this derivation are that

$$\int_0^1 \Lambda\left(s^*\left(\sigma'\right)\right) d\sigma' = L_t^{\Lambda}, \tag{85}$$

and that

$$\begin{aligned}
\int_0^1 P\left[d\left|\sigma \in \mathcal{S}\left(s^*\left(\sigma'\right)\right)\right.\right] d\sigma' &= \int_0^{\min\left\{\sigma_{l,t},\sigma_{h,t}\right\}} P\left[d\left|\sigma\right.\right] d\sigma \\
&+ \int_{\min\left\{\sigma_{l,t},\sigma_{h,t}\right\}}^{\min\left\{\sigma_m,\sigma_{h,t}\right\}} \left\{ \frac{\int_{\min\left\{\sigma_{l,t},\sigma_{h,t}\right\}}^{\min\left\{\sigma_m,\sigma_{h,t}\right\}} P\left[d\left|\sigma''\right.\right] d\sigma''}{\int_{\min\left\{\sigma_{l,t},\sigma_{h,t}\right\}}^{\min\left\{\sigma_m,\sigma_{h,t}\right\}} 1 d\sigma''} \right\} d\sigma \\
&+ \int_{\min\left\{\sigma_m,\sigma_{h,t}\right\}}^{\sigma_{h,t}} P\left[d\left|\sigma\right.\right] d\sigma \\
&+ \int_{\sigma_{h,t}}^{1} \left\{ \frac{\int_{\min\left\{\sigma_{l,t},\sigma_{h,t}\right\}}^{\min\left\{\sigma_m,\sigma_{h,t}\right\}} P\left[d\left|\sigma''\right.\right] d\sigma''}{\int_{\min\left\{\sigma_{l,t},\sigma_{h,t}\right\}}^{\min\left\{\sigma_m,\sigma_{h,t}\right\}} 1 d\sigma''} \right\} d\sigma.
\end{aligned} \tag{86}$$

The latter simplifies to

$$\begin{aligned}
\int_0^1 P\left[d\left|\sigma \in \mathcal{S}\left(s^*\left(\sigma'\right)\right)\right.\right] d\sigma' &= \int_0^{\min\left\{\sigma_{l,t},\sigma_{h,t}\right\}} P\left[d\left|\sigma\right.\right] d\sigma + \int_{\min\left\{\sigma_{l,t},\sigma_{h,t}\right\}}^{\min\left\{\sigma_m,\sigma_{h,t}\right\}} P\left[d\left|\sigma\right.\right] d\sigma \\
&\quad \int_{\min\left\{\sigma_m,\sigma_{h,t}\right\}}^{\sigma_{h,t}} P\left[d\left|\sigma\right.\right] d\sigma + \int_{\sigma_{h,t}}^{1} P\left[d\left|\sigma\right.\right] d\sigma \\
&= \int_0^1 P\left[d\left|\sigma\right.\right] d\sigma = P\left[d\right].
\end{aligned} \tag{87}$$

Substituting these last two results into (84) yields (27).

Derivation of optimality condition that determines $P_t^X(j)$:

Cost minimization problem of the intermediate good producer j.

We assume production of the intermediate good supplied to retailers of B_t and C_t happens through the same technology. Therefore any intermediate good producer can supply either of the two retail sectors, and we need to solve the profit maximization problem only once, with demand equal to the sum $X^d(j) = X^B(j) + X^C(j)$.

Firm J chooses factor demands in a perfectly competitive fashion. In particular, it chooses $L_t^X(j)$ and $K_t(j)$ to minimize:

$$\frac{W_t}{P_t^X} L_t^X(z) + \zeta_t K_t(z) \tag{88}$$

subject to

$$X_t^s(j) \leq Z_t L_t^X(j)^{1-\alpha} \overline{K}_t(j)^{\alpha}. \tag{89}$$

Capital is in fixed supply, and perfectly mobile across firms. Its real rental rate in units of X_t is ζ_t. The demand for labor and capital is given by the FOCs:

$$\frac{1}{\mathrm{MC}_t} \frac{W_t}{P_t^X} = (1-\alpha) \frac{X_t^s(j)}{L_t^X(j)} \tag{90}$$

$$\frac{1}{\mathrm{MC}_t} \zeta_t = \alpha \frac{X_t^s(z)}{K_t(z)}, \tag{91}$$

where MC_t is the real marginal cost of producing output. We may express MC_t as a function of prices, wages, and technology

$$\text{MC}_t = \frac{1}{Z_t}\left(\frac{W_t}{(1-\alpha)P_t^X}\right)^{1-\alpha}\left[\frac{\zeta_t}{\alpha}\right]^{\alpha}. \tag{92}$$

Given cost-minimization, the firm takes MC_t as given, when choosing its output price.

Optimal pricing problem of the intermediate good producer j.

Each firm purchases labor which it converts into a differentiated $X^s(j)$ good sold to retail firms. Intermediate firms adjust prices according to the Calvo updating model. Each period a firm can adjust its price with probability $1-\xi$. Since all firms that adjust their price are identical, they all set the same price. Given $MC_t^n = P_t^X MC_t$, the intermediate firm chooses $P_t^X(j)$, $X_t^s(j)$ to maximize

$$\sum_{k=0}^{\infty}(\xi\beta)^k E_t\left[\Lambda_{t,t+k}\frac{P_t^X(j) - MC_{t+k}^n}{P_{t+k}^X}X_{t+k}^s(j)\right] \tag{93}$$

subject to

$$X_{t+k}^s(j) = X_{t+k}^d(j) = \left[\frac{P_t^X(j)}{P_{t+k}^X}\right]^{-\varepsilon}X_{t+k}^d \tag{94}$$

where X_t^d is aggregate demand for the X goods basket and $\beta\Lambda_{t,t+1}$ is the stochastic discount factor. The firm's optimality condition can be written as:

$$P_t^X(j)E_t\sum_{k=0}^{\infty}(\xi\beta)^k\Lambda_{t,t+k}\left[\frac{P_t^X(j)}{P_{t+k}^X}\right]^{1-\varepsilon}X_{t+k}^d = \frac{\varepsilon}{\varepsilon-1}E_t\sum_{k=0}^{\infty}(\xi\beta)^k\Lambda_{t,t+k}MC_{t+k}^n\left[\frac{P_t^X(j)}{P_{t+k}^X}\right]^{1-\varepsilon}X_{t+k}^d \tag{95}$$

Write eq. (32) as:

$$P_t^X(j) = \frac{\hat{G}_t}{\hat{H}_t}, \tag{96}$$

where

$$\hat{G}_t = \mu MUC_{C,t}MC_t^N P_t^{X^{\varepsilon-1}}X_t^d + E_t\xi\beta\hat{G}_{t+1} \tag{97}$$

$$\hat{H}_t = MUC_{C,t}P_t^{X^{\varepsilon-1}}X_t^d + E_t\xi\beta\hat{H}_{t+1}. \tag{98}$$

and $MUC_{C,t}$ is the marginal utility of consumption in terms of the C_t good, which in equilibrium is homogeneous with the intermediate good X_t^d. Let $\mu = \frac{\varepsilon}{\varepsilon-1}$ be the flexible-price equilibrium markup. Divide $\hat{G}_t$ by $P_t^{X^\varepsilon}$ and $\hat{H}_t$ by $P_t^{X^{\varepsilon-1}}$ to obtain:

$$\tilde{G}_t \equiv \frac{\hat{G}_t}{P_t^{X^\varepsilon}} = \mu MUC_{C,t}\frac{MC_t^N}{P_{H,t}}X_t^d + E_t\xi\beta\frac{\hat{G}_{t+1}}{P_{t+1}^{X^\varepsilon}}\frac{P_{t+1}^{X^\varepsilon}}{P_t^{X^\varepsilon}} = \mu MUC_{C,t}MC_tX_t^d + E_t\xi\beta\tilde{G}_{t+1}(\Pi_{X,t+1})^{\varepsilon} \tag{99}$$

$$\tilde{H}_t \equiv \frac{\hat{H}_t}{P_t^{X^{\varepsilon-1}}} = MUC_{H,t}X_t^d + E_t\xi\beta\frac{\hat{H}_{t+1}}{P_{t+1}^{X^{\varepsilon-1}}}\frac{P_{t+1}^{X^{\varepsilon-1}}}{P_t^{X^{\varepsilon-1}}} = MUC_{C,t}X_t^d + E_t\xi\beta\tilde{H}_{t+1}(\Pi_{X,t+1})^{\varepsilon-1} \tag{100}$$

where Π_X is the steady state domestic good basket gross inflation rate. Using:

$$P_t^X(j) = \frac{\hat{G}_t}{\hat{H}_t} = \frac{\hat{G}_t/P_t^{X^\varepsilon}}{\hat{H}_t/P_t^{X^\varepsilon}} = \frac{\tilde{G}_tP_t^X}{\tilde{H}_t}. \tag{101}$$

the law of motion for the aggregate price index is:

$$\begin{aligned} P_t^{X^{\varepsilon-1}} &= \xi P_{t-1}^{X^{\varepsilon-1}} + (1-\xi)P_t^X(i)^{1-\varepsilon} = \xi P_{t-1}^{X^{\varepsilon-1}} + (1-\xi)\left[\frac{\hat{G}_t}{\hat{H}_t}\right]^{1-\varepsilon} \\ [\Pi_{X,t}]^{1-\varepsilon} &= \xi + (1-\xi)\left[\frac{P_t^X(i)}{P_{t-1}^X}\right]^{1-\varepsilon} = \xi + (1-\xi)\left[\frac{\tilde{G}_t}{\tilde{H}_t}\Pi_{X,t}\right]^{1-\varepsilon} \end{aligned} \tag{102}$$

Also we can write price dispersion as

$$\begin{aligned} f_t &\equiv \int_0^1 \left[\frac{P_t^X(j)}{P_t^X}\right]^{-\varepsilon} dj \\ &= (1-\xi)\left(\frac{\tilde{G}_t}{\tilde{H}_t}\right)^{-\varepsilon} + \xi \Pi_{X,t}^{\theta} f_{t-1}. \end{aligned} \tag{103}$$

Market Clearing Conditions and Aggregation:

Retail firms purchase output $X^s(j)$ from each j firm. Since production function of retail firms is simply $X^C = C$ and $X^B = B$ obtain that $P^B = P^C = P^X$ in perfect competition. Total intermediate demand is

$$X_t^d = X_t^C + X_t^B \tag{104}$$

Define L_t^X and K_t as

$$K_t = \int_0^1 K_t(z)dz, \tag{105}$$

and

$$L_t^X = \int_0^1 L_t^X(z)dz. \tag{106}$$

Using the definition of L_t^X, K_t and the FOC from the cost minimization problem:

$$\begin{aligned} Z_t L_t^{X\,1-\alpha} K_t{}^{\alpha} &= Z_t \left[\int_0^1 L_t^X(z)dz\right]^{1-\alpha} \left[\int_0^1 K_t(z)dz\right]^{\alpha} = \\ &= Z_t \left[(1-\alpha)MC_t \frac{P_t^X}{W_t}\right]^{1-\alpha} \left[\alpha MC_t \frac{1_t}{\zeta_t}\right]^{\alpha} \left[\int_0^1 X_t^s(z)dz\right]^{1-\alpha} \left[\int_0^1 X_t^s(z)dz\right]^{\alpha} \\ &= Z_t \left[(1-\alpha)MC_t \frac{P_t^X}{W_t}\right]^{1-\alpha} \left[\alpha MC_t \frac{1_t}{\zeta_t}\right]^{\alpha} \int_0^1 X_t^s(z)dz \end{aligned} \tag{107}$$

Then using

$$X_{t+k}^s(j) = X_{t+k}^d(j) = \left[\frac{P_t^X(j)}{P_{t+k}^X}\right]^{-\varepsilon} X_{t+k}^d \tag{108}$$

$$\begin{aligned} Z_t L_t^{X\,1-\alpha} K_t{}^{\alpha} &= Z_t \left[(1-\alpha)MC_t \frac{P_t^X}{W_t}\right]^{1-\alpha} \left[\alpha MC_t \frac{1_t}{\zeta_t}\right]^{\alpha} \left(\frac{P_t^X}{\left[\int_0^1 P_t^X(j)^{-\varepsilon} dj\right]^{-\frac{1}{\varepsilon}}}\right)^{\varepsilon} X_t^d \\ &= \left(\frac{P_t^X}{\left[\int_0^1 P_t(j)^{-\varepsilon} dj\right]^{-\frac{1}{\varepsilon}}}\right)^{\varepsilon} X_t^d \end{aligned} \tag{109}$$

where

$$X_t^d = \left[\int_0^1 X_t^d(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{110}$$

Then

$$\begin{aligned} Z_t L_t^{X\,1-\alpha} K_t^{\alpha} &= \int_0^1 \left(\frac{P_t^X(j)}{P_t^X}\right)^{-\varepsilon} dj X_t^d \\ &= f_t X_t^d \\ &= f_t(B_t + C_t) \end{aligned} \tag{111}$$

where latter comes from market clearing and production function for the retail sector.

Alternatively, can be rewritten as aggregate labor demand

$$\begin{aligned} L_t^X &= \int_0^1 L_t^X(j)dj \\ &= \left[f_t X_t^d \frac{1}{Z_t K_t^\alpha} \right]^{\frac{1}{1-\alpha}} \end{aligned} \tag{112}$$

Aggregating the budget constraint obtain:

$$P_t^B B_t + P_t^C C_t = W_t L_t^X + \Pi_t + \Gamma_t \tag{113}$$

Since payments to profits, capital and labor exhaust all output of intermediate sector, obtain in real terms

$$\begin{aligned} B_t + C_t &= \frac{W_t}{P_t} L_t^X + \Pi_t^r + \Gamma_t^r \\ &= \int X_t^s(j)dj \equiv X_t^s \end{aligned} \tag{114}$$

where the latter defines aggregate supply X_t^s and we used $P_t = P_t^C = P_t^B = P_t^X$. Then using

$$\begin{aligned} Z_t L_t^{X\,1-\alpha} K_t^\alpha &= f_t(B_t + C_t) \\ B_t + C_t &= X_t^s \end{aligned} \tag{115}$$

obtain that aggregate supply is equal to

$$X_t^s = \left(Z_t L_t^{X\,1-\alpha} K_t{}^\alpha \right) f_t^{-1} \tag{116}$$

and

$$X_t^d = X_t^s \tag{117}$$

Derivation of (39):

This follows from

$$\begin{aligned} L_t^X &= \int_0^1 L_{it}^X di = \int_0^1 \left(\frac{X_{it}}{Z_t} \right)^{\frac{1}{1-\alpha}} di \\ &= \left\{ \int_0^1 \left(\frac{1}{P_{it}^X} \right)^{\frac{\epsilon}{1-\alpha}} \right\} \left(P_t^X \right)^{\frac{\epsilon}{1-\alpha}} \left(\frac{X_t}{Z_t} \right)^{\frac{1}{1-\alpha}} \\ &= \left[\left(\frac{P_t^X}{\left\{ \int_0^1 (P_{it}^X)^{-\frac{\epsilon}{1-\alpha}} \right\}^{-\frac{1-\alpha}{\epsilon}}} \right)^{\epsilon} \left(\frac{X_t}{Z_t} \right) \right]^{\frac{1}{1-\alpha}} . \end{aligned} \tag{118}$$

Derivation of (41):

From (27), it can be seen that aggregate demand for C_t and B_t can be interpreted as the solution of the following intratemporal expenditure minimization problem

$$\{C_t, B_t\} = \underset{(C,B)\in\mathbb{R}_+^2}{\arg\min} \left\{ P_t^C C + \widetilde{P}_t^B B \text{ s.t. } Y_t \geq C^{1-\gamma} B^\gamma \right\}. \tag{119}$$

The price level that corresponds to this expenditure minimization problem is not $\overline{P}_t$, but equals

$$\begin{aligned}\widetilde{P}_t &= \left(\frac{P_t^C}{1-\gamma}\right)^{1-\gamma}\left(\frac{\widetilde{P}_t^B}{\gamma}\right)^{\gamma} \\ &= \left(\frac{1}{1-\gamma}\right)^{1-\gamma}\left(\frac{1}{\gamma}\right)^{\gamma}\left(\int_0^1\left(\frac{R_t}{R_t^B(\sigma)}\right)d\sigma\right)_t^{-\gamma}\overline{P} \\ &\sim \left(\int_0^1\left(\frac{R_t}{R_t^B(\sigma)}\right)d\sigma\right)_t^{-\gamma}\overline{P}.\end{aligned} \tag{120}$$

Derivation of equilibrium dynamics:

Full system of equilibrium equations:

Equilibrium in this economy can be written as a path of the following 21 variables

$$\left\{\begin{array}{c} Y_{t+s}, C_{t+s}, B_{t+s}, X_{t+s}, L_{t+s}, L_{t+s}^X, L_{t+s}^{\Lambda}, \sigma_{l,t+s}, \sigma_{h,t+s}, R_{t+s}^B(\sigma_l), R_{t+s}^B(\sigma_h), \\ W_{t+s}^r, MUC_{t+s}, MC_{t+s}, R_{t+s}, \Pi_{X,t+s}, \Pi_{t+s}, \tilde{G}_{t+s}, \widetilde{H}_{t+s}, f_{t+s}, \Omega_{t+s} \end{array}\right\}_{s=0}^{\infty}, \tag{121}$$

which, in any period t, satisfy the following 21 equations, where we assume all four loan market segments exist:

1. Marginal cost

$$W_t^r \frac{L_t^X/X_t}{(1-\alpha)} = MC_t \tag{122}$$

2. Wholesale production

$$X_t = \left(Z_t L_t^{X\,1-\alpha} K_t^{\,\alpha}\right) f_t^{-1} \tag{123}$$

3. Market clearing

$$X_t = B_t + C_t \tag{124}$$

4. Labor demand

$$L_t = L_t^{\Lambda} + L_t^X \tag{125}$$

5. PPI

$$[\Pi_{X,t}]^{1-\varepsilon} = \xi + (1-\xi)\left[\frac{\tilde{G}_t}{\tilde{H}_t}\Pi_{X,t}\right]^{1-\varepsilon} \tag{126}$$

6. intermediate firms pricing for $P_t^X(j) = \frac{\tilde{G}_t P_t^X}{\tilde{H}_t}$

$$\tilde{G}_t = \mu MUC_{C,t} MC_t X_t^d + E_t \xi \beta \tilde{G}_{t+1} (\Pi_{X,t+1})^{\varepsilon} \tag{127}$$

7. intermediate firms pricing for $P_t^X(j) = \frac{\tilde{G}_t P_t^X}{\tilde{H}_t}$

$$\tilde{H}_t = MUC_{C,t} X_t^d + E_t \xi \beta \tilde{H}_{t+1} (\Pi_{X,t+1})^{\varepsilon-1} \tag{128}$$

8. price dispersion for $f_t = \int_0^1 \left(\frac{P_t^X(j)}{P_t^X}\right)^{-\varepsilon}$

$$f_t = (1-\xi)\left(\frac{\tilde{G}_t}{\tilde{H}_t}\right)^{-\varepsilon} + \xi \Pi_{X,t}^{\varepsilon} f_{t-1}. \tag{129}$$

9. labor supply

$$W_t^r = \frac{W_t}{P_t} = \frac{\nu}{1-\gamma} C_t. \tag{E1}$$

10. aggregate consumption

$$Y_t = C_t^{1-\gamma} B_t^{\gamma}. \tag{E2}$$

11. marginal utility of consumption

$$MUC_t = \frac{1}{C_t} \tag{130}$$

12. consumption Euler equation

$$1 = \beta R_{t+1} E_t \left[\frac{C_t}{C_{t+1}} \frac{1}{\Pi_{X,t+1}} \right]. \tag{E3}$$

13. σ_{l_t}

$$\left(P\left[d \left| \sigma \in \left[\sigma_{l_t}, \sigma_m \right] \right. \right] - P\left[d \left| \sigma_{l_t} \right. \right] \right) = \frac{(1-\gamma)}{\gamma} \frac{\left(\overline{\Lambda} - \underline{\Lambda} \right)}{C_t} W_t^r \tag{131}$$

14. σ_{h_t}

$$\left(P\left[d \left| \sigma \in \left[\sigma_{h_t}, 1 \right] \right. \right] - P\left[d \left| \sigma_{h_t} \right. \right] \right) = \frac{(1-\gamma)}{\gamma} \frac{\underline{\Lambda}}{C_t} W_t^r \tag{132}$$

15. $R_t^B(\sigma_l)$

$$R_t^B\left(\sigma_l\right) = \frac{R_t}{\left\{ 1 - P\left[d|\sigma_l \right] \right\} - \frac{1-\gamma}{\gamma} \frac{\overline{\Lambda} W_t^r}{C_t}} \tag{133}$$

16. $R_t^B(\sigma_h)$

$$R_t^B\left(\sigma_h\right) = \frac{R_t}{\left\{ 1 - P\left[d|\sigma_h \right] \right\} - \frac{1-\gamma}{\gamma} \frac{\underline{\Lambda} W_t^r}{C_t}} \tag{134}$$

17. Labor demand for banking sector

$$L_t^{\Lambda} = \underline{\Lambda} \sigma_{h,t} + \left(\overline{\Lambda} - \underline{\Lambda} \right) \sigma_{l,t} + \Lambda_1 \left[1 - \sigma_{h,t} \right]. \tag{135}$$

18. Default wedge

$$\begin{aligned} \Omega_t &= \left\{ \left[1 - P\left[d \right] \right] - \frac{1-\gamma}{\gamma} W_t^r \frac{L_t^{\Lambda}}{C_t} \right\} \\ &= \int_0^1 \left(\frac{R_{t+1}}{R_{t+1}^B\left(\sigma\right)} \right) d\sigma \end{aligned} \tag{136}$$

19. Aggregate borrowing

$$B_t = \frac{\gamma}{1-\gamma} \frac{P_t^C}{P_t^B} C_t \left\{ \Omega_t \right\} \tag{137}$$

20. CPI

$$\Pi_t = \Pi_{X,t} \left(\frac{\Omega_t}{\Omega_{t-1}} \right)^{-\gamma} \tag{138}$$

21. Monetary Policy

$$R_t = f(\pi_t, Y_t, \varepsilon_t^M) \tag{139}$$

Table 1: Debt market equilibrium

Market	Securitized	Gross Nominal Interest Rate ($R_t^B(\sigma)$) and Market Segment
Prime	No	$\frac{R_t}{\{1-P[d\|\sigma]\}-\frac{1-\gamma}{\gamma}\frac{\overline{\Lambda}W_t}{P_t^C C_t}}$ for $\sigma \in [0, \min\{\sigma_{l,t}, \sigma_{h,t}\})$
Prime	Yes	$\frac{R_t}{\left\{1-P\left[d\middle\|\sigma\in\left[\min\left\{\sigma_{l,t},\sigma_{h,t}\right\},\min\left\{\sigma_m,\sigma_{h,t}\right\}\right]\right]\right\}-\frac{1-\gamma}{\gamma}\frac{\underline{\Lambda}W_t}{P_t^C C_t}}$ for $\sigma \in [\min\{\sigma_{l,t}, \sigma_{h,t}\}, \min\{\sigma_m, \sigma_{h,t}\}]$
Subprime	No	$\frac{R_t}{\{1-P[d\|\sigma]\}-\frac{1-\gamma}{\gamma}\frac{\underline{\Lambda}W_t}{P_t^C C_t}}$ for $\sigma \in [\min\{\sigma_m, \sigma_{h,t}\}, \sigma_{h,t})$
Subprime	Yes	$\frac{R_t}{1-P\left[d\middle\|\sigma\in\left[\sigma_{h,t},1\right]\right]-\frac{1-\gamma}{\gamma}\frac{\Lambda_1 W_t}{P_t^C C_t}}$ for $\sigma \in [\sigma_{h,t}, 1]$

Note: In Appendix A we show that if $\sigma_{h,t} \leq \sigma_m$ then $\sigma_{h,t} \leq \sigma_{l,t}$.

Table 2: Preference and Technology parameters

Parameter	Interpretation	Value
β	Discount factor	0.99
ν	Disutility of labor	0.5
ϵ	Demand elasticity for intermediates	6
α	Capital elasticity of output	1/3
ξ	Price stickiness (Calvo parameter)	0.7
ρ_z	Persistence of productivity shock	0.95

Note: parameter calibration based on quarterly frequency of observation.

All interest rates are annualized quarterly rates.

Table 3: Matched interest rate spreads

Spread	**Definition**	**Value**
safest-to-riskless	$R_t^B(0) - R_t$	5.00%
riskiest-to-safest	$R_t^B(1) - R_t^B(0)$	5.23%
subprime-jump	$\lim_{\sigma \downarrow \sigma_{m,t}} R_t^B(\sigma) - R_t^B(\sigma_{m,t})$	1.64%
subprime-premium	$\widetilde{R}_t^{subprime} - \widetilde{R}_t^{prime}$	3.50%

Note: all interest rates and spreads are annualized quarterly rates.

Table 4: Imperfect credit-market parameters

Parameter	**Interpretation**	**Value**
	Preferences	
γ	Utility weight of B_t	0.5
	Screening costs	
Λ_1	Fixed loan-underwriting cost	0.005
$\underline{\Lambda}$	Low-quality loan-screening cost	0.0088
$\overline{\Lambda}$	High-quality loan-screening cost	0.011
σ_m	Steady state Prime-subprime cut-off level	0.73
	Default-rate process	
ξ	Elasticity with respect to output	20
$\overline{\Xi}$	Average default rate	0.75%
ρ_Ξ	Persistence of default risk shock	0.95

Note: parameter calibration based on quarterly frequency of observation.

Table 5: Impact of securitization on steady state

Spread	**No Securitization**	**Securitization**
	Risk profile of interest rates	
R_t	4.00%	4.00%
$\widetilde{R}_t^{all}$	12.55%	11.51%
$\widetilde{R}_t^{prime}$		10.65%
$\widetilde{R}_t^{subprime}$		14.15%
	Market segments	
σ_l	0.73	0.50
σ_m	0.73	0.73
σ_h	1	0.80
	Default wedge	
Ω_t	0.9806	0.9830
	Securitization gain in default wedge	
all	0.26%	
prime	0.10%	
subprime	0.62%	

Note: all interest rates and spreads are annualized quarterly rates.

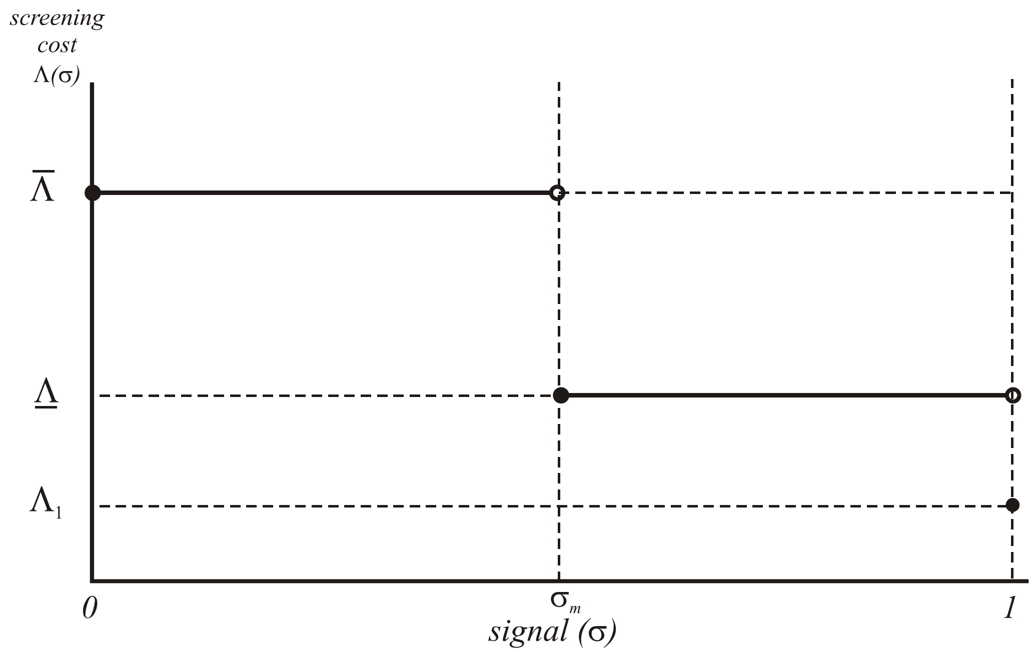

Figure 1: Screening cost function.

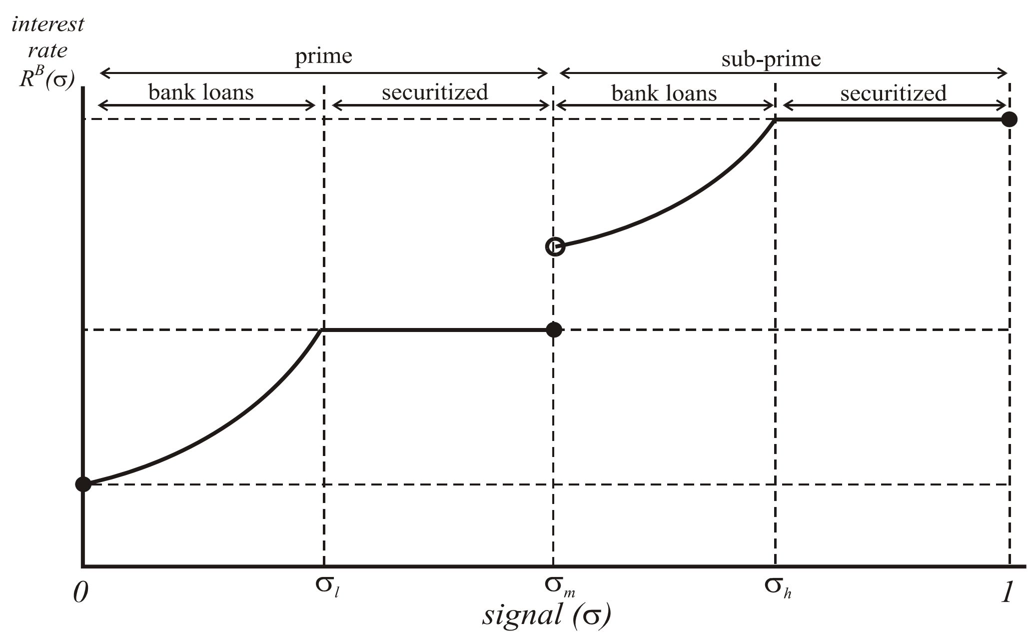

Figure 2: Equilibrium segmentation of debt market. Risky interets rate is measured as a spread relative to the riskless rate.

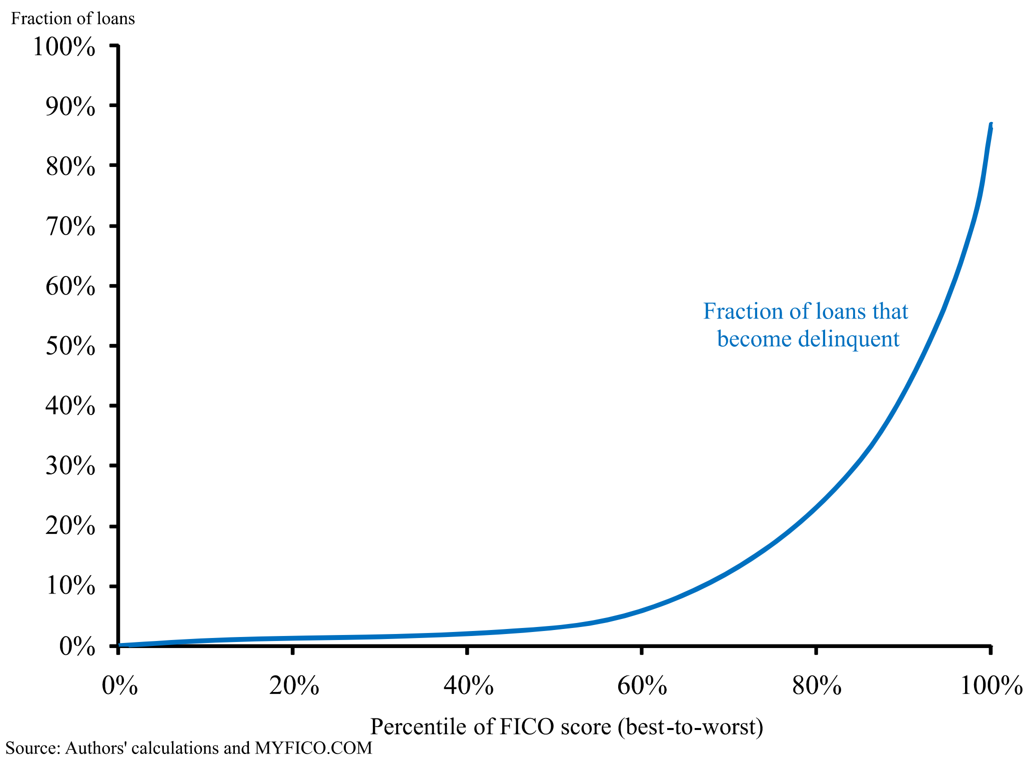

Figure 3: Empirical proxy for $\Pr[d\,|\sigma]$

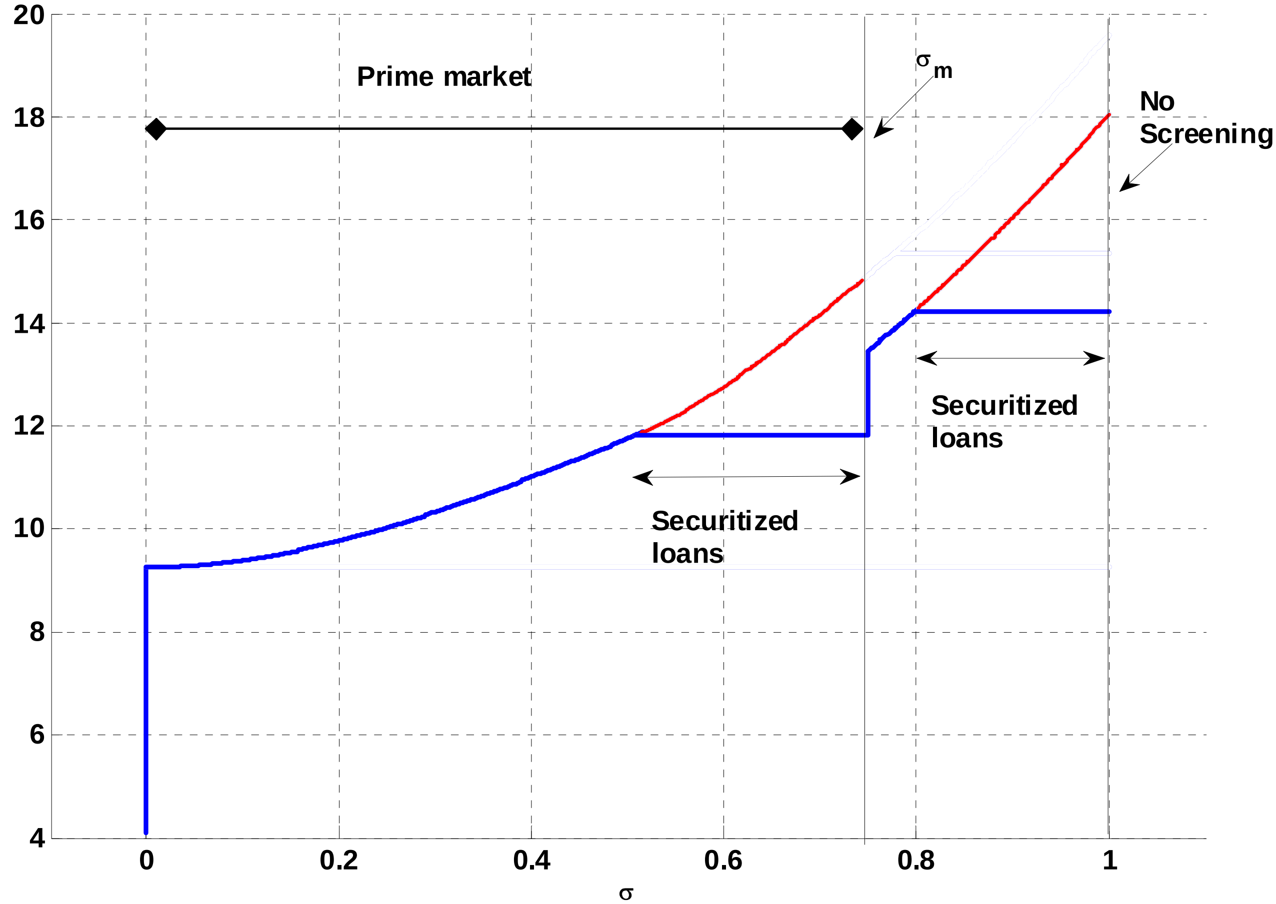

Figure 4: Steady state risk profile of interest rates as a function of the signal σ, with (dark line) and without (light line) securitization. The conditional probability of default $\Pr(\delta|\sigma)$ is quadratic in the signal σ.

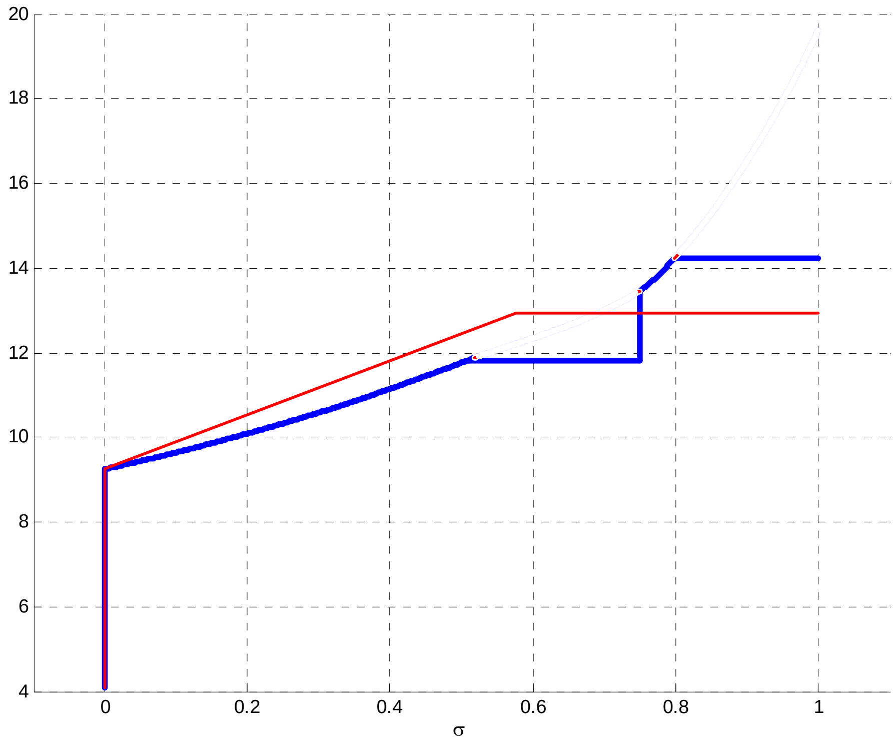

Figure 5: Steady state risk profile of interest rates as a function of the signal σ. Thick line: conditional probability of default $\Pr(\delta|\sigma)$ quadratic in the signal σ. Thin line: conditional probability of default $\Pr(\delta|\sigma)$ linear in the signal σ. Unconditional probability of default $\Pr(\delta)$ is identical across the two cases.

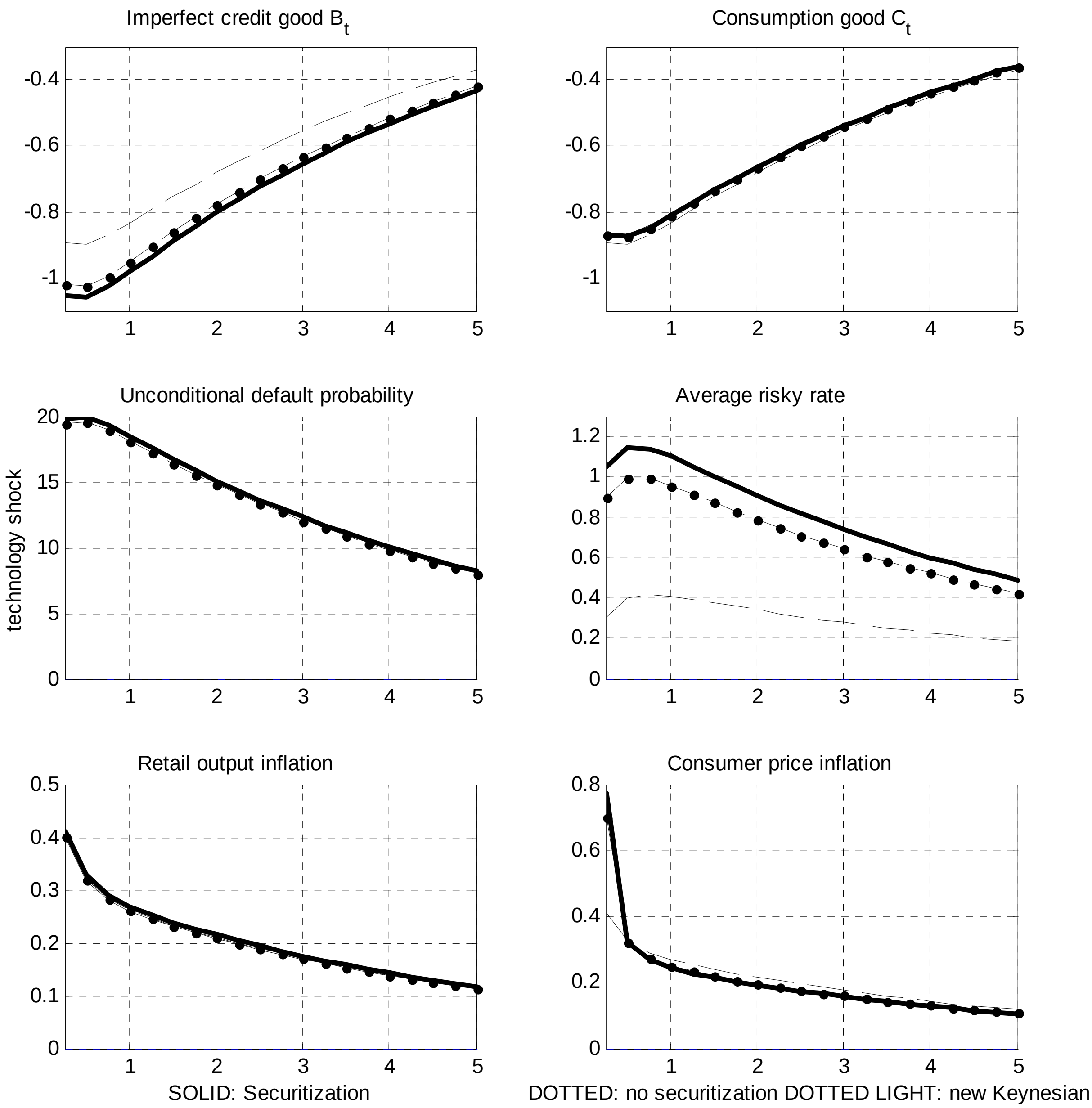

Figure 6: Impulse response function to a 1% fall in productivity. Interest rates are annualized.

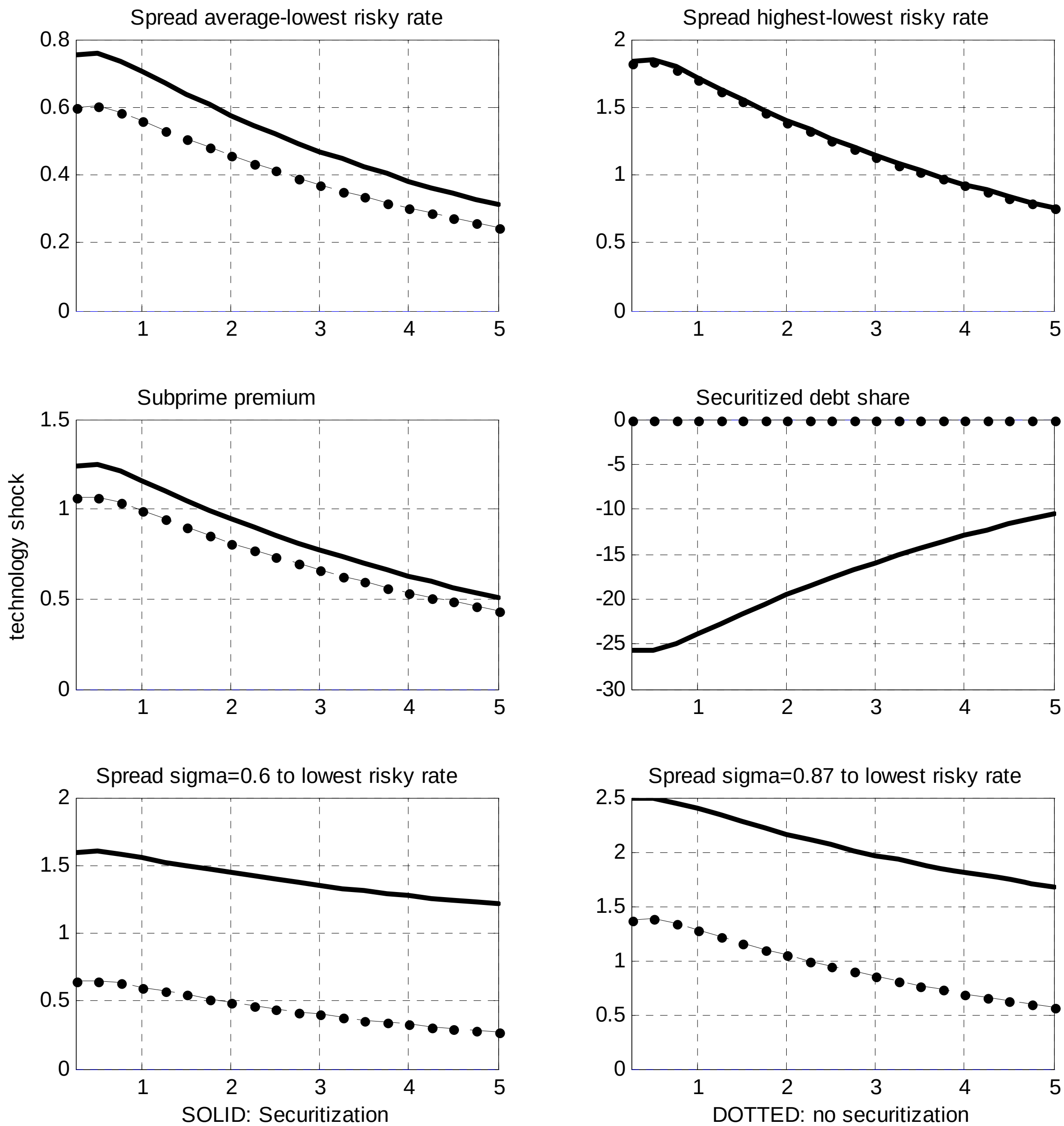

Figure 7: Impulse response function to a 1% fall in productivity. Interest rates are annualized.

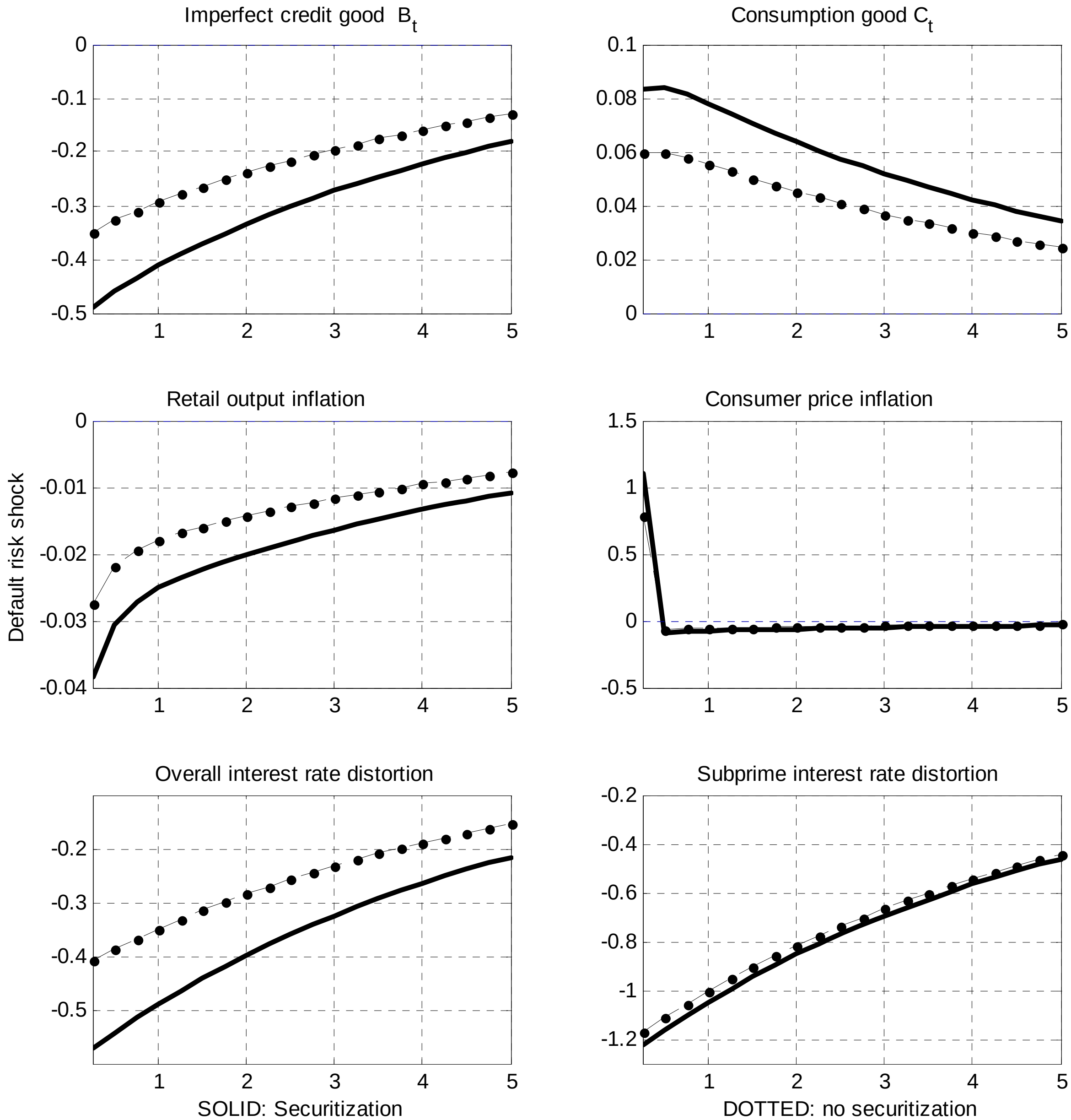

Figure 8: Impulse response function to financial disturbance. Unconditional default rate increases from 0.75% to 1.125%. Upper limit for risk in securitized prime portfolio σ_m increases by 15%. Interest rates are annualized.

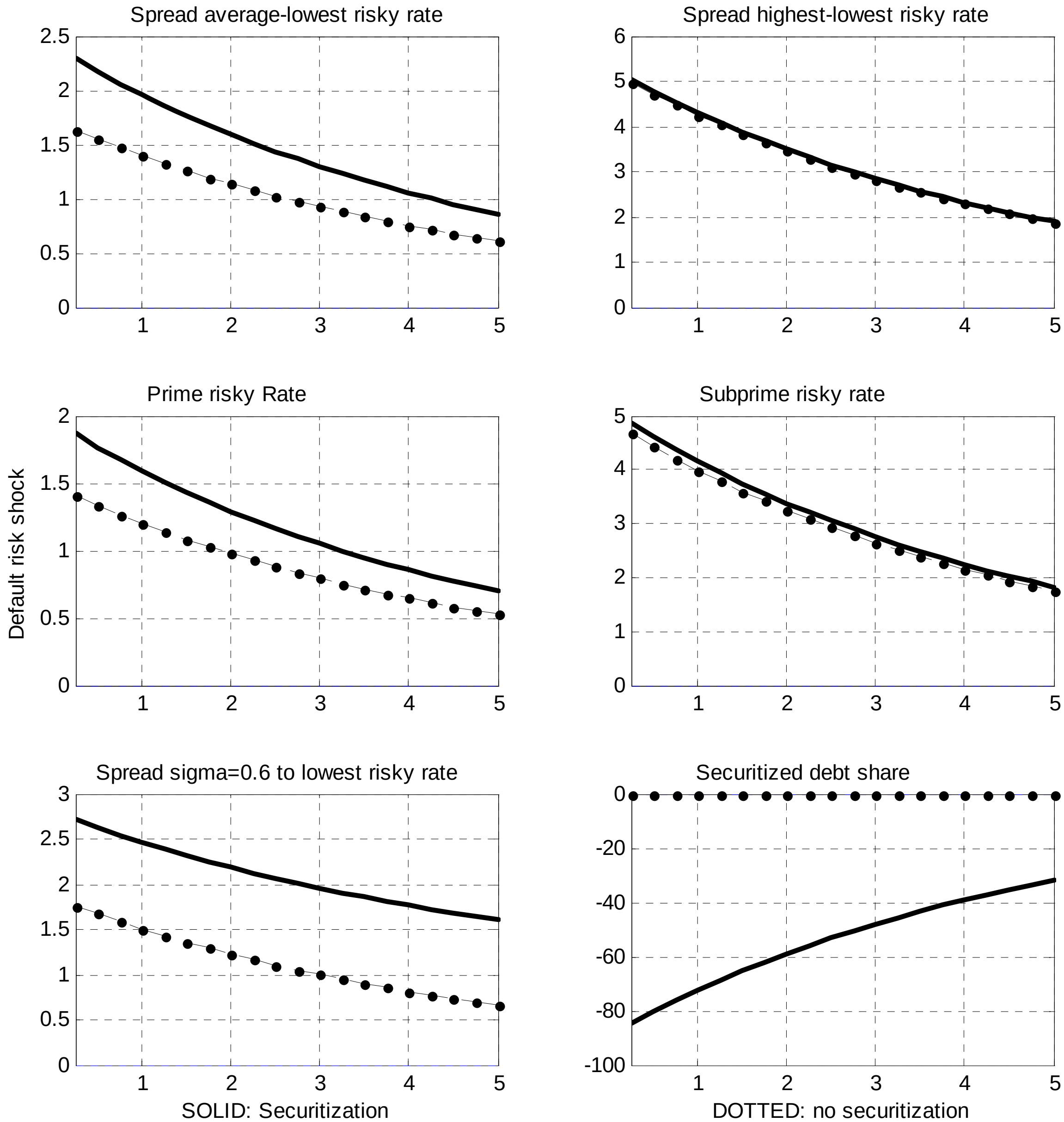

Figure 9: Impulse response function to financial disturbance. Unconditional default rate increases from 0.75% to 1.125%. Upper limit for risk in securitized prime portfolio σ_m increases by 15%. Interest rates are annualized.

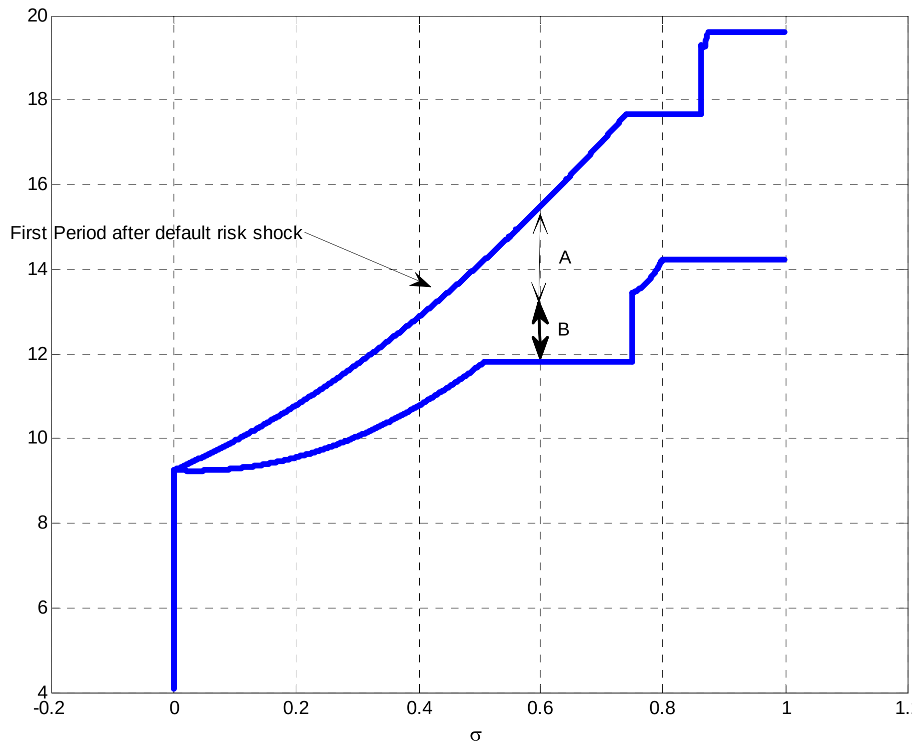

Figure 10: Impact at time $t = 1$ of financial disturbance on risk structure of interest rates. Unconditional default rate increases from 0.75% to 1.125%. Upper limit for risk in securitized prime portfolio σ_m increases by 15%. Interest rates are annualized. For loan with $\sigma = 0.6$: distance 'A' is the increase in rate due to change in default risk, distance 'B' is the increase in rate due to loss of the securitization discount.

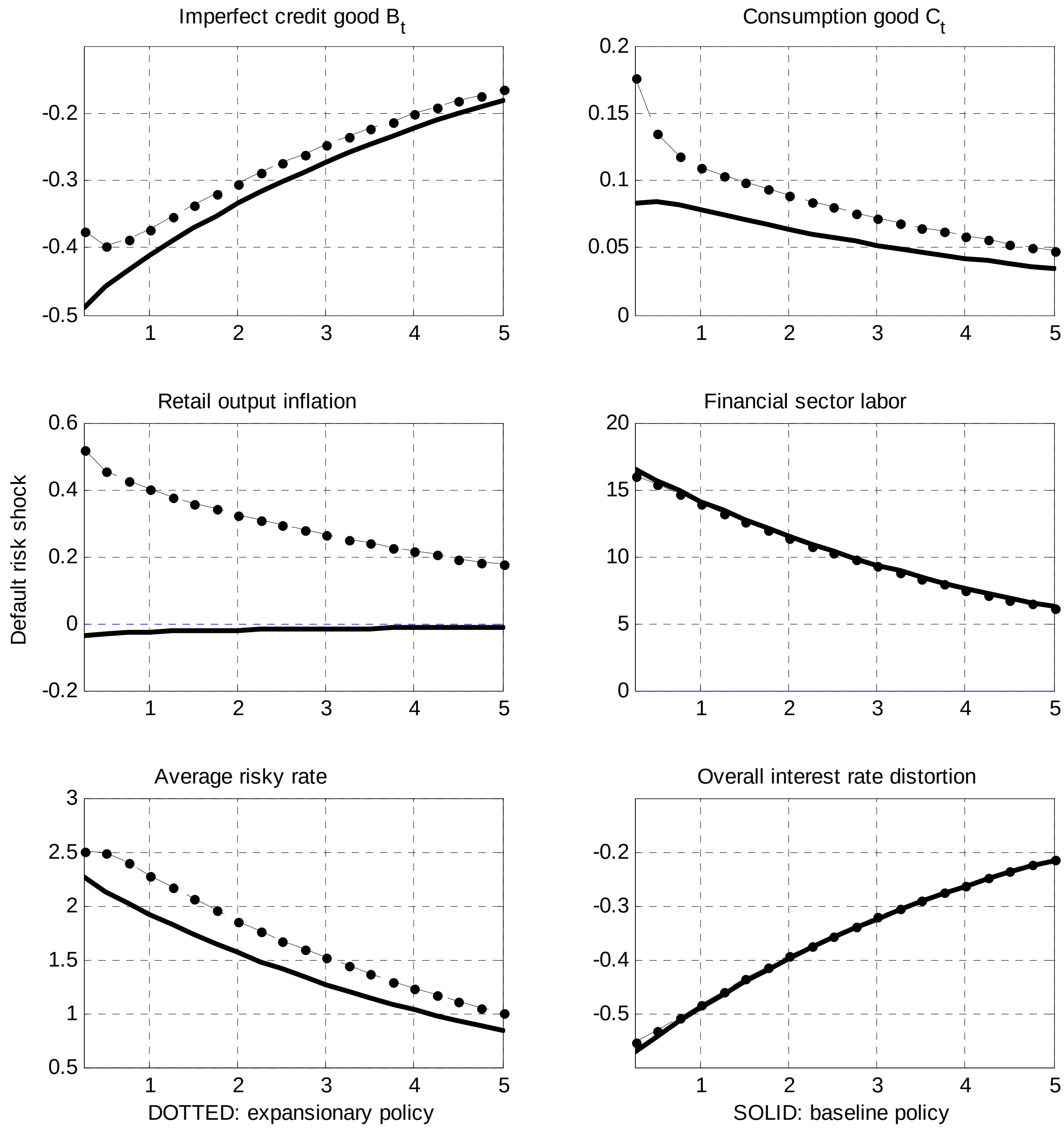

Figure 11: Impulse response function to financial disturbance. under two alternative interest rate policies. Baseline policy does not react to output. Expansionary policy implies interest rate decreases by 1% (annualized) for a 1% fall in output.

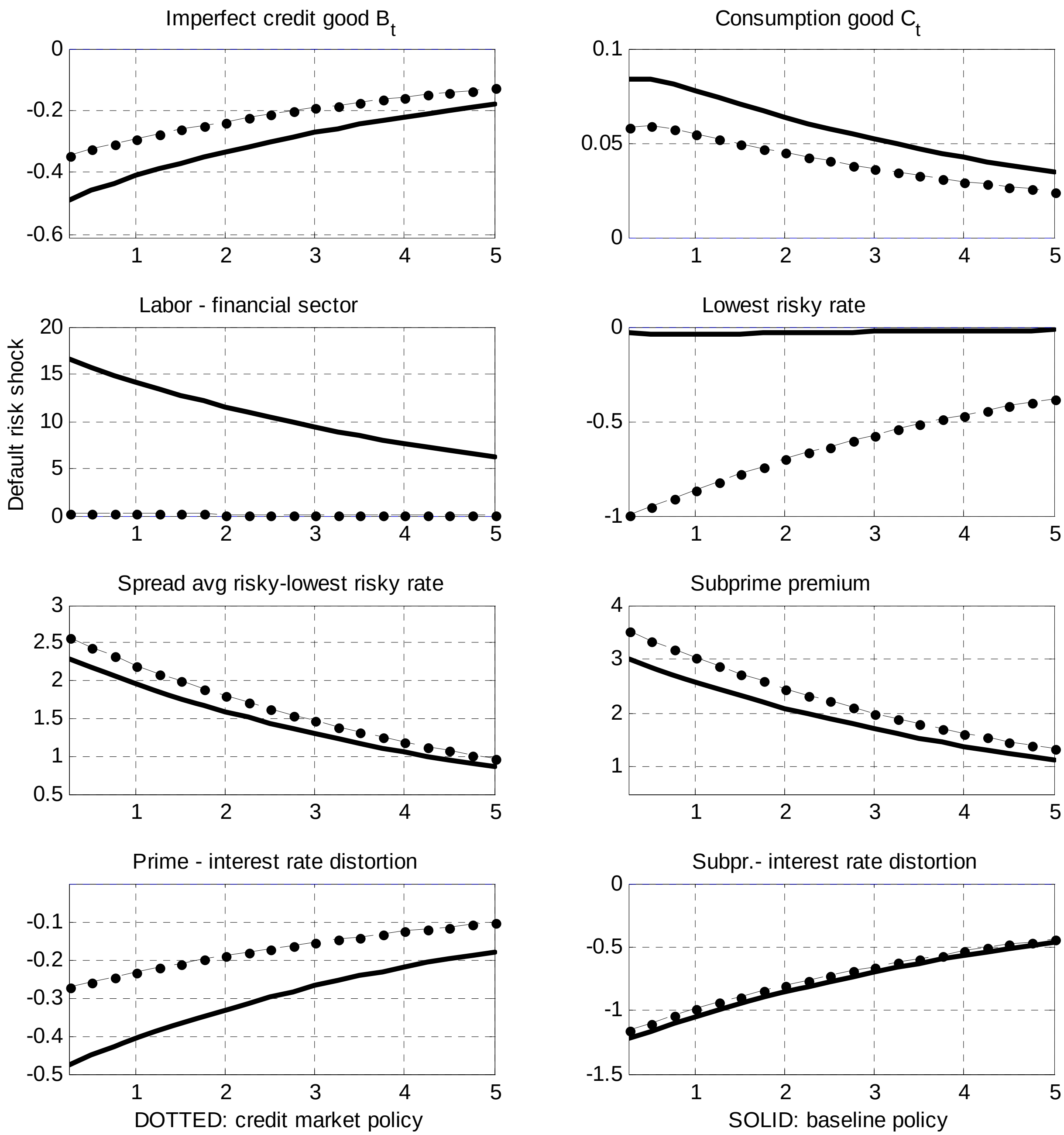

Figure 12: Impulse response function to financial disturbance. under two alternative policies. Baseline interest rate policy does not react to output. Credit market policy lowers cost of screening by 20% for all financial intermediaries.