# Consumers' Imperfect Information and Nominal Rigidities

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**Abstract**

This paper develops a model of slow macroeconomic adjustment to monetary shocks. I proceed in three steps. First, I develop a mechanism for price rigidities. My mechanism captures the notion that firms are reluctant to increase prices after an increase in demand or costs because it creates a disproportionate adverse reaction among consumers. These reactions arise endogenously for purely informational reasons. The key assumption is that some consumers are better informed than others about monetary shocks. If few consumers are informed, equilibria with nominal rigidity exist. In these equilibria firms do not change prices even though they are arbitrarily well informed, and have no menu costs. Moreover, if the proportion of informed consumers is low enough, these equilibria dominate equilibria with flexible prices. Second, I show that when firms do not change prices they inflict an informational externality on other firms. Consumers buy goods sequentially, one after the other, and change their beliefs about shocks when they see prices change. Therefore, when firms do not change prices, consumers do not learn. This hurts both firms and consumers. Third, I study the dynamic responses of output and inflation to shocks. Because of the informational externality learning is initially slow, the responses are delayed and hump-shaped. The responses are also asymmetric – prices increase faster than they decrease, and therefore negative shocks trigger larger output responses than positive shocks.

**Keywords**: signaling, distortion, decentralized trading, islands, dispersed information.

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# 1 Introduction

There is a long standing idea in economics that, when demand or costs increase, firms are reluctant to increase prices because it triggers a disproportionate adverse reaction among consumers. According to some authors, this idea could lie at the root of one of the main puzzles in macroeconomics, namely the existence of nominal price rigidities and real effects of money[1]. Interestingly, when asked to explain their reluctance to increase prices after an increase in costs, firms' managers usually answer that "price increases cause difficulties with customers" (Blinder et al. 1998). However, the reasons remain so far unclear. One possible explanation is that, for a variety of reasons, consumers question whether price increases are justified (Rotemberg 2005) or exhibit a strong distaste for them (Nakamura and Steinsson 2005; Heidhues and Köszegi 2005). One important issue left open by this solution, when considering price increases following an increase in money, is that rational consumers should understand that these increases are only nominal, not real. My paper addresses this issue and provides a model in which firms' difficulty to increase nominal prices arises for purely informational reasons.

I consider a dynamic economy that is subject to exogenous aggregate shocks. The key assumption is that there is heterogeneity of information regarding these shocks among consumers, some consumers being better informed than others. In my model, firms are able to learn the realization of shocks from the environment. By letting firms to have arbitrarily precise information, and without menu costs, I ask whether they adjust prices. My general goal is to study how information is transmitted in this economy, and its implications for output and prices. For concreteness, in this paper I focus on the case of monetary shocks, i.e. changes in the supply of money, but the basic idea can also be applied to other cases.

The environment is as follows. I consider an economy populated by firms and consumers. In this economy, goods markets are decentralized. More specifically, the economy is composed of islands, and on each there is a price-setting firm. Consumers travel from one island to the other, buying goods sequentially. Importantly, every period, the only price observable to consumers is the price of the island. After a finite number of periods, all consumers buy a good sold in a centralized and competitive economy-wide market. In a cash in advance environment, the price of this good is determined by the supply of money. Thus, this centralized market is meant to capture the idea that in the long run prices are flexible, and do not reflect any strategic concerns between suppliers and buyers.

After a monetary shock, an exogenous proportion of consumers learns its realization. At this point, all other consumers and firms are uninformed. Firms, being so far uninformed, keep

[1]From the literature it is possible to see that this idea goes back to at least Hall and Hitch (1939), and has been mentioned by many other authors. For some examples, see Okun (1981); Kahneman, Knetsch, and Thaler (1986); Greenwald and Stiglitz (1989); Fehr, Kirchsteiger, and Riedl (1993); Anderson and Simester (forthcoming).

their price unchanged but sell as much consumers demand. Because every island is visited by a representative sample of consumers, firms are able to learn the realization of the shock by observing total demand. This gives rise to an informational asymmetry between firms and uninformed consumers.

I obtain three sets of results. First, the informational asymmetry between firms and uninformed consumers gives rise to an interesting strategic tension. Uninformed consumers revise their beliefs about money shocks as a function of firms' prices. Firms make higher profits when consumers believe that monetary shocks are high, because they expect to face higher prices in the long run, and therefore are willing to spend more in the short run. These two facts lead to a *non existence* of equilibria mimicking perfect information when the proportion of informed is low. For the intuition, suppose that all consumers are uninformed, and that such an equilibrium exists. In this alleged equilibrium, prices are proportional to money supply – higher after positive money shocks than after negative shocks, and all consumers update their beliefs from prices. Thus, consumers spend more after positive shocks, and firms make higher nominal profits. But this means that when shocks are negative firms have a profitable deviation – to post high prices – and therefore this is not an equilibrium. Moreover, as it will become clear, the strategic tension between firms and uninformed consumers changes the nature of price increases, in particular, if only few consumers are informed, it is harder for firms to implement them.

More formally, because firms have private information, they play signaling games with uninformed consumers. These signaling games feature pooling equilibria – where the monopolist posts the same price independently of the realization of the shock – and also separating equilibria – where the monopolist posts a different price depending on the realization of the shock. If the proportion of informed is low, separating equilibria do not mimick perfect information allocations and prices – they are distortionary. If the proportion of informed consumers is low enough, pooling equilibria exist as well. As a way of comparison, which type of equilibria delivers higher (ex-ante) profits to the monopolist depends on the proportion of informed consumers. If the proportion of informed is low, pooling equilibria deliver higher profits. This result holds even if firms' nominal costs increase when there is a positive monetary shock.

Second, I show that whether firms adjust prices or not, by having an impact on consumer learning, they inflict an informational externality on other firms. In fact, when consumers see a price change, they change their beliefs about the state of the world, and this has an impact on the other firms' cost of changing prices. For instance, firms playing pooling equilibria prevent consumers from learning, and therefore make it hard to other firms to change prices. This lack of consumer learning hurts firms' profits, and at the same time reduces consumer welfare. Interestingly, the degree of price rigidity depends on firms' marginal costs, and there exist policies capable of improving welfare by bringing the economy closer to perfect information allocations.

More precisely, I show that a temporary tax on monopolists reduces the amount of rigidity, leading to faster learning among consumers and higher welfare.

Third, the dynamic responses of output and inflation are delayed, hump-shaped, and asymmetric. In the model, whether firms adjust to prices or not depends on firms' characteristics, such as firm specific productivity. Therefore, if firms are heterogeneous, some firms adjust prices before other firms. Price changes allow consumers to learn, and consumer learning makes other firms adjust prices in future periods. This reinforcement effect between learning and price changes leads, for a large set of parameter values, to a hump in the response of inflation, because an increasing number of firms change prices at later stages of the adjustment process.

The model also delivers hump-shaped responses of output. The procyclical effect of money arises each time an informed consumer buys from a firm playing a pooling equilibrium, for the usual reason: the informed knows that the price has not adjusted and therefore he buys, in the case of a positive (negative) monetary shock, more (less) than in the benchmark case. This means that two forces drive the size of the aggregate effect: the proportion of informed consumers, and the proportion of firms playing a pooling equilibrium. Initially, as long as the initial proportion of informed consumers is small, the effect on output is small. As the fraction of informed consumers increases, this effect grows. However, over time, as the number of firms playing a pooling equilibrium decreases, the procyclical effect of money dies out, leading to money neutrality in the long run[2]. The asymmetry of the responses of output and inflation relies on the fact that, in the aggregate, prices adjust faster to positive than to negative shocks, consistent with the evidence in Peltzman (2000). As a consequence, negative shocks trigger larger output responses than positive shocks. To summarize, the model delivers delayed, hump-shaped and asymmetric responses of both output and inflation. The first two characteristics seem consistent with the data according to Christiano, Eichenbaum, and Evans (2005), and the third according to Cover (1992).

**Related literature.** This paper is related to several strands of the literature. First, my paper is complementary to other explanations of the real effects of money through imperfect information (Lucas 1972; Woodford 2002; Mankiw and Reis 2002; Mackowiak and Wiederholt 2009b; Woodford 2009; among others). Most of these papers develop models where firms are imperfectly informed. In this literature, the most closely related papers to mine are Reis (2009) and Mackowiak and Wiederholt (2009a). Both of these papers develop important DSGE models where both firms and households have limited information. The mechanism generating rigidity in these papers is that, because of imperfect information, firms fail to recognize early enough that they should change prices. My model uses a different mechanism for nominal rigidities,

[2]A similar dynamic result is obtained by Mackowiak and Wiederholt (2009a).

based on the idea that at some point firms have better information than the most uninformed portion of consumers[3]. Hellwig and Venkateswaran (2009) develop a model with aggregate and firm-specific shocks. Firms learn from demand, and interestingly this leads to transient effects of money. My setup is more restrictive because I do not allow for firm-specific or idiosyncratic shocks, which facilitates learning on the side of firms. However, this allows to focus on the type of rigidity I want to highlight.

Second, my model is closely related to other economies with decentralized markets and private information (Golosov, Lorenzoni, and Tsyvinski 2009; Duffie and Manso 2007; Amador and Weill 2007). In these models, information about asset values spreads slowly among a population of traders. In a similar spirit, in my economy information about the aggregate state spreads slowly among firms and consumers.

Third, my paper is related to the literature on herding behavior. In that literature economies feature informational externalities in which information gets "trapped", leading to inefficient outcomes (Banerjee 1992; Bikhchandani, Hirshleifer, and Welch 1992; Chamley and Gale 1994; Chari and Kehoe 2003). A paper closely related to mine is Gorodnichenko (2009). This paper presents an interesting and complementary model where informational externalities also play an important role in delaying learning after monetary shocks. My paper differs in the basic mechanism leading to nominal rigidity.

Fourth, my paper is also related to other explanation of price stickiness based on "strong" reactions among consumers to price changes or strategic concerns. Rotemberg has a series of papers based on the idea of consumer anger (Rotemberg 2005), fairness (Rotemberg 2008), or other behavioral considerations (Rotemberg 2007) that create stickiness. Nakamura and Steinsson (2005) develop an interesting repeated game where, due to consumers' habit formation, firms need to commit to keep their price unchanged for long periods of time. Kimball (1995) explores a model in which demand curves are more elastic for price increases than for price decreases. Koszegi and Heidhues (2005) has shown that if consumers perceive price increases as a loss, firms choose a distribution of prices that exhibits stickiness. Maskin and Tirole (1988) have shown that oligopolistic competition also leads to price rigidities. My paper targets the same time of "consumer-based" origin for rigidities, but it differs with previous explanations in that the friction arising in my model is purely informational.

Finally, my paper is related to the industrial organization literature on prices as signals of quality (Wolinsky 1983; Bagwell and Riordan 1991). In those models, buyers are unsure about the quality of products, and therefore the price is a useful signaling device. In my model,

[3]At a conceptual level, my framework can be thought as an opposite model, in terms of agents' information sets, to Lucas (1972). In Lucas' framework, sellers do not know the price level, but buyers do. Here, the opposite happens: sellers know the price level, but some buyers do not.

uninformed buyers are uncertain about the value of goods with respect to goods bought in future periods. Firms, due to their superior information about the state of the economy, have information about future nominal prices and therefore they can signal it. A related interesting paper here is by Kamenica (2008), who presents a model where consumers make inferences on the basis of product lines.

The paper is organized as follows. In Section 2 I present a simple static model that shows the forces at play every time a firm and a group of consumers meet in the dynamic setup. This static model will illustrate how equilibria with nominal rigidity arise, and characterize them. In Section 3 I present the main dynamic results using a simple framework. I show that the fact that firms do not change prices inflicts an externality on other firms. I characterize welfare improving policies capable of correcting this externality. I then simulate the dynamic responses of the economy to a monetary shock. Section 4 develops a cash in advance dynamic model, to show that the insights of the dynamic model can be applied to a monetary framework. Section 5 concludes. Most of the proofs are relegated to the Appendix.

# 2 A Simple Static Model

This simple static model is meant to provide intuition for the main mechanisms at play in the more involved dynamic framework of the next Section. I consider the problem of a monopolist selling to uninformed consumers. These consumers buy first the good supplied by the monopolist, and then buy all other consumption. The price of all other consumption is exogenous, and can be thought as determined by the long-run macroeconomic state. For instance, after a monetary expansion, these (nominal) prices increase. Uninformed consumers do not know the state, implying that they do not know the price of all other consumption, and thus are uncertain about how much to buy for a given monopolist's price. The main result is that this can lead to rigidity in the monopolist's price.

More specifically, the setup is as follows. A monopolist sells to a unit mass consumers. Consumers are heterogeneously informed about the price of other goods in the economy, which I refer to as the price level. More specifically, consumers are either informed or uninformed. Informed consumers know the price level, uninformed consumers do not know the price level. To simplify, in this stark static model, the monopolist is informed and knows the price level[4]. All consumers observe the price posted by the monopolist before making a purchase decision. Because of this, uninformed consumers are able to update their beliefs about the price level upon observation of the monopolist's price. The main goal of this section is to show that the game between the firm and uninformed consumers has pooling equilibria, where the monopolist's

[4]This assumption will be relaxed in the dynamic model.

price does not depend on the price level, and that these pooling equilibria dominate separating equilibria, i.e. equilibria where the monopolist's price changes with the price level.

**Consumers.** There is a unit mass of consumers indexed by $i$. Consumer $i$ has the following utility function of consumption:

$$u(c_i) + C_i \quad . \tag{1}$$

I make the following assumptions concerning the utility function $u(c_i)$.

**Assumption 1** *The utility function $u(c_i)$ is assumed to be twice continuously differentiable on $R^2_{++}$, increasing, and strictly concave.*

The budget constraint is

$$pc_i + PC_i = \text{Income}_i \quad . \tag{2}$$

Utility (1) is linear in $C_i$ and therefore consumption of good $c$ is independent of income. A possible interpretation is that spending on $c$ is a small proportion of total income. Under this interpretation, I refer to good $c$ as a particular consumption good, and to good $C$ as all other consumption of the individual, and to its price $P$ as a "price level".

Goods $c$ and $C$ are bought sequentially. The consumer first buys good $c$. Then, the consumer buys all other consumption $C$. Consumers buy good $c$ from a monopolist who sets the price $p$. $P$ is drawn from a binary probability distribution over $\mathfrak{P} = \{P^h, P^l\}$, where $P^h > P^l$. I refer to $P = P^h$ as the high state, and to $P = P^l$ as the low state. I assume that both states are equally likely: $Pr(P = P^h) = Pr(P = P^l) = 1/2$. $\text{Income}_i$ is consumer $i$'s income.

**Information.** Informed consumers know the realization of the price level when buying from the monopolist. There is a proportion $\alpha$ of informed consumers. The complementary proportion $1 - \alpha$ of consumers is uninformed and does not know the price level when buying from the monopolist. Uninformed consumers know the distribution of possible realizations. All consumers know their income when buying from the monopolist. Also, all consumers observe the price set $p$ when deciding how much to buy from the monopolist.

The monopolist is informed, i.e., he observes the price level before setting his price. Also, to simplify the analysis, it is assumed the monopolist knows the proportion of informed consumers. However, the monopolist cannot discriminate between informed and uninformed consumers.

Informed consumers maximize (1) subject to (2) under perfect information. These consumers know $P$ and maximize their utility without uncertainty.

The monopolist and uninformed consumers play the following one-shot game. First, the monopolist observes the realization of the price level $P$. After having observed the price level, the monopolist posts a price $p$. Uninformed consumers observe $p$, form beliefs $\mu$ about the price level, and decide how much to demand from the monopolist[5]. Formally, this sequence of events define a signaling game. The sender of the signaling game is the monopolist. The type of the sender is defined by referring to different possible information sets he can access[6]. Therefore, there are two possible types of monopolist: the monopolist who observes a high realization of the price level, $P^h$, and the monopolist who observes a low realization of the price level, $P^l$. The message of the sender is the price $p$. The receiver is the set of uninformed consumers, whose action is their demand $c_i(\cdot)$, where $i$ belong to the set of uninformed. This action depends on beliefs $\mu_i$.

**Monopolist's Problem.** The monopolist is able to produce $c$ at zero cost. The monopolist chooses $p$ to maximize revenues:

$$\max_p pc(p, P, \mu_i(p)) \tag{3}$$

where $c(\cdot)$ is total demand for good $c$, to be derived below. The monopolist sets a price $p$. Consumers observe the price $p$ and submit their demand. Then, production takes place, and the monopolist sells as much as it is demanded. As it will become clear, total demand $c(p, P, \mu_i(p))$ depends on three objects. First, it depends directly on the price $p$. Second, it depends on the price level $P$, because the demand of informed consumers depends on $P$. Third, it depends beliefs held by the uninformed $\mu_i(p)$, which in turn depend on the monopolist's price $p$.

**Equilibrium definition.** I now define a perfect Bayesian equilibrium of the game between the monopolist and the uninformed consumers. I first describe the strategy of the monopolist. I focus on pure strategies. A pure strategy for the monopolist $p$ is a mapping

$$p : \mathfrak{P} \longrightarrow R_+ \quad , \tag{4}$$

that assigns a price $p$ to each state of nature $P \in \mathfrak{P}$. Next, I describe beliefs $\mu_i(p)$ of a given uninformed consumer $i$. I focus on symmetric beliefs of the uninformed. Beliefs are a probability distribution over $\mathfrak{P}$ defined by a mapping

$$\mu_i : R_+ \longrightarrow [0, 1] \quad , \tag{5}$$

[5]An interesting related paper here is by Jones and Manuelli (2002), where both the informed buyer and informed seller problem is considered.

[6]This is the standard definition of "type" in game theory.

that assigns a probability $\mu_i(p)$ to the high state of nature $P^h$. The mapping (5) is consistent with Bayes' rule on the path of equilibrium play. Because I focus on pure strategies for the monopolist, the requirement is simply that, for any equilibrium prices (4), denoted $p(P^h)$ and $p(P^l)$ for the high and low states respectively, if $p(P^h) \neq p(P^l)$ (a separating equilibrium), then $\mu_i(p(P^h)) = 1$ and $\mu_i(p(P^l)) = 0$. If instead $p(P^h) = p(P^l)$ (a pooling equilibrium), then $\mu_i(p(P^h)) = \mu_i(p(P^l)) = 1/2$. Beliefs $\mu_i(p)$ are unrestricted for other prices.

I now describe the strategy of uninformed consumers. I focus on symmetric pure strategies. A symmetric pure strategy $c_i$ for a given uninformed consumer $i$ is a mapping

$$c_i : R_+ \times [0,1] \longrightarrow R_{++} \quad ,$$

that assigns a demand $c_i$ to each price $p$ and beliefs $\mu_i(p)$. A perfect Bayesian equilibrium requires that both the firm and the uninformed consumers play a best response. Given this definitions, I can now define an equilibrium formally.

**Definition 1** *A Perfect Bayesian Equilibrium (PBE) is a list ($p(P)$,$\mu_i(p)$,$c_i$), for all $i$, such that*

1. *There is no profitable deviation from posting $p$, given consumers' play,*
2. *$\mu_i(p)$ is derived using Bayes' rule on the equilibrium path,*
3. *consumption decisions $c_i$ maximize utility (1), given the budget constraint (2), beliefs $\mu_i(p)$ and firm's play.*

Having defined an equilibrium of the game, it is now useful to present consumers' optimality conditions for good $c$. This will provide intuition for the fundamental strategic tension between the monopolist and uninformed consumers.

**Consumers' Optimality Conditions for good $c$.** In the case of informed consumers, marginal utility of $c_i$ is equated to the relative price of the goods:

$$u'(c_i) = \frac{p}{P} \quad . \tag{6}$$

This equation (6) pins down the demand for good $c$ by consumer $i$:

$$c_i\left(p\frac{1}{P}\right) \quad . \tag{7}$$

Notice that because of the quasilinearity of preferences, the demand (7) does not depend on income.

In the case of uninformed consumers, marginal utility of $c_i$ is equated to the expected relative price of the goods[7]:

$$u'(c_i) = E_{\mu_i(p)}\left[\frac{p}{P}\right] \quad , \tag{8}$$

where $E_{\mu_i(p)}\left[\frac{p}{P}\right]$ is simply the expectation of relative prices using beliefs $\mu_i(p)$, that is

$$E_{\mu_i(p)}\left[\frac{p}{P}\right] = \mu_i(p)\frac{p}{P^h} + (1-\mu_i(p))\frac{p}{P^l} \quad .$$

Because the expectation $E_{\mu_i(p)}\left[\frac{p}{P}\right]$ is conditional on $p$ (consumer $i$ observes it), the price $p$ can be taken out of the expectation operator, to obtain the demand function

$$c_i\left(pE_{\mu_i(p)}\left[\frac{1}{P}\right]\right) \quad . \tag{9}$$

At this point, it is important to notice that both the demand of the informed (7) and the demand of the uninformed (9) depends on the price chosen by the monopolist $p$ times a deflator. In the case of the informed, this deflator is equal to the inverse of the price level $1/P$. In the case of the uninformed, this deflator is equal to a belief about the inverse of the price level $E_{\mu_i(p)}[1/P]$. Since I focus on symmetric strategies for uninformed consumers, I write total demand as

$$c(p, P, \mu_i(p)) = \alpha c_i\left(p\frac{1}{P}\right) + (1-\alpha)c_i\left(pE_{\mu_i(p)}\left[\frac{1}{P}\right]\right) \quad . \tag{10}$$

An interesting feature of (10) is that is it increasing in $\mu_i(p)$. This fact will be key for the form of equilibria under imperfect information. It implies that the firm prefers uninformed consumers to believe that the price level is high, because in that case the deflator $E_{\mu(p)}\left[\frac{1}{P}\right]$ is low. Notice that this fact is independent of the state of the world.

## 2.1 Perfect Information Benchmark

To develop intuition, here I consider a perfect information benchmark where all consumers are informed, i.e. when $\alpha = 1$. In this case, total demand is

$$c(p, P, \mu_i(p)) = c_i\left(p\frac{1}{P}\right) \quad . \tag{11}$$

Plugging (11) into (3), the monopolist's problem becomes

[7] To get this expression, substitute $\text{Income}_i$ from (2) into (1) and then take the first order condition with respect to $c_i$.

$$\max_p pc_i\left(p\frac{1}{P}\right) \quad . \tag{12}$$

**Lemma 1** *When all consumers know the value of the price level, the monopolist's price is proportional to the price level and demand is the same in both states of nature.*

**Proof.** Taking the first order condition for the problem (12) and rearranging, get

$$c_i\left(p\frac{1}{P}\right) + p\frac{1}{P}c_i'\left(p\frac{1}{P}\right) = 0 \quad . \tag{13}$$

From condition (13) it is easy to see that the monopolist's optimal price is proportional to the price level. Since total demand depends on the monopolist's price times the realization of the price level, this demand will be the same in both states of nature.

■

## 2.2 Heterogeneous Information

In this case, there is a proportion of consumers that do not know the realization of the price level $P$, and therefore $\alpha < 1$. Here I will analyze the equilibria of the game between the firm and uninformed consumers[8]. I will show that, for low enough values of $\alpha$ this game admits equilibria with price rigidity, i.e. pooling equilibria in which the firm posts the same price in both states of the world. I will also show that, for low enough values of $\alpha$, these equilibria are deliver higher (ex-ante) profits for the monopolist.

This signaling game belongs to the well-known class of monotonic signaling games (Cho and Sobel 1990) (the proof is in Appendix A.1). This has two implications. First, the firm is better off if uninformed customers believe that the state of the world is high, independently of the actual realization of the state. This characteristic follows from the fact that total demand and profits are decreasing in uninformed consumers' beliefs, as explained above. Second, the game has the single-crossing property. This means that the high type is more at ease in posting high prices than the low type. The reason is that informed consumers know that he is the high type,

[8] An interesting, non-strategic, feature of this model without marginal costs is worth highlighting. Suppose no consumer is informed ($\alpha = 0$), and fix consumers' beliefs $E_{\mu_i(p)}\left[\frac{1}{P}\right] \equiv \Xi$. Then, to maximize its revenue, the monopolist posts a price $p$ inversely proportional to consumers' beliefs. To see this, take the first order condition

$$c_i(p\Xi) + p\Xi c_i'(p\Xi) = 0 \quad ,$$

which implies that $p$ is inversely proportional to $\Xi$. The optimality result of pooling equilibria for few informed consumers presented later in this section hinges on this fact, but is valid even in the presence of marginal costs that are proportional to the price level $P$.

and their demand is less sensitive to price increases compared to the demand from the low type's informed consumers. Together, these two properties make this game tractable.

Signaling games usually have many equilibria, and this game is no exception. I will first characterize separating equilibria. The following lemma characterizes a benchmark separating equilibrium, the one where both types get the highest (ex-post) profits possible.

**Proposition 1 (Best Separating Equilibrium)** *The following is the Best Separating Equilibrium. Define* $\overline{\alpha}$ *by*

$$p^l c_i\left(p^l \frac{1}{P^l}\right) = p^h\left(\overline{\alpha} c_i\left(p^h \frac{1}{P^l}\right) + (1-\overline{\alpha}) c_i\left(p^h \frac{1}{P^h}\right)\right) \quad . \tag{14}$$

*Then,*

- *If* $\alpha \geq \overline{\alpha}$*:*
  - *The firm posts the same prices as in the perfect information benchmark:*

$$p^h = \arg\max_p pc_i\left(p \frac{1}{P^h}\right) \quad , \tag{15}$$

$$p^l = \arg\max_p pc_i\left(p \frac{1}{P^l}\right) \quad . \tag{16}$$

*For a given equilibrium set of prices* $p(P)$*, define ex-ante real profits as*

$$\Pi(p(P)) = \frac{1}{2}\frac{1}{P^h}\pi(P^h) + \frac{1}{2}\frac{1}{P^l}\pi(P^l) \quad . \tag{17}$$

*where* $\pi(P) = pc(p, P, \mu_i(p))$*. In this case, ex-ante real profits* $\Pi(p(P))$ *are equal to ex-ante real profits in the perfect information benchmark:*

$$\Pi^* = \frac{1}{2}\frac{1}{P^h}\pi(P^h) + \frac{1}{2}\frac{1}{P^l}\pi(P^l) \quad , \tag{18}$$

*where* $\pi(P^h) = \max_p pc_i\left(p \cdot 1/P^h\right)$ *and* $\pi(P^l) = \max_p pc_i\left(p \cdot 1/P^l\right)$*.*

- *If* $\alpha < \overline{\alpha}$*:*
  - *The firm posts* $p^l$ *and* $\overline{p} > p_h$ *s.t.*

$$p^l c_i\left(p^l \frac{1}{P^l}\right) = \overline{p}\left(\alpha c_i\left(\overline{p}\frac{1}{P^l}\right) + (1-\alpha) c_i\left(\overline{p}\frac{1}{P^h}\right)\right) \quad . \tag{19}$$

*In this case,* $\overline{p}$ *is strictly decreasing and, if* $u(c_i)$ *is such that the profit function is single-peaked, then* $\Pi(p(P))$ *is strictly increasing in* $\alpha$*.*

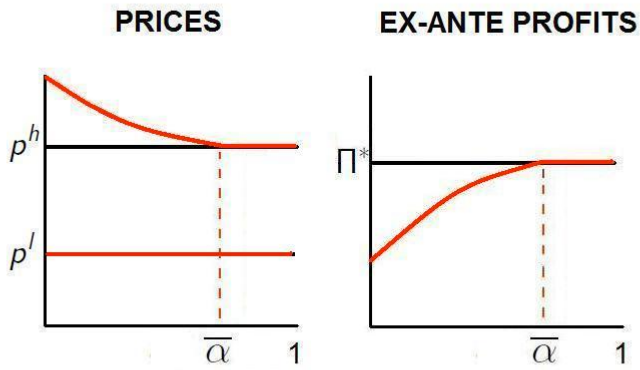


Figure 1: Best Separating Equilibrium (proposition 1) in the case of a quadratic $u(c_i)$.

The proof of this proposition is in the appendix. This lemma shows that when the proportion of informed consumers is high enough, the high type can separate from the low type by posting the perfect information prices. The reason is that, in this case, the proportion of informed consumers is high enough to discourage the low type from imitating him: if the low type posts $p^h$, the informed know that his price is too high and they buy less than the perfect information quantity $c^{ss}$, hurting the low type's profits. When the proportion of informed is lower, the only way a separating equilibrium possible is one where the high type posts a price strictly higher than $p^h$. This makes sure that the low type does not imitate him. Figure 1 is a graphical illustration of this proposition for the case of a quadratic utility function $u(c_i)$. It is shown that ex-ante profits are increasing in $\alpha$, and reach $\Pi^*$ when $\alpha \geq \overline{\alpha}$.

Having characterized a benchmark separating equilibrium, I will now characterize a benchmark pooling equilibrium. Pooling equilibria are interesting for the study of nominal rigidities since in these equilibria the firm sets the same price independently of the state of the world. Pooling equilibria exist when the proportion of informed is low. The following proposition characterizes a benchmark pooling equilibrium. This is the pooling equilibrium at the price corresponding to profit maximization when $\alpha$ is equal to zero and no consumer knows the state of the world. In the dynamic model of Section 4, this equilibrium corresponds to keeping the price unchanged after a monetary shock. I also show that when $\alpha$ is equal to zero, this equilibrium reaches the perfect information level of ex-ante profits $\Pi^*$[9].

**Proposition 2 ($p^*$-pooling Equilibrium)** *Consider $p^*$ s.t.*

$$p^* = \arg\max pc_i \left( p \left[ \frac{1}{2} \cdot \frac{1}{P^h} + \frac{1}{2} \cdot \frac{1}{P^l} \right] \right) \quad , \tag{20}$$

[9]This holds also in the presence of marginal costs proportional to the price level $P$.

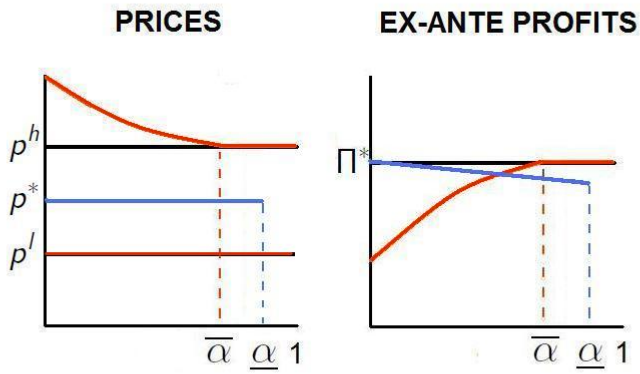


Figure 2: Best Separating Equilibrium and $p^*$-pooling Equilibrium (Propositions 1 and 2) in the case of a quadratic $u(c_i)$.

*and consider $\underline{\alpha}$ s.t.*

$$p^*\left(\underline{\alpha}c_i\left(p^*\frac{1}{P^h}\right)+(1-\underline{\alpha})c_i\left(p^*\left[\frac{1}{2}\cdot\frac{1}{P^h}+\frac{1}{2}\cdot\frac{1}{P^l}\right]\right)\right)=\arg\max_p p\left(\underline{\alpha}c_i\left(p\frac{1}{P^h}\right)+(1-\underline{\alpha})c_i\left(p\frac{1}{P^l}\right)\right) \quad . \tag{21}$$

*For $\alpha \leq \underline{\alpha}$, there exists a pooling equilibrium at $p^*$. Moreover, if $\alpha = 0$, this equilibrium reaches ex-ante profits $\Pi^*$. Also, if $u(c_i)$ is such that the profit function is single-peaked, ex-ante profits $\Pi(p^*)$ are strictly decreasing in $\alpha$.*

The proof is in the appendix. When the proportion of informed consumers is low this equilibrium exists because both types do not want to deviate. In the general equilibrium model this pooling equilibrium will be particularly interesting because, for a strictly positive proportion of informed consumers, monetary shocks will have a procyclical effect on demand. Figure 2 is a graphical illustration of this proposition for the case of a quadratic utility function $u(c_i)$. It is shown that ex-ante profits are increasing in $\alpha$, and that there is a unique $\alpha^{***}$ where the ex-ante profit functions cross.

An interesting feature of this pooling equilibrium is that it yields higher ex-ante profits for the firm when the proportion of informed consumers is low than any separating equilibrium. The next proposition develops this result in the case of any utility function satisfying assumption 1.

**Proposition 3** *If $\alpha \leq \alpha^*$, the $p^*$-pooling Equilibrium delivers higher ex-ante profits than any separating equilibrium.*

The proof is in the appendix. The intuition for this result is the following. There is an ex-ante trade-off between two possible distortions. The first distortion arises in separating equilibria: in any separating equilibrium, when the proportion is informed consumers is low enough, there is a distortion at the top, because the firm needs to post a very high price to be able to credibly signal the state of the world to uninformed consumers. This distortion hurts ex-ante profits. On the other hand, another type of distortion arises in pooling equilibria: in any pooling equilibrium the price posted does not correspond to beliefs of the informed in all states of nature, making them buy either too much or little with respect to $c^{ss}$, the demand under perfect information, creating a distortion that hurts ex-ante profits. The first type of distortion is bigger the lower the proportion of informed consumers: in this case, it is easy for the low type to imitate the high type, and therefore the high type needs to post very high prices to separate. The opposite happens in the second type of distortion: this distortion is bigger the higher proportion of informed consumers, in this case the proportion of sales at a wrong price being important. Thus, each of these distortions varies monotonically but in opposite directions with the proportion of informed consumers, creating a trade-off for the firm, which favors pooling equilibria when few consumers are informed, and separating equilibria when many consumers are informed. As shown in the appendix, this holds even in the presence of marginal costs proportional to the price level $P$.

A symmetric result follows when the proportion of informed consumers is high enough.

**Proposition 4** *If $\alpha \geq \alpha^{**}$, the Best Separating Equilibrium delivers higher ex-ante profits than any pooling equilibrium.*

The proof is in the appendix. The intuition for this result is the same as for the previous result. There is a trade-off between two types of distortions. The distortion arising in the Best Separating Equilibrium is small when the proportion of informed consumers $\alpha$ is high, and therefore in this case this equilibrium ex-ante dominates any pooling equilibrium.

In order to apply this framework to a dynamic study of the responses to a monetary shock, an issue that I need to confront is equilibrium selection. Given propositions 3 and 4, it is tempting to pick, for each $\alpha$, the equilibrium that provides maximum ex-ante real profits to the firm. Thus, when I study these dynamic responses in the next Section (p. 20) I select, for each proportion of informed $\alpha$, among the Best Separating Equilibrium and the $p^*$-equilibrium, the one in which firms have higher ex-ante profits. This selection criterion can be justified by assuming that the firm has a commitment device that allows to choose, before the state is realized, a pricing plan. However, these pricing plans have to be 'credible', i.e. the consumers need reasons to believe that, ex-post, the firm will keep its promise. In other words, the firm can only commit to prices that satisfy a PBE and therefore, when the state of the world is low, the firm will not be tempted to fake that the state of the world is high[10]. This type of criterion is useful for my purposes given that for the macroeconomic application of this simple model I am specially interested in

[10] A formal way of justifying this criterion is to modify the game as in Maskin and Tirole (1992).

pooling equilibria, and a particular pooling equilibrium – the $p^*$-pooling equilibrium – ex-ante dominates all separating equilibria when the proportion of informed is low[11].

However, it is important to emphasize that the above proposed criterion is not crucial for any of the dynamic results of Section 3, including the informational externality and the general shape of the responses to a monetary shock. As long as a non-zero fraction of firms play a pooling equilibrium, similar results would be obtained. What is crucial is that, for low proportion of informed $\alpha$, the pooling equilibrium exists, and that for high values of $\alpha$ only separating equilibria exist, so that when enough consumers become informed all firms change prices and the economy returns to steady state. Thus, any selection criterion that allows to have a non-zero fraction of firms play pooling equilibria would work[12].

## 2.3 Comparative Statics in the Presence of Marginal Costs

In a more general model, all the cutoffs presented above should depend on firm specific characteristics. To illustrate this point, let me consider the case where the monopolist has a marginal cost of production $kP$. I analyze which among the Best Separating vs. the $p^*$-pooling Equilibrium are ex-ante optimal. The following numerical result follows.

**Result 1 (Optimality of $p^*$-pooling vs. Best Separating Equilibria under Marginal Costs)** *Assume $u(c_i) = ac_i - \frac{1}{2}c_i^2$, and consider the Best Separating Equilibrium and the $p^*$-pooling equilibrium. For $k \leq \hat{k}$, there is $\alpha \leq \hat{\alpha}$ where both equilibria exist. In this region:*

- *for $\alpha \geq \alpha^{***}(k)$, ex-ante profits are higher in the separating equilibrium,*
- *for $\alpha < \alpha^{***}(k)$, ex-ante profits are higher in the pooling equilibrium.*

*Moreover, $\alpha^{***}(k)$ is decreasing with $k$.*

As this result shows, which equilibrium delivers higher ex-ante profits depends on firms' marginal cost $kP$. The higher $k$, the lower the critical value of $\alpha$ at which firms prefer playing separating equilibria. Figure 3 plots this cutoff as a function of $k$ and shows that it is decreasing. The region below the curve is where pooling equilibria deliver higher ex-ante profits, then region above the curve is where separating equilibria deliver higher ex-ante profits. The intuition is that higher marginal costs need to create less of a distortion in separating equilibria.

[11]In the literature, there is no consensus on how to select equilibria in signaling games. A popular criterion is the intuitive criterion. However, in a richer model with more states of the world – clearly a relevant extension of this model for the analysis of monetary policy – this criterion looses its bite – in the sense that it fails to select a unique equilibrium – and therefore would not be useful (Cho and Kreps 1987, p. 212).

[12]For instance, if one were to Pareto rank equilibria, this would also favor pooling equilibria for low $\alpha$s and separating equilibria for high $\alpha$s. The reason is that pooling equilibria are less distortionary in the former case – and thus provide higher welfare, and separating equilibria in the latter.

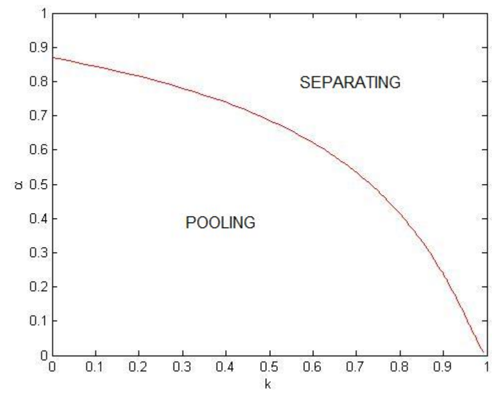


Figure 3: Cutoff $\alpha^{***}(k)$, and regions where $p^*$-pooling/best separating equilibria deliver highest ex-ante profits ($P^h = 1$,$P^l = .9$,$a = 1$).

This result has an interesting application in a macroeconomic model. Indeed, one can write a model where firms have heterogeneous productivity, giving rise to heterogeneity in $k$. The presence of firms playing separating equilibria allows for the possibility of consumer learning. This, in turn, has implications for the proportion of firms willing to play separating equilibria, because if the proportion of informed is higher, according to the above result the Best Separating Equilibrium is more profitable when the proportion of informed is higher, or according to Proposition 2, the $p^*$-equilibrium stops existing. Thus, it seems, a dynamic model can deliver interesting feedback effects between learning and the proportion of firms playing separating equilibria. I explore these themes in the next section.

# 3 Dynamics Under Heterogeneous Information

In this Section I present and discuss the main dynamic implications of the nominal rigidity developed in Section 2. The discussion will be centered around the informational implications of pooling and separating equilibria. As explained above, uninformed consumers learn the state of the world by observing firms' prices. Therefore, whether firms change prices or not has informational implications for consumers, and therefore, in a dynamic setting, has implications for price setting of other firms. Thus, a given pricing strategy of firms creates an externality on other firms. In particular, firms playing pooling equilibria prevent consumers from learning the state of the world. I first discuss the normative implications and then positive implications of this externality.

## 3.1 Simple Dynamic Setup

For presentational purposes here I present a simple dynamic setup. This is a simplified version of more involved model of next Section.

Time is discrete, indexed by $t$, and runs from $t = 1$ to a final period $t = T + 1$. There is a unit mass of consumers. Consumers enter this economy with an initial amount of cash $M_0$. Consumers buy an initial set of goods $c_1, \ldots, c_T$ using credit, and a good $c_{T+1}$ using cash.

An exogenous monetary shock $\nu$ occurs. This shock is a transfer to consumers' cash holdings, whose value is stochastic and observed by consumers only at $T+1$. The sum of the initial amount of cash and the transfer is equal to money supply $M$:

$$M \equiv M_0 + \nu \quad ,$$

which is high $(M^h)$ or low $(M^l)$, both outcomes equally likely. An exogenous proportion $\alpha_1$ of consumers learns the amount of the transfer at $t = 1$.

This economy is divided into a unit mass of islands. On every island there is a firm. This firm is a monopolist on the island and sets a price. In this simple dynamic model firms have perfect information and know money supply $M$. This assumption is relaxed in the more explicit model of Section 4, where an additional initial period is introduced where firms can learn $M$ by observing the amount of sales. This reveals $M$ because of the proportion of informed consumers $\alpha_1$.

Consumption happens in the following way. Each consumer buys goods $c_1, \ldots, c_T$ sequentially, one after the other, and on a different island each time. More specifically, consumers are divided into representative samples of unit mass. Every period $t$, each of these samples is sent to a different island for consumption[13].

All firms produce the same good $c_t$, which is the consumption good of period $t$. Firm located in island $j$ produces this good with a linear technology of labor

$$c_t = A_j L_t \quad .$$

In the final period $T+1$, consumers receive the cash transfer, and then buy a good $C_{T+1}$ sold by an economy-wide competitive firm. Utility is linear in this good. The price of this good, $P$, is proportional to total money supply $M$. Firms hire labor from an economy-wide competitive market at a wage[14] $W$, also proportional to money supply $M$.

[13] There is a unit mass of these samples.

[14] In Section 4 I show that, after a normalization, $P = W = M$ in a cash in advance general equilibrium model.

## 3.2 Informational Externality

I now analyze the normative implications of the nominal rigidity presented in Section 2. Recall that uninformed consumers learn the state of the world by observing firms' prices. Therefore, firms that play a pooling equilibrium will prevent consumers from learning. In this dynamic setting, this inflict an informational externality on firms in the next period. Here I show that this type of informational externality hurts firms' profits and reduces consumers' welfare over periods $t = 1, \dots, T+1$.

For the purpose of the analysis of the informational externality, I will suppose, without loss of generality, that all firms are identical, thus $A_j = A$ for all $j$ (there is need to relax this assumption to analyze the positive implications of the model, see below). This homogeneity assumption implies that all firms have the same marginal costs of production $kW \equiv \frac{1}{A}W$. Suppose that the initial exogenous proportion of informed consumers $\alpha_1$ is low enough so that the $p^*$-equilibrium exists, and that it is played at least on some islands at $t = 1$. The following proposition states that in this case firms' (ex-ante) profits and consumers' (ex-ante) welfare are lower than under perfect information.

**Proposition 5 (Welfare Effect of Informational Externality)** *Assume $0 < \alpha_1 \leq \underline{\alpha}(k)$ and that at least on one island the $p^*$-equilibrium is played at time $t = 1$. Then, if the profit function is single-peaked, the sum of firms' ex-ante profits over $t = 1, \dots, T+1$ is lower than under perfect information. Moreover, if $u(c_i)$ is quadratic, the sum of consumers' ex-ante welfare over $t = 1, \dots, T+1$ is lower than under perfect information.*

The proof is in the appendix. Intuitively, the profit loss comes from the presence of a non-zero proportion of informed consumers. Firms' prices are not optimal for this segment of the market, leading to a loss in profits. The welfare loss comes from the informed consumers as well: their consumption varies across states, and given the concavity of the utility function $u(x_i)$ and linearity of marginal utility, by Jensen's inequality ex-ante welfare is lower. Perhaps surprisingly, uninformed consumers have the same ex-ante welfare as in the perfect information benchmark. The reason is that the price $p^*$ corresponds to their belief and therefore their consumption bundle is not distorted.

Notice also that in the situation described by Proposition 5, some firms do not change prices and uninformed consumers buying from them do not learn. Since all firms are identical, if no firm changes its price, this is an extreme example in which information gets "trapped" forever and the economy does not return to its benchmark level production even if $T$ is arbitrarily large. This result has the flavor of an information cascade à la Banerjee (1992) or Bikhchandani et al. (1992), where informational "cascades" prevent agents from fully aggregating information in the long run.

Interestingly, in this model nominal rigidity is endogenous and, as shown previously, depends on firms' productivity $A$ and by implication on its marginal cost $kW$. Thus, it is possible to make prices more flexible using a tax proportional to sales. The tax can increase profits and welfare by allowing uninformed consumers to learn. The following proposition makes this statement precise.

**Proposition 6 (Welfare Improving Policy)** *Assume $0 < \alpha_1 \leq \underline{\alpha}(k)$ and that at least on one island the $p^*$-equilibrium is played at period $t = 1$. Then, if $T$ is big enough and the profit function is single-peaked, a social planner can increase the sum of firms' ex-ante profits over $t = 1, \ldots, T + 1$ by implementing a proportional tax rate $\psi P$ on firms' sales at period $t = 1$. If $u(c_i)$ is quadratic, the social planner can increase the sum of welfare over $t = 1, \ldots, T + 1$ as well.*

**Proof (sketch).** Consider $\psi$ financed lump-sum s.t. pooling equilibria do not exist anymore and therefore all firms play a separating equilibrium at $t = 1$. Then, from $t = 2$ on, the economy is in perfect information and ex-ante welfare is maximum. Thus, if $T$ is big enough, this policy improves the sum of welfare over all periods.

■

In the presence of marginal costs of production, the cutoff $\underline{\alpha}(k)$ for existence of pooling equilibria depends on the parameter $k$ (equivalently on productivity $A$). This cutoff is decreasing in $k$. A tax $\psi P$ increases firms marginal costs and therefore, for high enough $\psi$ only separating equilibria exist. Thus, all consumers learn at $t = 1$. From $t = 2$ on the economy is in perfect information, profits and consumer welfare are higher.

## 3.3 Learning Dynamics and Responses of the Economy to Monetary Shocks

I now analyze the positive implications of the nominal rigidity presented in Section 2. In particular, I will describe the dynamics of learning, output, and prices. These dynamics are meant to represent the responses of the economy to a monetary shock in an infinite horizon model, as the one developed in the next Section.

One of the issues highlighted when describing the normative implications of the informational externality is that, if the exogenous initial proportion of informed consumers is low, firms are homogeneous and equilibrium selection is such that no firm changes its price, then consumers never learn. This implies that the economy is stuck in an equilibrium in which there is no adjustment back to the perfect information level of production. To solve this issue, and allow for learning for arbitrarily low initial proportion of informed, I introduce firm heterogeneity.

Recall the equilibrium selection criterion proposed in Section 2 – among the Best Separating Equilibrium and the $p^*$-pooling Equilibrium, pick the one that provides higher ex-ante profits to the firm. Indeed, as shown by result 1, if the utility function is quadratic, according to this criterion the cutoff at which firms change prices depends on firms' productivity. In an economy where firm heterogeneity gives rise to different $A_j$ across firms, low productivity firms change prices for lower proportion of informed consumers. This allows consumers buying from those firms to learn the state, and therefore the proportion of informed consumers is always increasing over time. It is important to notice that the use of the equilibrium selection criterion proposed in the last Section is not crucial for this result, and all other results in this Section. Indeed, it is possible to show that all cutoffs $\overline{\alpha}$, $\underline{\alpha}$, $\alpha^*$, $\alpha^{**}$, and $\alpha^{***}$ presented in Section 2 depend on firm productivity, and more generally on other firm specific characteristics. In particular, this is the case for the cutoff of existence of pooling equilibria $\underline{\alpha}$, and thus this equilibrium would exist or not for a particular firm, and this can be used to generate consumer learning. What is important is only that, when it exists, it is not completely ruled out for some firms.

To illustrate consumer learning under heterogeneity, Figure 4 plots the evolution of the proportion of informed consumers ($\alpha_t$) for a small initial (exogenous) proportion of informed consumers (6 percent). The ratio of money supply $M$ in the two states of the world is equal to $M^l/M^h = .95$, and the number of periods is 13 ($T = 12$). The dynamics of learning are the same in the case of a positive or negative monetary shock. The Figure shows that learning is slow at the beginning, and accelerates, leading, in the case of these parameter values, to a convex curve. Let me now turn to the evolution of the proportion of firms with rigid prices. In a benchmark case where the proportion of firms playing the separating equilibrium is constant, this curve would be concave, as consumers would learn at a constant rate, with fast learning at the beginning, and slow learning towards the end. However, generally, the proportion of firms playing the separating equilibrium is increasing over time. To illustrate, Figure 5 plots the proportion of firms playing the pooling equilibrium. Thus, learning is generally slow at the beginning and it accelerates towards the end, leading to a convex curve. The reason for this is the informational externality in the model. As learning increases, the separating equilibrium becomes more attractive and more firms play it. This increases the rate at which consumers learn. Therefore, there is a positive feedback effect between learning and the proportion of firms playing the separating equilibrium which tends to be convex. This feature is relatively robust to using other parameter values.

The dynamic responses of output the economy are determined by the two forces represented in Figures 4 and 5. Each time an informed consumer buys from a firm playing a pooling equilibrium he creates a procyclical effect of money, i.e. higher output than under perfect information when money supply is high, and lower otherwise. Thus, one the hand, other things equal the higher the

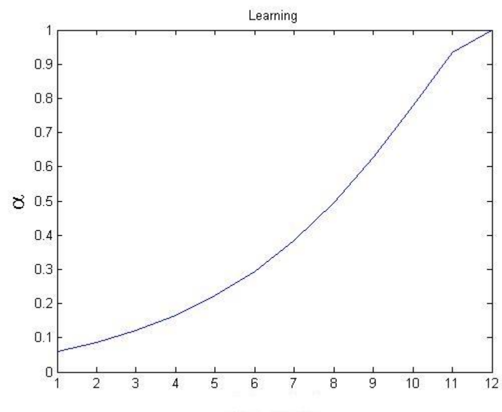


Figure 4: Learning

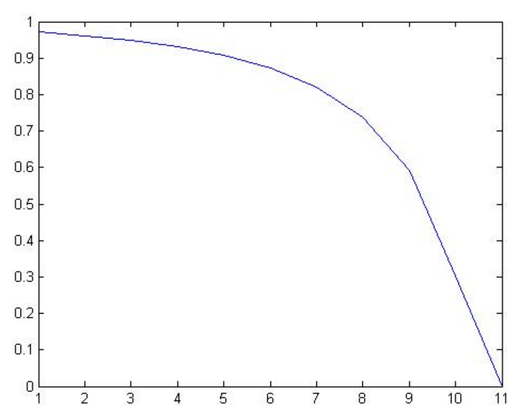


Figure 5: Proportion of firms playing the $p^*$-pooling equilibrium

proportion of informed, the stronger this effect. On the one other hand, the lower the proportion of firms pooling, the weaker this effect. It is clear that in the long run the fraction firms pooling is zero, and therefore the effect dies out, and the economy returns to the benchmark level of output (informed consumers buying from firms setting perfect information prices do not create a procyclical effect of money). Therefore, as long as there is an initial effect, it will be dampened in the long run. If the initial proportion of informed if small, the effect will be initially and arbitrarily small as well. Thus, learning can create a hump-shaped effect of output, a feature that is shown in Figure 6 as percentage deviations from the perfect information benchmark. This feature is pretty robust to other parameter values. The model is clearly too stylized to be brought to the data, but it is interesting to note the resemblance of these dynamics with its empirical counterparts (Christiano, Eichenbaum, and Evans 2005).

Another feature worth highlighting, regarding the responses of output, is that they are asymmetric – the response is bigger (in absolute value) and arrives a bit earlier when the when the shock is negative. In the case of a negative shock, the effect peaks at around -3 percent, whereas in the case of a positive shock, it peaks at around 2.5 percent.The reason is that sometimes firms increase prices and create a distortion that decreases sales below the benchmark level. This happens when $\alpha_t$ has not yet reached $\overline{\alpha}$, the cutoff for having separating equilibria replicating perfect information (see proposition 1)[15]. This asymmetry in the responses to money shocks is interesting and has been found in the data (Cover 1992).

I now describe the dynamics of prices and inflation. Prices are increasing in the case of high money supply and decreasing in the case of low money supply. This is shown in Figure 7. Notice also that the effect is delayed significantly – as it is in the case of output – because learning is slow at the beginning. This leads to responses of inflation that hump-shaped because, as long as the initial proportion of informed consumers is low enough, the proportion of firms changing prices is increasing over time[16]. More precisely, if the distribution of 'real marginal costs' $k_j = 1/A_j$ is uniform, if the learning curve is convex, the proportion of firms playing a pooling equilibrium has a decreasing slope over time, and there is a hump in inflation. This is the case in this simulation, and this feature is robust to other parameter values. Notice also that the dynamics of prices and inflation are asymmetric. In particular, prices adjust faster in the high state, consistent a response of output that is smaller than in the low state.

To summarize, the model delivers interesting responses of both output and inflation. In the

[15] The reason this effect is small and only leads to a slightly dampened positive shock is that only a few firms cause it, precisely because, according to the selection criterion used firms avoid this distortion. I have experimented with many parameter values and I have never found that this effect dominates, leading positive shocks to have negative effects on output.

[16] This is related to an interesting paper by Cavallo (2009) who, using daily frequency data on price changes, finds empirical evidence that firms synchronize price changes.

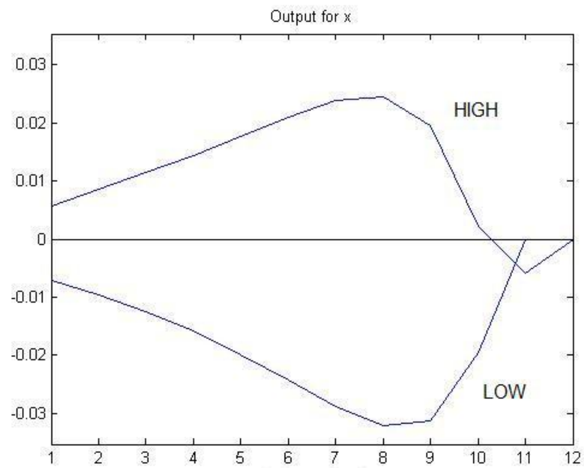


Figure 6: Dynamics of Output in High/Low State

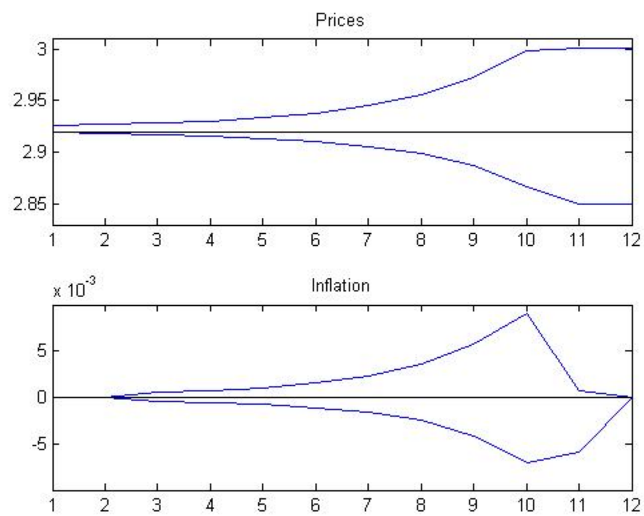


Figure 7: Dynamics of Prices and Inflation in High/Low State

data, these responses have been estimated as delayed and hump-shaped as well. In the model, the informational externality and the implied the slow nature of learning deliver naturally these features, as long as the initial proportion of informed consumers is small.

# 4 The Dynamic Cash in Advance Framework

The goal of this Section is to show that the simple dynamic model of last Section is compatible with a cash in advance, general equilibrium, framework.

In this model money has an explicit role due to a standard cash in advance constraint (Lucas and Stokey 1987). In each island there is a price-setting firm. Firms are heterogeneous in that they will have different productivity levels. As shown by Result 1, heterogeneity implies that after a monetary shock firms with low productivity change prices earlier. Consumers learn endogenously from these price changes. As consumers become informed, more and more firms change prices, slowly bringing the economy to a long run equilibrium.

**Time.** Time is discreet and every period indexed by $t = 0, \ldots$. Every period is subdivided into $N+2$ subperiods, indexed by $\tau = 0, 1, \ldots, N+1$. For the analysis of the dynamics after a monetary transfer, I am interested in the dynamics across subperiods.

**Geography.** There is a unit mass of islands indexed by either $j \in [0,1]$. Every island there is populated by a unit mass of households, indexed by $i$. Therefore, a household in this economy is identified by a double index $ij$. On every island, households own a firm that produces a consumption good $c$. The firm is a monopolist on the island where it operates. Each one of these firms produce the same good.

**Shoppers and Workers.** Every household is divided into a shopper and a worker.

**Consumption.** In the first $N+1$ subperiods of every period $t$ shoppers are randomly sent to islands for consumption of goods $c_\tau$. Therefore, in every subperiod $\tau \leq N+1$, every monopolist receives a new mass of shoppers that demand good $c_\tau$. Goods $c_\tau$ are sold by the monopolists on credit. For this reason, in the rest of the paper I will refer to these goods as credit goods. In the last subperiod $N+2$ of every period $t$, shoppers buy a cash good $C$ from a centralized, representative competitive firm. As in the partial equilibrium model, goods bought in decentralized markets – credit goods – are denoted in lowercase, and goods bought in centralized markets – the cash good – are denoted in uppercase. As it will become clear, goods bought in centralized markets will have prices that will be proportional to the money supply at all times and will be denoted in uppercase. (Goods bought in decentralized markets need not have prices proportional to the money supply at all times, and will be denoted in lowercase.)

**Goods and Labor Markets.** Goods $c_\tau$ are sold on every island by a price-setting monopolist. Good $C$ is sold in a centralized competitive market by a representative firm. There is a centralized competitive labor market where all workers supply labor $L$. Because labor is supplied in a centralized market it is denoted in uppercase. All firms in the economy hire labor from this market.

**Timing and Information Assumptions.** At the beginning of every period $t$ there is a monetary transfer $\nu_t$ into the economy. I will refer to $\nu_t$ as the "monetary transfer" or the "monetary shock" interchangeably. This monetary transfer is not immediately revealed to households.

At the first subperiod $\tau = 0$, every island receives a representative sample of shoppers in the economy. An exogenous proportion of shoppers $\alpha_{0t}$ knows the monetary transfer $\nu_t$ when buying credit good $c_0$. The complementary proportion $1 - \alpha_{0t}$ does not know the transfer when buying the credit good. In this period, firms are uninformed, but can learn the state by observing total demand.

At every subperiod $\tau = 1, \ldots, N$, again, every island receives a representative sample of shoppers in the economy. In the first subperiod $\tau = 1$, the proportion of $\alpha_{1t}$ knows the monetary transfer $\nu_t$ when buying credit good $c_1$. The complementary proportion $1 - \alpha_{1t}$ does not know the transfer when buying the credit good. At this point firms are informed about the monetary shock, and therefore, on some islands it is possible to learn the monetary transfer from prices set by monopolists. Therefore, the proportion of shoppers that know the monetary shock grows endogenously: $\alpha_{2t} \geq \alpha_{1t}$, and more generally, $\alpha_{\tau+1t} \geq \alpha_{\tau t}$, $\tau = 1, \ldots, N$.

Meanwhile, workers supply labor in the centralized labor market. In equilibrium the wage $W_t$ is perfectly flexible and proportional to the money supply, and for this reason, the wage immediately reveals the transfer to workers.

At the last subperiod $\tau = N + 1$, each shopper receives the monetary transfer $\nu_t$ and buys the cash good $C$ using cash in a centralized market. At the end of the period, the worker comes back home bringing labor income, the household pays consumption of credit goods, profits from firms are received, and financial markets open. At this point, all agents in the economy know the monetary transfer $\nu_t$. This implies that all agents know the money supply in the economy.

**Games Between Monopolists and Uninformed Shoppers.** Every time a particular informed firm and a sample of the shoppers population are matched, the firm and the proportion of uninformed shoppers play the following signalling game. Firms' type is determines by the monetary shock $\nu_t$. Therefore, there two possible types for the firm: the "high" type, i.e. the firm that observed a high monetary shock; and the "low" type, i.e. the firm that observed a low monetary shock. After observing the monetary shock, the firm posts a price $p_{j\tau t}$ in a store for the good it sells. Uninformed shoppers observe this price, form beliefs about the monetary shock $\mu(p_{j\tau t})$, and decide how much to demand[17]. Notice that shoppers and firms are matched at every subperiod $\tau t$, and also that an infinite number of these matches happen at every $\tau t$.

**Stochastic Process for Money.** Money supply evolves as:

$$\log M_t = \log M_{t-1} + \nu_t \quad , \tag{22}$$

where $\nu_t$ is shock that can take two values, $\nu_h > 0$ or $\nu_l < 0$, both outcomes with equal probability. I further impose that

$$E\left[e^{-\nu_t}\right] = 1 \quad . \tag{23}$$

This centering assumption implies that the the inverse of the money supply, which I interpret as the real value of a 1 dollar bill, is a martingale:

$$E\left[\frac{1}{M_t}\right] = E\left[\frac{e^{-\nu_t}}{M_{t-1}}\right] = \frac{1}{M_{t-1}} \quad . \tag{24}$$

This assumption will be convenient when characterizing the equilibria of the games played between the monopolists and the shoppers.

**Households' Problem.** Household $i$ of island $j$ faces the problem

$$\max E_{ij\tau t}\left[\sum_{t=0}^{\infty}\beta^t\left(\sum_{\tau=0}^{N}u(c_{ij\tau t}) + v(C_{ijt}) - L_{ijt}\right)\right] \quad , \tag{25}$$

where $c_{ij\tau t}$ is consumption of good $x$ at subperiod $\tau$ time $t$, produced by a randomly matched firm $\hat{j}$ of island $\hat{j}$, $C_{ijt}$ is consumption of the credit good, and $L_{ijt}$ is labor supplied by the worker. $\hat{j}(i,j,\tau,t)$ is a function designates the firm $\hat{j}$, that sells to household $ij$ at subperiod $\tau$ time $t$. $E_{ij\tau t}$ is an expectation that is taken at different stages of each period, and using the relevant decision maker's (shopper or worker) specific information set. This maximization is subject to the budget constraint

$$\sum_{\tau=1}^{N} p_{\hat{j}\tau t}c_{ij\tau t} + P_tC_{ijt} + M_{ijt} + B_{ijt} = (1+R_t)B_{ijt-1} + M_{ijt-1} + \tilde{\nu}_t + W_tL_{ijt} + s_{ij}\Pi_{jt} \quad , \tag{26}$$

[17]Notice that all uninformed consumers play the same strategy, and therefore have the same beliefs about the monetary shock.

where $\tilde{\tau}_t$ is a lump-sum transfer consistent with the process of money[18].

The cash-in-advance constraint for consumption of good $y$ is

$$P_t C_{ijt} \leq M_{ijt-1} + \tilde{\nu}_t \quad . \tag{27}$$

At this point it is important to notice that households' preferences are quasilinear. By eliminating wealth effects on household choices, this assumption simplifies the analysis in three ways. First, similar to Lagos and Wright (1995), it allows to handle the heterogeneity in households' information. These heterogeneity causes different shoppers to make different choices for credit goods $c_\tau$. However, the absence of wealth effects implies that all other choices are homogenous across households, and therefore the cross-sectional distribution of money and bond holdings collapses into a degenerate distribution. For this reason, I do not need to keep track of it. A second implication of the absence of wealth effects is that there is no option value in waiting to gather more information before consuming credit goods, which simplifies the games between firms and shoppers enormously. Third, linearity in labor supply implies that every set of equilibria of these games is consistent with a general equilibrium in which the labor market clears.

**Monopolists.** On every island $j$ there is a monopolist that produces good $c_\tau$ using technology

$$c_{j\tau t} = A_j L_{j\tau t} \quad , \tag{28}$$

where $L_{j\tau t}$ is labor hired by firm $j$ at subperiod $\tau t$. Firms are heterogenous with respect to their productivity. Write the firm's real marginal costs as

$$k_j = \frac{\frac{W_t}{M_t}}{A_j} \quad , \tag{29}$$

where $W_t$ is the nominal wage. I assume that firm heterogeneity is such that there is a distribution $A_j$ that yields a uniform distribution of real marginal costs

$$k_j \sim U[0, a] \quad . \tag{30}$$

**Representative Firm Producing $C$.** There is a representative firm that hires labor and produces good $C$ according to a simple linear technology. The labor productivity if this firm is normalized to 1:

$$C = L \quad .$$

[18] More specifically, $\tilde{\nu}_t$ is such that $\tilde{\nu}_t = M_t - M_{t-1}$. I will show that since in equilibrium all agents have the same money holdings and therefore this way of defining the transfer is possible.

**Definition of Equilibrium.** A general equilibrium of this economy are allocations $\{c_{ij\tau t}, C_{ijt}\}$, beliefs $\mu_{ij\tau t}(p_{j\tau t})$, labor supply decisions $\{L_{ijt}\}$, prices $\{p_{j\tau t}, P_t\}$, nominal wages, $\{W_t\}$, interest rates $\{1 + R_t\}$, for all $i$, $j$, $\tau$, $t$, s.t.

1. The households' conditions for optimality and associated constraints are satisfied;
2. the equilibrium strategies for the games played between monopolists and shoppers satisfy Bayesian Perfection:
   - monopolists post prices to maximize profits, given consumers' play,
   - uninformed shoppers use Bayes' rule on the path of equilibrium play,
   - shoppers' demand satisfies the condition for optimality;
3. the representative firm maximizes profits taking the price as given;
4. goods, labor, bonds, and money markets clear.

## 4.1 General Equilibrium.

**Household $ij$'s Optimality Conditions.** The conditions for optimality are computed as follows. Each time the shopper of household $ij$ is matched with a monopolist, he observed the price that was posted and computes the first order condition for consumption of the good sold by the monopolist:

$$\beta^t u'(c_{ij\tau t}) = p_{\hat{j}\tau t} E_{\mu_{ij\tau t}}[\lambda_{ijt}] \quad . \tag{31}$$

$E_{ij\tau t}[\cdot]$ is an expectation computed using the shopper's information set. This information set contains information he has previously collected plus the information revealed by the price of the monopolist.

When the shopper finally arrives to buy the cash good, he computes a first order condition for consumption of this cash good after observing its price. This good is sold by a representative competitive firm in a centralized market, and therefore its price reveals the realization of the monetary shock to the shopper, in case he did not know it already. Therefore, at this point the shopper does not face any uncertainty, and the first order condition is:

$$\beta^t v'(C_{ijt}) = P_t(\lambda_{ijt} + \psi_{ijt}) \quad . \tag{32}$$

The worker arrives at the market for labor and computes a first order condition for labor supply after observing the equilibrium wage. This is a centralized and competitive market, and therefore this wage reveals the realization of the monetary shock to the worker. Therefore, the worker does not face any uncertainty, and the first order condition is:

$$\beta^t = W_t \lambda_{ijt} \quad . \tag{33}$$

The first order condition for money holdings is computed at financial markets at the end of the period, and therefore under perfect information:

$$\lambda_{ijt} = E_t[\lambda_{ijt+1} + \psi_{ijt+1}] \quad . \tag{34}$$

The first order condition for bond holdings is also computed under perfect information:

$$\lambda_{ijt} = (1 + R_{t+1}) E_t[\lambda_{ijt+1}] \quad . \tag{35}$$

**Cash Good and Labor Markets.** It is possible to show that every set of PBE of the games played between monopolists and shoppers is consistent with a general equilibrium. The reason is the linearity in the disutility of labor, which implies that individual labor choices are always consistent with the resource constraint of the economy.

In equilibrium, the price of the cash good $C$ is pinned down by the cash in advance constraint, and therefore it is proportional to the money supply. Optimality of production for the representative firm producing the cash good immediately implies that the wage $W_t$ is also proportional to the money supply $M_t$. After a normalization, we have that all of these three quantities are always equal:

$$P_t = W_t = M_t \quad . \tag{36}$$

Also, the nominal interest rate is determined in expectations of next period's shock, and therefore always constant. Consumption of the credit good is also constant every period.

**Demand for Credit Good $c_{\tau t}$ by Shopper $ij$.** Substituting (33) into (31):

$$u'(c_{ij\tau t}) = p_{\hat{j}\tau t} E_{\mu_{ij\tau t}} \left[ \frac{1}{M_t} \right] \quad . \tag{37}$$

From this equation get the demand:

$$c_{ij\tau t} \left( p_{\hat{j}\tau t} E_{\mu_{ij\tau t}} \left[ \frac{1}{M_t} \right] \right) \quad . \tag{38}$$

As in the partial equilibrium model of section 2, the demand for credit goods depends on a deflated version of the price posted by firms. Here, the deflator is a belief about the inverse of the real value of a unit of money, and therefore the consumer basically transforms the nominal price posted by the monopolist into a real price. If this estimate is low (corresponding to a belief that the supply of money is high), a given nominal price is deflates into a low real price, increasing the shopper's demand. As it will become clear, this effect will limit firms credibility to signal that there has been a high monetary shock in the economy.

**Aggregate Demand for $c_{\tau t}$.** At every subperiod $\tau t$ a proportion $\alpha_{\tau t}$ of shoppers know the monetary aggregate. Therefore, the aggregate demand is

$$\int_{\mathcal{M}_{\hat{j}\tau t}} c_{ij\tau t}(p, M_t, \mu_{ij\tau t}) didj = \alpha_{\tau t} c_{ij\tau t}\left(p_{\hat{j}\tau t} \cdot \frac{1}{M_t}\right) + (1-\alpha_{\tau t}) c_{ij\tau t}\left(p_{\hat{j}\tau t} \cdot E_{\mu_{ij\tau t}}(p_{\hat{j}\tau t})\left[\frac{1}{M_t}\right]\right) \quad , \tag{39}$$

where $\mathcal{M}_{\hat{j}\tau t} \in [0,1] \times [0,1]$ is the subset of consumers matched with firm $\hat{j}$ at subperiod $\tau t$. A proportion $\alpha_{\tau t}$ of the shoppers arriving to firm $\hat{j}$ will know the monetary aggregate and therefore will have the correct beliefs, and the complementary proportion will form beliefs using Bayes' rule on the equilibrium path.

At this point, notice that the total demand (39) that every firm faces is the same as (10) in Section 2. This shows that result 1 applies in the case of this cash in advance model[19]. Also, all the dynamic results of Section 3 apply here. Thus, it is possible to write down a cash in advance general equilibrium framework compatible with all the results of this paper.

# 5 Conclusion

This paper proposes an explanation to nominal rigidities that is complementary to other explanations based on imperfect information[20]. It analyzes an environment where consumers' information about aggregate shocks is heterogenous. Some consumers are informed while others are uninformed. In the model, firms can learn the realization of aggregate shocks by observing total demand. This leads to an asymmetry of information between firms and uninformed consumers. I show that, because of this asymmetry of information, equilibria with rigid prices arise and are, for some parameter values, more profitable to firms than equilibria with flexible prices. In these equilibria firms do not change prices even though they have perfect information and face no menu costs.

This result, together with the strategic tension arising between firms and consumers in my model, accommodate some anecdotal evidence related to price rigidity. For instance, when asked about their reluctance to change prices, often firm managers declare to be concerned about

[19]See page 16.

[20]For a more detailed exposition of the relationship of my paper with this literature, see page 4.

consumers' reactions to price changes (Blinder 1991). Also, it is customary for firms to justify the reasons behind price increases or explicitly apologize once an increase has been implemented.

This type of nominal rigidity has interesting normative and positive dynamic implications. On the normative side, the rigidity leads to an informational externality that hurts firms' profits and reduces consumer welfare. This type of externality has been found on other settings (Banerjee 1992; Chamley and Gale 1994; Gorodnichenko 2009). I show that a temporary tax on monopolists reduces the amount of rigidity and improves welfare. On the positive side, the dynamic responses of output and inflation after a shock are delayed and hump-shaped. The reason is that initially few consumers know about the shock, but learning accelerates as firms start changing prices. These features seem consistent with the data (Christiano, Eichenbaum, and Evans 2005). Moreover, the responses are asymmetric: bigger effects on output for negative shocks, and faster increase in prices after a positive shock. These asymmetric features also seem realistic (Cover 1992; Peltzman 2000).

Further extensions of this setup are needed to understand several issues left open. For instance, a model with a richer type space for monetary policy is a clear extension to understand how the degree of rigidity changes for different shocks. Also, the cash in advanced setup can be further enriched. The first extension that comes to mind is staggering of households so that different set of households make use of financial markets at different times. I leave these extensions for future work.

# A Appendix

## A.1 Characterization of the Game.

First, it is necessary to define the following well-known property for a function of two variables.

**Definition 2 (Increasing Differences Property)** *A function $f(x, y)$ has strict increasing differences in $(x, y)$ if, for $x' > x$ and $y' > y$,*

$$f(x, y') - f(x, y) < f(x', y') - f(x', y) \quad . \tag{40}$$

The following lemma shows that under some conditions this game is a standard monotonic signaling game (Cho and Sobel 1990).

**Lemma 2 (Characterization of the Game)** *If $\alpha > 0$ and $pc_i(p \cdot 1/P)$ has increasing differences in $(p, P)$, this is a monotonic signaling game. It satisfies:*

1. *Monotonicity.*

   *Let $\mu_i'(p)$ and $\mu_i(p)$ be two possible beliefs by the uninformed. If $\mu_i'(p) > \mu(p)$, then, for all $p$, $pc(p, P, \mu_i'(p)) > pc(p, P, \mu_i(p))$.*

2. *Single-crossing. For any $p' > p$, we have that, for arbitrary demand by the uninformed, $p'c(p', P^l, \mu_i(p)) \geq pc(p, P^l, \mu_i(p)) \implies p'c(p', P^h, \mu_i(p)) > pc(p, P^h, \mu_i(p))$*

**Proof.** I first prove monotonicity, and then single-crossing.

1. Monotonicity. $u'(c_i)$ being a strictly decreasing function, the demand of the uninformed $c_i(pE_{\mu_i(p)}[1/P])$ is strictly increasing in $\mu_i(p)$. Therefore, for any $\mu_i'(p) > \mu(p)$, $pc'(p, P, \mu_i'(p)) > pc(p, P, \mu_i(p))$.

2. Single-crossing.

   Consider $p$, $p'$, such that $p < p'$, and assume

   $$p'c(p', P, \mu_i(p)) \geq pc(p, P, \mu_i(p))$$

   This is equivalent to

   $$p'c(p', P, \mu_i(p)) - pc(p, P, \mu_i(p)) \geq 0$$

   Since $c(p, P, \mu_i(p))$ has strict increasing differences in $(p, P)$,

$$p'c(p', P^h, \mu_i(p)) - pc(p, P^h, \mu_i'(p)) > p'c(p', P^l, \mu_i(p)) - pc(p, P^l, \mu_i'(p)) \geq 0$$

and therefore

$$p'c(p', P^h, \mu_i(p)) > pc(p, P^h, \mu_i(p))$$

■

## A.2 Proof of Proposition 1

The cutoff $\overline{\alpha}$ is obtained using the Incentive Compatibility (IC) constraint for the low type (14). Off equilibrium path beliefs are $\mu_i(p) = 0$.

Once the cutoff obtained, there are two cases:

- $\alpha \geq \overline{\alpha}$.

  In this case, (14) is satisfied at $p^h$ and $p^l$ defined by (15) and (16). Firms optimization in each state yields $p^h$ and $p^l$. Therefore, this is the best separating equilibrium. Since in this outcome consumption is $c^{ss}$, in this outcome ex-ante profits are $\Pi^*$.

- $\alpha < \overline{\alpha}$.

  In this case, the IC constraint for the low type is satisfied for a price $\overline{p}$ defined by (19). In all separating equilibria the low type posts $p^l$, otherwise he has a profitable deviation. Moreover, from (14) $\overline{p}$ is decreasing in $\alpha$, and therefore $\Pi(p(P))$ is increasing in $\alpha$.

■

## A.3 Proof of Proposition 2

The cutoff $\underline{\alpha}$ is obtained from the IC constraint for the high type (21). Off equilibrium path beliefs are $\mu_i(p) = 0$. Given this and (21), the IC constraint for the low type

$$p^* \left( \underline{\alpha} c_i \left( p^* \frac{1}{P^h} \right) + (1 - \underline{\alpha}) c_i \left( p^* \left[ \frac{1}{2} \cdot \frac{1}{P^h} + \frac{1}{2} \cdot \frac{1}{P^l} \right] \right) \right) = p^l \left( \underline{\alpha} c_i \left( p^l \frac{1}{P^h} \right) + (1 - \underline{\alpha}) c_i \left( p^l \frac{1}{P^l} \right) \right) \quad ,$$

is satisfied $\forall \alpha \leq \underline{\alpha}$.

I now show that if $\alpha = 0$, $\Pi(p(P)) = \Pi^*$. For $\alpha = 0$,

$$\Pi(p^*) = \frac{1}{2} \frac{1}{P^h} p^* c_i \left( p^* \left[ \frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l} \right] \right) + \frac{1}{2} \frac{1}{P^l} p^* c_i \left( p^* \left[ \frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l} \right] \right) \quad . \tag{41}$$

From the FOC of (20), (15) and (16), notice that

$$p^* \left[ \frac{1}{2} \cdot 1/P^h + \frac{1}{2} \cdot 1/P^l \right] = p^h \cdot 1/P^h = p^l \cdot 1/P^l \quad . \tag{42}$$

Also,

$$c_i \left( p^* \left[ \frac{1}{2} \cdot 1/P^h + \frac{1}{2} \cdot 1/P^l \right] \right) = c^{ss} \quad . \tag{43}$$

Together, (42) and (43) imply that the right hand side of (41) is equal to $\Pi^*$.

More generally,

$$\Pi(p^*) = \alpha \left( \frac{1}{2} \frac{1}{P^h} p^* c_i \left( p^* \frac{1}{P^h} \right) + \frac{1}{2} \frac{1}{P^l} p^* c_i \left( p^* \frac{1}{P^l} \right) \right) + (1-\alpha)\Pi^* \quad , \tag{44}$$

which is decreasing in $\alpha$.

■

## A.4 Proof of Proposition 3

In the case of the best separating equilibrium, $\Pi(p(P))$ is continuous and strictly increasing in $\alpha$. In the case of the $p^*$-pooling equilibrium, from (44) $\Pi(p^*)$ is continuous at $\alpha = 0$, reaches $\Pi^*$ at $\alpha = 0$, and it is strictly decreasing thereafter. Thus, there is a boundary $[0, \alpha^*]$ away from $\alpha = 0$ where $\Pi(p(P))$ is higher in the $p^*$-pooling equilibrium.

■

As argued in the body of the text, this proposition can be extended to the case of marginal costs proportional to the price level $P$. It is required that these are low enough so that the firm makes profits in the high state of the world. As long as $\alpha = 0$ ex-ante real costs are the same as under perfect information, and there it is very easy to obtain the alleged result.

## A.5 Proof of Proposition 4

Pick $\alpha^{**} = \overline{\alpha}$. For $\alpha \geq \alpha^{**}$, $\Pi(p(P)) = \Pi^*$ in the separating equilibrium. In the pooling equilibrium, if it exists for $\alpha \geq \overline{\alpha}$, the right hand side (RHS) of (44) is strictly below $\Pi^*$, and decreasing.

■

## A.6 Proof of Proposition 5

The RHS of (44) is strictly smaller than ex-ante profits in the perfect information benchmark, $\Pi^*(k)$. This proves the statement on ex-ante profits.

I will now prove the result on ex-ante welfare. Given separability of the utility function, consider the case of good $c_1$ (the argument is identical in the case of the other goods $c_2, \ldots, c_T$). The demand for this good is

$$c_{i1} = a - p_1^* E_{i1}\left[\frac{1}{P}\right]$$

In case of uninformed, consumption of good $c_1$ in the $p^*$-pooling Equilibrium is

$$c_{i1}^h = c_{i1}^l = a - p_1^*\left[\frac{1}{2}\frac{1}{P^h} + \frac{1}{2}\frac{1}{P^l}\right]$$

Because $p_1^*\left[\frac{1}{2}\cdot 1/P^h + \frac{1}{2}\cdot 1/P^l\right] = p^h \cdot 1/P^h = p^l \cdot 1/P^l$, $c_{i1}^h = c_{i1}^l = c^{ss}$. Thus, in the $p^*$-pooling Equilibrium, the uninformed get the same ex-ante utility as under perfect information.

In the case of the informed, consumption of good $c_1$ in this equilibrium is either

$$c_{i1}^h = a - p_1^*\frac{1}{P^h} > c^{ss} \quad ,$$

or

$$c_{i1}^l = a - p_1^*\frac{1}{P^l} < c^{ss} \quad .$$

Using Jensen's inequality, for an informed consumer:

$$\frac{1}{2}u(c_{i1}^h) + \frac{1}{2}u(c_{i1}^l) < u(c^{ss})$$

Thus, the informed get strictly less ex-ante utility than under perfect information.

Therefore, welfare is strictly smaller than under perfect information.

■

# References


Amador, M. and P. O. Weill (2007). Learning from private and public observation of other's actions. *mimeo, Stanford University*.

Anderson, E. and D. Simester. Price stickiness and customer antagonism. *Quarterly Journal of Economics*. forthcoming.

Bagwell, K. and M. H. Riordan (1991). High and declining prices signal product quality. *American Economic Review 81*(1), 224–239.

Banerjee, A. V. (1992). A simple model of herd behavior. *Quarterly Journal of Economics 57*(3), 797–817.

Bikhchandani, S., D. Hirshleifer, and I. Welch (1992, October). A theory of fads, fashion, custom, and cultural change as informational cascades. *Journal of Political Economy 100*(5), 992–1026.

Blinder, A. (1991). Why are prices sticky? Preliminary results from an interview study. *American Economic Review 89-96*.

Blinder, A., E. Canetti, D. Lebow, and J. Rudd (1998). *Asking About Prices: A New Approach to Understanding Price Stickiness*. New York. Russell Sage Foundation.

Cavallo, A. (2009). Scraped data and sticky prices: frequency, hazards, and synchronization. *mimeo, Harvard University*.

Chamley, C. and D. Gale (1994). Information revelation and strategic delay in a model of investment. *Econometrica 62*(5), 1065–1085.

Chari, V. V. and P. J. Kehoe (2003). Hot money. *Journal of Political Economy 111*(6), 1262–1292.

Cho, I.-K. and J. Sobel (1990). Strategy stability and uniqueness in signaling games. *Journal of Economic Theory 50*, 381–413.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1), 1–45.

Cover, J. P. (1992). Asymmetric effects of positive and negative money-supply shocks. *Quarterly Journal of Economics 107*(4), 1261–1282.

Duffie, D. and G. Manso (2007). Information percolation in large markets. *American Economic Review Papers and Proceedings 97*, 203–209.

Fehr, E., G. Kirchsteiger, and A. Riedl (1993). Does fairness prevent market clearing? *Quarterly Journal of Economics 108*.

Fuhrer, J. C. (2000). Habit formation in consumption and its implications for monetary-policy models. *American Economic Review 90*, 367–90.

Golosov, M., G. Lorenzoni, and A. Tsyvinski (2009). Decentralized trading with private information. *mimeo, MIT*.

Gorodnichenko, Y. (2009). Endogenous information, menu costs and inflation persistence. *mimeo, University of California, Berkeley*.

Greenwald, B. and J. E. Stiglitz (1989). Toward a theory of rigidities. *American Economic Review Papers and Proceedings 79*(2), 364–369.

Hall, R. and C. Hitch (1939). Price theory and business behavior. *Oxford Economic Papers 2*, 12–45.

Heidhues, P. and B. Köszegi (2005). The impact of consumer loss aversion on pricing. *mimeo, University of Bonn and University of California, Berkeley*.

Hellwig, C. and V. Venkateswaran (2009). Setting the right prices for the wrong reasons. *mimeo, UCLA*.

Jones, L. and R. E. Manuelli (2002). Volatile policy and private information: The case of monetary shocks. *Journal of Economic Theory 99*, 265–296.

Kahneman, D., J. L. Knetsch, and R. H. Thaler (1986). Fairness as a constraint on profit seeking: Entitlements in the market. *American Economic Review 76*(4), 728–741.

Kamenica, E. (2008). Contextual inference in markets: On the informational content of product lines. *American Economic Review 98*(5), 2127–2149.

Kimball, M. S. (1995). The quantitative analytics of the basic neomonetarist model. *Journal of Money, Credit and Banking Part 2: Liquidity, Monetary Policy, and Financial Intermediation 27*(4), 1241–1277.

Koszegi, B. and P. Heidhues (2005). The impact of consumer loss aversion on pricing. *Working paper, Berkeley*.

Lagos, R. and R. Wright (1995). A unified framework for monetary theory and policy analysis. *Journal of Political Economy 113*(31), 463–484.

Lucas, R. E. (1972). Expectations and the neutrality of money. *Journal of Economic Theory 4*, 103–124.

Lucas, R. E. and N. L. Stokey (1987). Money and interest in a cash-in-advance economy. *Econometrica 55*(3), 491–513.

Mackowiak, B. and M. Wiederholt (2009a). Business cycle dynamics under rational inattention. *mimeo*.

Mackowiak, B. and M. Wiederholt (2009b). Optimal sticky prices under rational inattention. *American Economic Review 99*(3), 769–803.

Mankiw, G. and R. Reis (2002). Sticky information versus sticky prices: a proposal to replace the New Keynesian Phillips curve. *Quarterly Journal of Economics 117*(4), 1295–1328.

Maskin, E. and J. Tirole (1988). A theory of dynamic oligopoly, II: Price competition, kinked demand curves, and Edgeworth cycles. *Econometrica 56*(3), 571–599.

Maskin, E. and J. Tirole (1992). The principal-agent relationship with an informed principal, II: Common values. *Econometrica 60*(1), 1–42.

Nakamura, E. and J. Steinsson (2005). Price setting in a forward-looking customer markets. *mimeo, Columbia University*.

Okun, A. M. (1981). *Prices and Quantities: A Macroeconomic Analysis.* Brookings Institution.

Peltzman, S. (2000). Prices rise faster than they fall. *Journal of Political Economy 108*(3), 466–502.

Reis, R. (2006). Inattentive producers. *Review of Economic Studies 73*, 793–821.

Reis, R. (2009). A sticky-information general-equilibrium model for policy analysis. K. Schmidt-Heubel and C. Walsh (Eds.). Monetary Policy under Uncertainty and Learning, Central Bank of Chile: forthcoming.

Rotemberg, J. J. (2005). Customer anger at price increases, changes in the frequency of prices changes and monetary policy. *Journal of Monetary Economics* (52), 829–852.

Rotemberg, J. J. (2007). Behavioral aspects of price setting, and their policy implications. *mimeo, Harvard University*.

Rotemberg, J. J. (2008). Fair pricing. *mimeo, Harvard University*.

Wolinsky, A. (1983). Prices as signals of product quality. *Review of Economic Studies, 50*(4), 647–658.

Woodford, M. (2002). Imperfect common knowledge and the effects of monetary policy. P. Aghion, R. Frydman, J. Stiglitz, and M. Woodford (Eds.). Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps, Princeton Univ. Press, 2002.

Woodford, M. (2009). Information-constrained state-dependent pricing. *mimeo, Columbia University*.