Building Reputation for Contract Renewal: Implications for Performance Dynamics and Contract Duration*

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Abstract

Due to technological progress, investment or learning, recent performance is often more informative about future performance prospects than is older performance. We incorporate information decay in a career concern model in which nonverifiable quality depends on the agent's innate productivity but the agent can invests to improve his type. In contrast with the career concerns literature (e.g. Lewis, 1986; RJE), the agent's incentives are stronger and quality is higher when the project approaches renewal date. We prove this result also in the case where performance depends on type and effort. We further show that long-term investment is crowded out by short-term effort in late contract periods, but it is boosted in early periods. More frequent contract renewals strengthen reputational effects and improve performance if the relative cost of investment is low, but otherwise longer-term contracts lead to higher quality. Our results are corroborated by some empirical studies showing that performance improves as the contract approaches renewal date.

Keywords: Career concerns, contract duration, contract renewal, reputation and dynamic incentives.

1 Introduction

Casual observation and empirical evidence suggest that contract renewal can be a powerful motivation device which, whenever renewal decisions depend upon past performance, can provide incentives to exert noncontractible effort. This has been observed in a variety of

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sectors. Using a panel of the 25 franchisees providing passenger services in the UK railway industry in 1997-2000, Affuso and Newbery (2000) found for example that voluntary investment by the contractors increased as the contract renewal date became nearer. In a study of the French water industry, Chong, Huet and Saussier (2006) found that contracts near to expiry date were characterized by lower prices compared to other contracts, all things being equal. While the study relies on a cross-section analysis of 1102 French local public authorities in 2001, their finding suggests that operators tend to reduce their prices as expiry dates approach.[1] In this paper, we provide a rationale for such performance patterns and derive implications for optimal contract duration.

The economics literature has approached the incentive power of contract renewal in two ways. The first approach relies on hidden action models of relational contracting.[2] For example, Kim (1998) considers an infinitely repeated setting in which effort is sustained by an implicit agreement between the principal and the agent. The agent exerts nonverifiable effort as long as the long-term gain from contract renewal is greater than the one-shot saving on the cost of effort. Conversely, the principal renews the contract with a well-performing agent as long as the value of future cooperation is greater than the one-shot gain from reneging on the promised rent. So far, however, the literature has mainly focused on stationary environments and thus tended to ignore performance dynamics within the contracting period. The principal usually retaliates (e.g., does not renew the contract) whenever she observes a deviation from the implicit agreement, regardless of when the deviation occurred, which in turn leads the agent to exert the same amount of effort in every period.[3]

The second rationale for the incentive power of contract renewal hinges on career concerns models. In the classic model of Holmström (1982), the market uses the agent's current output to update its belief about the agent's ability (type) and then bases future wages on these updated beliefs. The agent increases performance by taking actions that the market cannot observe, in an attempt to influence the market's belief about his type.

[1] A similar response to incentives appears in Ichino and Riphahn (2005) who, using personnel data from a large Italian bank, find that absenteeism of white collar workers is lower during probation – with an abrupt jump right after tenure is granted.

[2] The idea builds on the repeat-purchase mechanism, first explored by Klein and Leffler (1981). In their model, the firm provides nonverifiable quality if the discounted stream of profit from quality provision is greater than the one-shot gain from underperformance.

[3] See MacLeod (2007) for a survey.

Reputation building then makes the agent work harder.[4] Again, this framework does not explain why performance improves as the renewal date approaches. To the contrary, since information accumulates as time passes by, additional observations have less impact on market estimates, which (together with discounting and shorter remaining careers) reduces the incentives to exert effort.[5] In a multiperiod procurement setting, Lewis (1986) shows indeed that effort is smaller and costs are higher in later stages of the contract, since bad information is then less likely to induce the principal to terminate the project.

The type of an agent is not necessarily a static concept. An agent who is skilled today may become a low performer when a new technology comes along. Conversely, agents may invest in training or new technologies in order to acquire skills that improve their ability; alternatively, the process of learning through experience may change the agent's type over time. Recent performance is then more informative about the agent's future performance than older performance, which could potentially lead to performance patterns such as mentioned above. But whilst the career concerns literature has explored information decay due to switching types, its focus has been on the persistence of incentives[6] rather than on performance pattern, in settings where performance evaluations take moreover place at exogenous dates. The effects of contract renewal and contract duration on performance dynamics have to our knowledge remained unexplored.

Starting from these observations, in this paper we incorporate information decay in a model of career concerns and find that contractual performance then indeed improves as the contract approaches the renewal date.[7] In the most basic framework, we consider

[4]The incentive power of contract renewal is also discussed by Laffont and Tirole (1993, Chapter 8), who consider an adverse selection model with repeated auctions of incentive contracts, and show that favouring the incumbent at the renewal stage can improve incentives. In Dalen, Espen and Riis (2006), quality is nonverifiable but competitors can be ranked according to their quality performance. Tournaments are then used at renewal in order to reward noncontractible quality.

[5]A number of empirical papers have tested the predictions of the career concerns literature, but their focus has mainly been on the effect of reputation on incentives, the effect of performance information on the probability of termination and the effect of experience on the value of performance information (see e.g. Chevelier and Ellison (1999) and Hong, Kubik and Solomon (2000).

[6]See for example Malaith and Samuelson (2001) and Phelan (2006). As discussed in depth by Bar-Isaac and Tadelis (2009), for reputational concerns to be sustained over time, the market must never fully learn the type of the agent. This may hold for example if types exogenously change over time or if there is finite memory.

[7]Information decay and career concerns have been used to explain performance dynamics in other contexts, with different insights. In Fudenberg and Tirole (1995) information decay and career concerns lead managers to smooth earnings across periods in order to avoid interference. In Cukiermann and Meltzer (1986), elected officials bias pre-election policies in order to signal their competence. The more recent literature on political budget cycle (see e.g. Rogoff (1990)) is also related to our work, though

a three-period setting where performance (quality) is observable but nonverifiable and depends on the agent's unobservable innate productivity (type). The agent's productivity may change across periods due to exogenous factors. In each period, the agent can also increase his productivity, and thus his quality, by making an unobservable investment. The principal cannot precommit to a renewal policy and uses past performance to infer the agent's productivity and update her belief about the agent's future performance. At the time of renewal, asymmetric information on operating costs confers a rent to the agent; this rent is however endogenously determined, and increases with expected quality: the principal is more eager to deal with the agent when she expects him to deliver a high quality, which leads her to make a more generous offer at renewal.

In this setting, the agent invests to build a good reputation and improve his bargaining position at the renewal stage (we refer to this as the '*contract renewal effect*'). Due to information decay, the incentive to invest moreover increases as the renewal date approaches (we refer to this as the '*information-decay effect*'). When the investment technology exhibits constant returns to scale, the agent then never invests in the first period of the contract whilst he invests in the second period if the cost is not high. With decreasing returns to scale, the agent may undertake some investment in the first period, so as to reduce the overall cost of reputation building, but investment incentives remain stronger in the second period. A crowding-out effect moreover occurs, as the second-period investment reduces the value of investing in the first period.

The incentive generated by the contract renewal (weakly) increase with information persistence, the magnitude of agency rents and the level of the discount factor. But while this incentive increases welfare, it remains insufficient and underinvestment prevails, as the agent focuses on future rents and ignores the current value of quality for the principal. (Weakly) higher renewal prices help alleviate this underinvestment problem, as they can act as a commitment device to reward good performance. Since investment only takes place in the last period when investment costs are linear, any factor that increases investment incentives then exacerbates the performance pattern. By contrast, when the investment cost is convex, and thus investment can take place in both periods, only higher agency rents then unambiguously exacerbate the performance pattern.

We also consider the case where the agent can exert effort to improve his current

there pre-election myopia is assumed to justify improvements just before renewal (election).

quality, without affecting his underlying productivity and thus future quality. Signal jamming then arises: upon observing good performance, the principal is now unable to disentangle whether the productivity has increased or the agent is just working harder. We find that incentives remain stronger before renewal but also identify an additional effect, as (contrary to what standard career concern models suggest) agents with better reputation have more incentives to exert effort. We call this the 'reputation effect'.[8]

We also consider the case where the agent can choose between investing in productivity, which has a lasting impact on quality, and effort, which is less costly but only has a short-term impact on current performance. We show that effort then crowds out investment in the second period of the contract; the resulting signal jamming however creates incentives to invest in the first period. Thus, in sectors where the potential for improvement is high, we can expect both long-term investment and short-term effort to take place – the former at the beginning of the contractual relationship whilst the latter towards the end.

Finally, we stress that renewal decisions and information decay have quite different impact on cost and quality dynamics. Indeed, the agent is more likely to engage in cost-cutting activities at the beginning of a contract, so as to benefit from lower costs over a longer horizon; in addition, reducing his cost exposes the agent to a ratchet effect at the renewal stage, as the principal is more likely to insist on lower prices when she expects the agent to benefits from lower costs, which further tends to discourage cost-cutting activities before renewal.[9]

In the second part of the paper, we study the implications of these insights for contract design. Specifically, we consider an infinite repetition of the basic framework, which we use to compare two-period contracting with one-period contracting. In the case of one-period contracting, future investment curb incentives for current investment. With two-period contracting, investment is lower at the beginning of the contract, but this limits the crowding-out effect and fosters investment at the end of the contract. As a result, two-period contracting improves average performance when investment incentives

[8]In the original Holmström (1982) paper, agents with better reputation have weaker incentives to invest. Later papers have obtained opposite results – see e.g. Diamond (1989), and the survey by Bar-Isaac and Tadelis (2008).

[9]Our paper is therefore also linked to the literature on the ratchet effect in regulatory settings – see e.g. Pint (1992) and Laffont and Tirole (1993). There, the firm reduces productivity efforts before a price review, fearing that exhibiting low costs induces the regulator to tighten regulation. Dick and Di Tella (2002) shows that price cap regulation in Chile has led to cost reductions that are U-shaped: strong initial cost reductions reverse every four years, coinciding with regulatory reviews.

are otherwise weak, that is, for low values of the agent's future benefit from contract renewal or low degrees of information persistence.

Apart from explaining performance dynamics, our paper contributes to the literature on career concerns by allowing for endogenous type-switching;[10] this enables us to study the choice between such (long-term) productivity investment and (short-term) performance improvement effort, as well as their interaction over time. Endogenizing the impact of past performance on the expected rent from renewal moreover allows us to stress the role of the type of performance (e.g., cost versus quality). Our paper also contributes to the literature on the effects of contract duration, or the frequency of evaluations, on incentives. The existing literature has mostly focused on moral hazard issues. On the one hand, longer contracts alleviate moral hazard problems by facilitating consumption smoothing (Lambert (1983)) and ease hold-up and ratchet effects in the presence of specific investment (Laffont and Tirole (1993)). On the other hand, shorter contracts increase the flexibility to use new information as it comes along (Ellman (2006)) and reduce the gain from defection from implicit agreements (Shapiro (1983), Cesi, Valentini and Iozzi, (2009), and Calzolari and Spagnolo (2009)).[11] By considering instead adverse selection and type-switching investment, we highlight a different trade-off resulting from information decay and career concerns.

The structure of the paper is as follows. In Section 2 we discuss the basic model where the agent can invest to enhance his innate productivity and the principal can neither commit to a renewal policy nor to a pricing policy. Section 3 then provides several robustness checks. We first allow for some commitment over future contracting terms, before considering the possibility that the agent exerts effort to temporarily enhance quality, and investigating the interaction between investment and effort over the contract life. Finally, we contrast the impact of contract renewal and career concerns on quality and cost performance. In Section 4 we analyze the implications for contract duration in an infinite horizon setting. Section 5 concludes. All proofs are in an Appendix.

[10] In a paper written at the same time as ours, Board (2009) also introduces endogenous switching types in a career concerns settings. He focuses however on the value of reputation under different market learning hypotheses, rather than on contract renewal and the frequency of evaluations.

[11] Evidence on the determinants of contract duration shows that contracts are longer when relationship specific investment is important (Joskow, 1987) and shorter in periods of higher uncertainty (Crocker and Masten's (1988) and Saussier (1999)), which is consistent with the benefit of shorter contracts in the presence of information decay. Bandiera's findings (2002) are also consistent with the idea that contract length is chosen to provide incentives.

2 Basic model

We start by investigating the nature of incentives provided by contract renewal and its implications for performance dynamics, in situations where reputation building enhances the agent's bargaining power for future contract negotiations. To this purpose we consider a basic model, in which a principal (she) has delegated to an agent (he) the provision of a good or service for 2 periods. Service quality is observable but nonverifiable, and depends on the agent's innate productivity, which for technological reasons may change over time. In each period, a low-productivity agent can moreover make a nonverifiable investment to enhance his productivity. When the contract expires, having observed the agent's performance in the past two periods, the principal offers a new price to the agent or opts instead for in-house provision. The principal cannot precommit herself to a renewal policy or to a price offer. Agency problems, in the form of asymmetric information on operating costs, generate a rent for the agent if he continues to provide the service.

After describing this basic model, in the subsequent two subsections we discuss the renewal decision of the principal and its implications for performance dynamics.

2.1 Framework

Agent. The agent's productivity (type) is uncertain and may change over time, due to technological progress and/or investment. More precisely, in each period t the quality can take two values: high (H) or low (L); we will denote by $\Delta \equiv H - L$ the quality differential and by q_t^e the expected quality in period t. When the quality is initially L, the agent can however upgrade it to H by investing c. For the sake of exposition, we will refer to the initial quality in period t as the agent's "productivity" or "type", and will denote it by θ_t; we will use the term "quality" and the notation q_t to refer to the quality eventually provided in period t. By assumption, $q_t = H$ if $\theta_t = H$; if instead $\theta_t = L$, then $q_t = H$ if the agent invests c, and $q_t = L$ otherwise. We denote by $i_t \in [0, 1]$ the probability that the agent invests in period t when the quality is initially low.

The initial productivity of the agent, θ_1, is randomly drawn and is equally likely to be H or L; in the subsequent periods, the productivity follows a stationary first-order Markov process based on the agent's quality at the end of the previous period: $\theta_t = q_{t-1}$ with probability $\rho > 1/2$. The parameter ρ captures some information decay: a higher value of ρ denotes slower changing technologies and therefore a higher probability that, absent any investment, the agent's quality in period t remains the same as in period $t-1$.[12]

The agent's operating cost in period t, C_t, is also random and can take two values, $\underline{C}$ and $\overline{C}$, with respective probabilities α and $1-\alpha$.[13] Under delegation, the agent's per period payoff is $\pi_t = p_t - c_t - i_t c$, where p_t denotes the price paid to the agent in period t. The realization of the cost is privately observed by the agent, and not by the principal. As we will see, this information asymmetry generates a rent for the agent, who therefore gains from convincing the principal to renew the contract.

Principal. Under delegation, the principal's per period payoff is $u_t = q_t - p_t$. In-house provision generates instead a per period payoff of V, which is random and uniformly distributed over a range $\left[\underline{V}, \overline{V}\right]$.

At the beginning of period 3, having observed the qualities q_1 and q_2 provided in the first two periods as well as the realization of V, the principal makes a take-it-or-leave-it offer (a price p_3) to the agent for the future provision of the service.[14]

Timing. The timing of the game is as follows.

- Periods $t=1$ and $t=2$:
  - θ_t is realized and observed by the agent;
  - if $\theta_t = L$, chooses whether to invest;
  - q_t is realized and observed by both parties.[15]
- Period $t=3$:

[12] The analysis would apply to alternative processes determining current productivity as a function of performance history, such as moving averages or other higher-order Markov process, as long as recent performance provides more reliable information about future productivity.

[13] Our qualitative results remain unchanged when operating costs depend on types, as long as some uncertainty on costs remains present.

[14] In this section, we focus on the renewal decision (including the price p_3) and its impact on the agent's behavior during the initial contract; we will not need to discuss the determination of the prices p_1 and p_2. In section 4, we consider an infinite repetition of this basic framework and analyze the determination of prices in all periods.

[15] The analysis does not depend here on whether the principal observes the productivity θ_t and/or the investment decision: observing the quality q_t eliminates any relevant information asymmetry. In later sections, in which the agent may temporary increase the perceived quality, observing quality only maintains some ambiguity.

– V is realized and observed by the principal;

– the principal offers a price p_3;[16]

– C_3 is realized and observed by the agent, who then accepts or rejects the principal's offer.

We will assume that the principal and the agent use the same discount factor δ when evaluating multiperiod payoffs.

2.2 Contract renewal

At the beginning of period 3, the principal observes the realized value of V and chooses the price p_3. Since this is the last contracting period, the agent has no incentive to invest in case of low productivity; therefore, if delegating the provision to the agent, the principal expects a quality:

$$q_3^e = E\left[q_3\right] = \begin{cases} L + \rho\Delta & \text{if} \quad q_2 = H, \\ L + (1-\rho)\,\Delta & \text{if} \quad q_2 = L. \end{cases}$$

Given that the agent's cost is either $\overline{C}$ or $\underline{C}$, the principal will offer $p_3 = \overline{C}$, $p_3 = \underline{C}$, or make an unacceptable offer ($p_3 < \underline{C}$). The last option yields a payoff V. If instead the principal offers $p_3 = \underline{C}$, then with probability α, the agent observes $C_3 = \underline{C}$ and accepts the offer, whilst with probability $1-\alpha$, the agent observes $C_3 = \overline{C}$ and rejects the offer, in which case in-house provision yields V for the principal. Thus, by offering $p_3 = \underline{C}$, the principal obtains an expected payoff equal to:

$$E\left[U_3 \mid p_3 = \underline{C}\right] = \alpha\left(q_3^e - \underline{C}\right) + (1-\alpha)\,V.$$

If instead the principal offers a high price, $p_3 = \overline{C}$, the agent always accepts the offer and the principal thus obtains $q_3^e - \overline{C}$. It can be checked that this exceeds $E\left[U_3 \mid p_3 = \underline{C}\right]$ when:

$$V < \hat{V} \equiv q_3^e - \underline{C} - \frac{\overline{C} - \underline{C}}{1-\alpha}, \tag{1}$$

in which case it also exceeds V. Therefore, when $V < \hat{V}$, the principal offers a high price $\left(p_3 = \overline{C}\right)$, in which case the agent obtains a positive payoff $\overline{C} - \underline{C}$ with probability α, and just covers his cost otherwise. If instead $V > \hat{V}$, then the principal either offers $p_3 = \underline{C}$,

[16] As usual, there is no loss of generality assuming that the principal always make an offer; offering a price p lower than $\underline{C}$, which will always be rejected, amounts to making no offer.

which may be accepted if $C = \underline{C}$, or an even lower price which is never accepted; in both cases, the agent obtains zero payoff. Therefore, the expected profit of the agent is equal to:

$$E[\Pi_3] = \Pr\left(V \leq \hat{V}\right) B = \frac{\hat{V} - \underline{V}}{\overline{V} - \underline{V}} B,$$

where $B \equiv \alpha\left(\overline{C} - \underline{C}\right)$ denotes the agent's expected rent from a high price $p_3 = \overline{C}$. Since the threshold $\hat{V}$ increases with the expected quality q_3^e, which itself is higher when the agent previously provided a good quality ($q_2 = H$ rather than $q_2 = L$), we have:

Proposition 1 *The expected profit of the agent in period* 3 *increases with his previous performance.*

Indeed, the better the previous performance of the agent, the greater the principal's expected payoff (net of the price) from delegation in period 3 and thus the higher the incentive of the principal to make a high-price offer. Since the agent earns a rent only when (his cost is low and) he receives such a high-price offer, a better past performance raises the expected rent of the agent.[17]

For the sake of exposition, we will normalize the distribution of V as follows:

Assumption 1: $\underline{V} \equiv L - \underline{C} - \left(\overline{C} - \underline{C}\right)/(1-\alpha)$ and $\overline{V} \equiv H - \underline{C} - \left(\overline{C} - \underline{C}\right)/(1-\alpha)$.

With this normalization, the probability of a high-price offer simply coincides with the probability that the principal assigns to quality being high in period 3; the agent's expected profit from renewal is thus equal to:

$$E[\Pi_3] = \begin{cases} \rho B & \text{if} \quad q_2 = H, \\ (1-\rho) B & \text{if} \quad q_2 = L. \end{cases}$$

2.3 Performance dynamics

We now analyze the agent's behavior during the initial contract. In period 2, the agent must decide whether to invest in case of low productivity (that is, if $\theta_2 = L$). Investing costs c but upgrades the current quality q_2 from L to H, and thus increases the expected rent in period 3 from $(1-\rho) B$ to ρB. Therefore, the agent will invest if:

$$c < c^* \equiv \delta (2\rho - 1) B.$$

[17] As already noted, the agent's rent derives here from asymmetric information about the operating cost. Absent such private information, at the renewal stage the principal would offer a cost-based contract extracting the whole surplus; this would nullify the potential role of the contract renewal as an incentive device.

Thus, if $c < c^*$, the agent invests in period 2 whenever $\theta_2 = L$, and thus always delivers a high quality: $q_2 = H$; we show in the appendix that this, in turn, implies that the agent never invests in period 1: this would have no impact on the final quality q_2 (and thus on the principal's belief and the agent's associated rent at renewal), and it does not pay to invest in period 1 merely to reduce the probability of having to do so in period 2. Conversely, when $c > c^*$, the agent does not invest in period 2; in that case, it does not pay to invest in period 1 either, since the benefit is even lower (and comes later). Therefore:

Proposition 2 *The agent never invests in period* 1 *whilst he invests in period* 2 *if (he has a low type,* $\theta_2 = L$*, and)* $c < c^*$*. The incentive to invest in period* 2 *thus increases with the relative benefit of renewal,* $\delta B/c$*, and with the degree of information persistence,* ρ*.*

Proof: see the Appendix.

Since quality is noncontractible, the principal cannot incentivize the agent to provide high quality via explicit contractual terms. However, the *contract-renewal effect* creates an implicit incentive: past performance affects the principal's belief about the agent's productivity, and thus the agent's expected rent from renewal, which encourages the agent to invest when needed in order to improve his performance.

However, the *information-decay effect* weakens this incentive as time moves away from renewal date. This effect is here extreme and nullifies the investment incentives in the first period: quality in period 1 has indeed no informational value for the renewal decision, since quality in period 2 provides a sufficient statistic for the agent's type at the end of the initial contract. More generally (see below), information decay reduces incentives in early periods whenever recent performance is more relevant than prior performance for the renewal decision.

Information decay affects not only performance dynamics but also the strength of the incentive generated by contract renewal. In case of full decay ($\rho = 1/2$), past performance tells nothing about future productivity and thus has no impact on the renewal decision; the agent then never seeks to invest in order to improve performance. As ρ increases, past performance becomes more and more informative about the agent's future productivity, which in turn gives the agent greater incentives to invest in period 2 when needed.

An implication of Proposition 2 is that, on average, the agent's performance improves as the renewal stage approaches:

Corollary 1 *The expected quality (weakly) increases as the contract approaches the renewal date, and it does so strictly when $c < c^*$.*

Indeed, the expected quality in period 1 is simply $q_1^e = (L + H)/2$, since the agent never invests in that period; in the second period, the quality is instead equal to H with probability 1 when $c < c^*$ (it remains equal to $(L + H)/2$ otherwise).

The incentive power of contract renewal in multiperiod bilateral relationships was first analyzed by Lewis (1986). In his model the principal decides in each period whether to continue to finance the project or to terminate it. The threat of termination and the asymmetry of information on the project type between the principal and the agent may induce the agent to exert effort to limit costs. The benefit from the project accrues to the principal only when the project is completed; thus the threat of termination weakens as the project nears completion. This induces the agent to work less hard over time. In our model instead the benefit from the project accrues to the principal in each period whilst the renewal decision only occurs at the end of the contract. Together with the presence of information decay, absent in Lewis, this makes the agent work harder as the project nears completion.

Remark: welfare analysis. Investing in period t in case of low productivity not only increases current quality, q_t, but also enhances future expected quality $q_{t+\tau}^e$ and thus expected welfare in period 3, which is of the form:

$$E[W_3; q_3^e] = \int_{\underline{V}}^{\hat{V}(q_3^e)} (q_3^e - C^e) \frac{dV}{\Delta} + \int_{\hat{V}(q_3^e)}^{\min\{\tilde{V}(q_3^e), \overline{V}\}} [\alpha (q_3^e - \underline{C}) + (1-\alpha) V] \frac{dV}{\Delta} + \int_{\min\{\tilde{V}(q_3^e), \overline{V}\}}^{\overline{V}} V \frac{dV}{\Delta}, \quad (2)$$

where $C^e \equiv \alpha \underline{C} + (1-\alpha) \overline{C}$ denotes the agent's expected cost, $\hat{V}(q_3^e)$ is given by (1) and $\tilde{V}(q_3^e) \equiv q_3^e - \underline{C}$ denotes the threshold above which the principal favors in-house provision. Investment incentives are clearly insufficient in period 1, since the agent never invests – even if $c < \Delta$, in which case the short-term impact on quality would already suffice to make the investment efficient. Whilst contract renewal provides a positive incentive in period 2, this incentive remains insufficient there as well: the agent focuses on his future rent, which ignores the impact on current quality, and only partially reflects the impact on future quality:

Corollary 2 *Investing in case of low productivity in period* 2 *is socially desirable whenever:*

$$c < c^S \equiv c^* + \Delta + \delta \hat{U},$$

where Δ *reflects the improvement on current quality and* $\hat{U} > 0$ *reflects the principal's additional benefit from quality in period* 3*, which the agent fails to internalize.*

Proof: see the Appendix.

Whenever $c^* < c < c^S$, the agent does not invest in period 2 even though doing so would be socially desirable. When for example delegation to a low-cost agent is always more efficient than in-house provision:

$$L + (1 - \rho)\, \Delta - \underline{C} > \overline{V} \iff \overline{C} - \underline{C} > (1 - \alpha)\, \rho \Delta, \tag{C}$$

then the principal always offers at least equal to $\underline{C}$, even upon observing a low quality in period 2 ($q_2 = L$), and $\hat{U} = \frac{1+\alpha}{2} \delta\, (2\rho - 1)\, \Delta$.

3 Robustness

The above analysis shows that the combination of the effects of contract renewal and information decay induce the agent to invest more in quality as the contract renewal date approaches. This takes an extreme form in the previous framework: the agent never invests in the first period, since the quality observed in period 2 fully reveals anyway the agent's type at the end of the contract, and this is what matters for the renewal decision. We now test the robustness of the insights with respect to the underlying assumptions.

First, we show that these insights carry over to situations where the principal can precommit herself to a pricing policy at renewal stage; in addition, however, adopting higher prices can then help enhance investment incentives. We then consider several variants in which the agent may find it desirable to maintain a good quality in the first period. First, we allow for decreasing returns to scale in investment. As we shall see, the agent may then invest in period 1, but incentives remain stronger in period 2. Second, we introduce the possibility that the agent exerts effort to improve his current performance, with no effect on innate productivity. We show that our main insights continue to hold in this extended model, although contract renewal generates additional effects on performance dynamics. Finally, we show that the effects of contract renewal and information decay on performance dynamics are quite different for investment in cost-reducing activities.

3.1 Limited commitment

So far we have assumed that the principal cannot commit to any renewal or pricing policy. As a result, while the renewal process does give the agent some incentives to invest, these are insufficient and may fail to induce the agent to invest whenever it is efficient to do so. Therefore, the principal would benefit from committing in advance to a renewal or pricing policy that enhances the agent's incentives to invest.

To see this, we now introduce some commitment ability for the principal, in that she can precommit over the prices that she may propose in period 3. Ex post, the principal can opt for three types of offers: (i) an offer that the agent will accept whatever his cost, (ii) an offer that the agent will accept only if he faces a low cost, (iii) an offer that is never acceptable (equivalently the principal could choose not to make any offer), Clearly, among the prices falling in each of the first two categories, the principal will chose the most favorable one. Therefore, without loss of generality, we can restrict attention to two relevant prices: a "high price" $\overline{p} \geq \overline{C}$ designed to be accepted by the agent whatever his cost, and a "low price" $\underline{p} \in \left[\underline{C}, \overline{C}\right]$ that the agent will accept only if he faces a low cost $(C = \underline{C})$.[18]

Committing to prices $\overline{p}$ and $\underline{p}$ higher than the corresponding costs, $\overline{C}$ and $\underline{C}$, generates two effects: keeping constant the probability of renewal, it increases the agent's expected rent; however, it also reduces the probability that the principal will wish to renew the contract. The thresholds $\hat{V}(q_3^e)$ and $\tilde{V}(q_3^e)$ are now defined by:

$$\begin{aligned} \hat{V}\left(q_3^e\right) &: \quad q_3^e - \overline{p} = \alpha\left(q_3^e - \underline{p}\right) + (1-\alpha)\hat{V} \Leftrightarrow \hat{V}\left(q_3^e\right) = q_3^e - \underline{p} - \frac{\overline{p} - \underline{p}}{1-\alpha}, \\ \tilde{V}\left(q_3^e\right) &: \quad \alpha\left(q_3^e - \underline{p}\right) + (1-\alpha)\tilde{V} = \tilde{V} \Leftrightarrow \tilde{V}\left(q_3^e\right) = q_3^e - \underline{p}. \end{aligned}$$

As before, a low-productivity agent never invests in period 1 $(i_1 = 0)$, whilst he may

[18]The principal might also commit to a "severance pay" – giving a compensation if the contract is not renewed. The analysis remains similar, adjusting the prices $\overline{p}$ and $\underline{p}$ by the same amount. It therefore amounts to increasing the expected rent at the renewal stage, which the principal can however retrieve ex ante by reducing the price for the first contract.

invest in period 2 ($i_2 > 0$) when this has a large enough impact on the expected rent from renewal, which is now given by:

$$E[\Pi_3; q_3^e] = \int_{\underline{V}}^{\min\{\hat{V}(q_3^e), \overline{V}\}} (\overline{p} - C^e) \frac{dV}{\Delta} + \int_{\min\{\hat{V}(q_3^e), \overline{V}\}}^{\min\{\tilde{V}(q_3^e), \overline{V}\}} \alpha \left(\underline{p} - \underline{C}\right) \frac{dV}{\Delta}. \tag{3}$$

Investing in case of low productivity in period 2 increases the expected quality q_3^e by $(2\rho - 1)\Delta$, from $\underline{q}_3^e \equiv L + (1-\rho)\Delta$ to $\overline{q}_3^e \equiv L + \rho\Delta$; it thus increases the thresholds $\tilde{V}(q_3^e)$ and $\hat{V}(q_3^e)$ by the same amount and enhances the agent's expected rent by:

$$\Delta\Pi_3 = (2\rho - 1)\left[\hat{\xi}(\overline{p} - C^e) - \tilde{\xi}\alpha\left(\underline{p} - \underline{C}\right)\right],$$

where $\hat{\xi}$ and $\tilde{\xi}$ lie between 0 and 1, depending on the position of the above thresholds with respect to $\overline{V}$.[19] It follows that, in order to enhance the agent's incentive to invest in quality, the principal should keep $\underline{p}$ as low as possible (i.e., $\underline{p} = \underline{C}$) and instead increase $\overline{p}$ above $\overline{C}$.

As the principal can retrieve ex ante the agent's expected payoff from renewal through the price of the first contract, she will seek to maximize total expected welfare, $E[W_3; q_3^e]$. It is thus optimal to adopt $\underline{p} = \underline{C}$, since this not only maximizes the agent's incentive to invest, but moreover ensures that in-house provision is adopted only when it is efficient, that is, exactly when $V < \overline{q}_3^e - \underline{C}$.[20] By contrast, increasing $\overline{p}$ above $\overline{C}$ enhances investment incentives; while this also distorts the renewal decision, the principal may choose to do so – and will then adopt the lowest level necessary to provide adequate incentives – when the distortion remains limited:

Proposition 3 *If the principal can pre-commit herself to specific price offers, she always finds it optimal to adopt* $\underline{p} = \underline{C}$ *but may increase* $\overline{p}$ *above* $\overline{C}$ *in order to foster the agent's*

[19] $\hat{\xi}$ is equal to 0 if $\hat{V}(\underline{q}_3^e) > \overline{V}$, to 1 if $\hat{V}(\overline{q}_3^e) < \overline{V}$, and to $\left(\overline{V} - \hat{V}(\underline{q}_3^e)\right)/(2\rho - 1)\Delta$ otherwise. $\tilde{\xi}$ is instead equal to 0 if $\tilde{V}(\overline{q}_3^e) < \overline{V}$, to 1 if $\tilde{V}(\underline{q}_3^e) > \overline{V}$, and to $\left(\tilde{V}(\overline{q}_3^e) - \overline{V}\right)/(2\rho - 1)\Delta$ otherwise.

[20] Given $\overline{q}_3$, consider an increase in $\underline{p}$, which thus decreases $\tilde{V} = \overline{q}_3 - \underline{p}$, adjusting $\overline{p}$ so as to keep $\hat{V}$ constant; as long as $\tilde{V} \leq \overline{V}$ we have:

$$\frac{\partial E[W_3]}{\partial \underline{p}} = -\frac{\partial E[W_3]}{\partial \tilde{V}} = -\alpha\left(\overline{q}_3 - \underline{C} - \tilde{V}\right) = -\alpha\left(\underline{p} - \underline{C}\right).$$

It is thus efficient to adopt $\underline{p} = \underline{C}$.

incentive to invest in quality: there exists $\hat{c}^s \in (c^*, c^s)$ such that this is (strictly) profitable if and only $c^* < c < \hat{c}^s$.

Proof: see the Appendix.

Thus, when private incentives are insufficient to induce the agent to invest, the principal may find it optimal to raise the agent's expected rent from contract renewal by raising $\overline{p}$. Raising $\overline{p}$ however further distorts the principal's ex post choice between delegation and in-house provision and thus reduces expected welfare: since the principal does not internalize ex post the agent's rent at the time of renewal, increasing $\overline{p}$ encourages the principal to offer a low price rather than a high price, thereby raising the probability of inefficient in-house provision when the agent faces a high cost. This reduction in expected welfare also implies that the principal may choose not to provide incentives even though investment would be socially desirable (i.e., $\hat{c}^s < c^s$).

3.2 Variable investment

We assumed so far that productivity investment exhibited constant returns to scale; we now consider the case of decreasing returns to scale and suppose instead that, when $\theta_t = L$, upgrading quality with probability $i \leq 1$ costs $c(i) = ki^2/2$. Building on the above analysis, in period 2 a low-type agent ($\theta_2 = L$) will choose i_2 so as to maximize:

$$\max_i -c(i_2) + [i_2\rho + (1-i_2)(1-\rho)]\,\delta B = -\frac{k}{2}(i_2)^2 + i_2\delta(2\rho-1)B + (1-\rho)\delta B = \underline{\Pi}_2$$

That is, by investing $c(i_2)$ the agent increases the probability of earning the rent B in period 3 by $i_2(2\rho-1)$; the agent will thus choose:

$$i_2^* = \min\left\{(2\rho-1)\frac{\delta B}{k}, 1\right\}.$$

By doing so, the agent obtains an expected payoff equal to:

$$\underline{\Pi}_2 = \begin{cases} \rho\delta B - \dfrac{k}{2} & \text{if} \quad k \leq \delta(2\rho-1)B, \\ (1-\rho)\delta B + (2\rho-1)^2\dfrac{\delta^2 B^2}{2k} & \text{if} \quad k > \delta(2\rho-1)B. \end{cases}$$

The expected payoff of a high-type agent ($\theta_2 = H$) is instead equal to $\overline{\Pi}_2 = \rho\delta B$. Therefore, in period 1 a low-productivity agent ($\theta_1 = L$) will choose i_1 so as to maximize:

$$\begin{aligned} & -c(i_1) + \delta\left\{i_1\left[\rho\overline{\Pi}_2 + (1-\rho)\underline{\Pi}_2\right] + (1-i_1)\left[(1-\rho)\overline{\Pi}_2 + \rho\underline{\Pi}_2\right]\right\} \\ = \; & -\frac{k}{2}(i_1)^2 + i_1\delta(2\rho-1)\left(\overline{\Pi}_2 - \underline{\Pi}_2\right) + (1-\rho)\overline{\Pi}_2 + \rho\underline{\Pi}_2. \end{aligned} \tag{4}$$

The analysis of these investment opportunities yields:

Proposition 4 *When investment costs are convex, a low-type agent invests in both periods, and investment levels increase in both periods with the relative benefit of renewal* (B/k)*, the weight put on the future* (δ) *and the degree of information persistence* (ρ)*. The agent however invests less in period* 1 *than in period* 2*, and the expected quality thus still increases as the contract approaches the renewal date* ($q_1^e < q_2^e$)*. The ratio* i_1^*/i_2^* *moreover (weakly) decreases with* B/k *but increases with* δ *and* ρ*.*

Proof: see the Appendix.

In contrast with the case of constant returns to scale, with decreasing returns it is worth investing also in period 1 in order to smooth investment cost over the lifetime of the contract. The first two terms in expression (4) can be rewritten as:

$$-\frac{k}{2}(i_1)^2 + i_1\delta(2\rho-1)\left[\delta(2\rho-1)B - \left(i_2^*\delta(2\rho-1)B - \frac{k}{2}(i_2^*)^2\right)\right]$$

The incentive to invest in period 1 thus combines two effects. First, an increase in $\delta, B/k$ or ρ raises the benefit from investing in period 1, and thus have a direct, positive impact on i_1^*; this impact is however lower than in the second period, due in particular to information decay (reflected in the term $\delta^2(2\rho-1)^2 B < \delta(2\rho-1)B$). Second, an increase in these variables also increases the second-period investment i_2^*, which indirectly reduces the incentive to invest in period 1 through a *crowding-out effect.* In particular, the agent never invests maximally in period 1: he would *a fortiori* invest $i_2 = 1$ in period 2, in which case the sole benefit of the first-period investment is to lower the probability of having to invest again in the second period, which induces $i_1^* < 1/2$.[21] This crowding-out effect is limited and does not suffice to offset the direct positive impact on i_1^*; it however explains why an increase in B/k, which boosts i_2^*, reduces the ratio i_1^*/i_2^* and thus exacerbates the performance temporal profile. Finally, an increase in persistence (that is, a higher value for δ or ρ) allows the investment to have more lasting effect on productivity, which in turn encourages the agent to invest in early periods. As a result, and despite the crowding-out effect, it has a positive impact on the ratio i_1^*/i_2^* and thus tends to flatten the performance profile over time.

[21] The agent would then maximize $i_1\delta(2\rho-1)k/2 - ki_1^2/2$ and thus choose $i_1^* = (2\rho-1)\delta/2 < 1/2$.

3.3 Effort

We now revert to the case of no-commitment but suppose that, in each period t, a low-type agent ($\theta_t = L$) can exert an effort that, at cost γ, improves the quality ($q_t = H$) perceived (correctly or wrongly) by the principal. In contrast with the above investment technology, this effort does not change the agent's underlying "type" θ: θ_{t+1} coincides with θ_t with probability ρ, whatever the agent's effort.

If $q_2 = L$, the principal infers that the agent has a low productivity in period 2. By contrast, $q_2 = H$ entertains ambiguity, since high quality can be the result of productivity or effort; let ν_{q_1} denote the principal's corresponding belief, as a function of the quality $q_1 \in \{H, L\}$ observed in period 1. The higher the principal's belief, the more likely she is to offer a high price $p_3 = \overline{C}$, which in turn can encourage the agent to exert effort. Using the same normalization as before, the agent's expected profit from contract renewal is given by:

$$E[\Pi_3] = \begin{cases} (1 - \rho + (2\rho - 1)\,\nu_{q_1})\,B & \text{if} \quad q_2 = H, \\ (1 - \rho)\,B & \text{if} \quad q_2 = L. \end{cases}$$

Since the agent's strategy depends on how the principal will interpret performance, it may depend on the quality observed in period 1 (q_1), but not on the unobservable type θ_1. In particular, the agent faces the same incentive to exert effort in period 2, whether $q_1 = H$ was the result of high productivity ($\theta_1 = H$) or effort ($\theta_1 = L, q_1 = H$). We will denote by $e_1 \in [0, 1]$ the probability that a low-type agent exerts effort in period 1 and by e_{q_1} the probability that he exerts effort in period 2, as a function of the quality q_1 observed in period 1.

Consider first the effort decision of a low-type agent in period 2 ($\theta_2 = L$). Exerting effort costs γ but improves the quality to $q_2 = H$, inducing the principal to offer a high price with an additional probability $(2\rho - 1)\,\nu_{q_1}$ in period 3. The agent will thus exert effort if:

$$\gamma \leq (2\rho - 1)\,\nu_{q_1}\delta B. \tag{5}$$

Consider now the effort decision of a low-type agent in period 1 ($\theta_1 = L$). If the agent does not exert effort, $q_1 = L$ and, in period 2, the principal will observe $q_2 = H$ with probability $1 - \rho + \rho e_L$, leading her to believe that the agent is productive with probability ν_L. With complementary probability, $q_2 = L$ will be observed, revealing a low productivity ($\nu_2 = 0$).

Thus, the agent's expected payoff from not exerting effort in period 1 is:

$$[1-\rho+(1-\rho+\rho e_L)(2\rho-1)\nu_L]\,\delta^2 B-\delta\rho e_L\gamma. \tag{6}$$

Similarly, the expected payoff of the agent from exerting effort in period 1 is

$$[1-\rho+(1-\rho+\rho e_H)(2\rho-1)\nu_H]\,\delta^2 B-\delta\rho e_H\gamma-\gamma. \tag{7}$$

Comparing (6) and (7), the agent will exert effort in period 1 if:

$$\gamma\leq\frac{(1-\rho+\rho e_H)\,\nu_H-(1-\rho+\rho e_L)\,\nu_L}{1+\rho\,(e_H-e_L)\,\delta}\delta^2\,(2\rho-1)\,B. \tag{8}$$

As before, the agent never exerts any effort in case of full information decay ($\rho = 1/2$), since reputation then does not matter ($\nu_2 = 1/2$ whatever the past performance). Conversely, in the absence of any information decay ($\rho = 1$), it does not make sense for a low-productivity agent to exert effort in only one period: the principal would observe a low quality in the other period and then perfectly infer the agent's type. Therefore, a low-productivity agent either always exerts effort or never does so; in both cases, the agent's performance is constant over time. We now characterize the agent's behavior when some information decay is present:

Proposition 5 *(i) The incentive to exert effort increases as the contract gets closer to the renewal date:* $e_1 > 0$ *implies* $e_H = 1$ *and* $e_L > 0$*, whereas* e_H *and* e_L *can be positive when* $e_1 = 0$*; as a result, the expected quality still (weakly) increases as the contract approaches the renewal date* ($q_1^e \leq q_2^e$)*. (ii) Incentives moreover increase with existing reputation:* $e_H \geq e_L$ *with strict inequality whenever* $0 < e_L < 1$*. This incentive also increases with the net benefit of renewal* (B/γ) *and with the weight on the future* (δ)*.*

Proof: see the Appendix.

Contract renewal provides here again an effective incentive device. The strength of this implicit incentive depends on the expected rent from renewal and a number of other factors. First, there is again an information-decay effect: the incentive to exert effort increases as the renewal date approaches since recent performance provides better information than past performance about the agent's underlying productivity.[22] This effect

[22] Our formulation allows for "asymmetric ambiguity": a bad quality L « reveals » a low productivity, whereas a good quality H entertains some ambiguity, as it may be the result of the agent's effort. More generally, incentives to exert effort remain stronger in the second period whenever, due to information decay, productivity estimates are ranked as "$LL \leq HL < LH \leq HH$".

takes a less brutal form here, since the agent may exert effort in the first period as well as in the second one; yet, a necessary condition for effort to be exerted in period 1 is that it is exerted in period 2.[23] Second, a "*reputation effect*" appears: in the second period, the agent's incentive to exert effort increases with the reputation acquired in the first period: $e_H \geq e_L$. That is, the agent has more incentives to hide bad news when he is supposed to be good.

As in the basic model of productive investment – and contrary to Lewis (1986) –, we thus find again that performance improves as the contract approaches the expiry date. But incentives moreover increase with existing reputation – which also goes in the opposite direction as Lewis' findings. As low quality reveals a low productivity and thus ruins the agent's reputation, the greater this reputation, the stronger the incentive to exert effort.

3.4 Investment and effort

Suppose now that both effort and investment are possible in each period and consider the agent's choice between investment and effort.

It is natural to assume that pure effort is less costly than actual investment: $\gamma < c$. If follows that, in period 2, no investment ever takes place, since effort provides a less costly way to achieve the same result, namely, delivering a high quality in order to increase the expected rent from contract renewal.

In period 1, the agent's expected payoff is given by (6) if he does not invest or exert effort, and by (8) if he exerts effort. If instead the agent invests to improve his current type, his expected payoff becomes:

$$[1 - \rho + (\rho + (1 - \rho) e_H) (2\rho - 1) \nu_H] \delta^2 B - \delta (1 - \rho) e_H \gamma - c. \tag{9}$$

Comparing these options yields:

Proposition 6 *As c tends towards γ: (i) in the first period of the contract, the agent has relatively more incentives to invest than to exert effort; (ii) the opposite holds in the second period.*

[23] The role of information persistence is less clear-cut since both recent and past history matters. For example, when $q_1 = L$, an increase in ρ has a positive direct impact on the incentives to invest, taking as given the principal's belief at the beginning of period 2, but it tends to decrease this belief. In contrast, when $q_1 = H$, both the direct and the indirect effects enhance the incentives to invest.

Proof: see the Appendix.

In the second period, the agent favors effort over investment, since the former is a cheaper way to deliver high quality. This, however, makes quality less informative about the agent's underlying productivity; as a result, showing good performance becomes valuable also in period 1. Since investment brings longer-term benefits, in the first period the agent then favors investment over effort when the cost difference is small. As in the previous instances, performance improves over time:

Proposition 7 *Performance weakly increases over time.*

Proof: see the Appendix.

3.5 Cost performance and ratchet effect

In this section we extend the analysis to discuss how contract renewal and career concerns affect incentives to invest in cost-reducing activities. To this purpose, assume that quality is fixed at q and that the cost of production is determined by the agent's type: it is $\underline{C}$ if $\theta_t = H$ and $\overline{C}$ if $\theta_H = L$. In that latter case, however, the agent can invest c to reduce his cost down to $\underline{C}$. The payoff of the principal is $q - p_t$ if the service is provided by the agent and V if the service is taken in-house. The payoff of the agent is $p_t - C$ when he provides the service.

Following the same reasoning as in Section 2.2, consider the pricing offers made by the principal in period 3, given the probability μ_3 that quality will be high. An offer $p_3 = \overline{C}$ is always accepted and gives the principal a payoff $q - \overline{C}$, whereas an offer $p_3 = \underline{C}$ is accepted only when the agent's cost is low, and thus yields an expected payoff equal to:

$$E\left[U_3 \mid p_3 = \underline{C}; \mu_3\right] = \mu_3\left(q - \underline{C}\right) + \left(1 - \mu_3\right)V.$$

It can be checked that $q - \overline{C} > E\left[U_3 \mid p_3 = \underline{C}; \mu_3\right]$ when

$$V < \hat{V}\left(\mu_3\right) \equiv q - \underline{C} - \frac{\overline{C} - \underline{C}}{1 - \mu_3},$$

in which case $q - \overline{C}$ also exceeds V. Therefore, when $V < \hat{V}\left(\mu_3\right)$, the principal offers a high price $\left(p_3 = \overline{C}\right)$; the agent then obtains a payoff $\overline{C} - \underline{C}$ with probability μ_3 and just covers his cost otherwise. If instead $V > \hat{V}\left(\mu_3\right)$, the principal either offers $p_3 = \underline{C}$ or an even lower price which is never accepted; in both cases, the agent obtains zero payoff. The expected rent of the agent in period 3, given μ_3, is thus equal to:

$$E\left[\Pi_3 \mid \mu_3\right] = \Pr\left(V \leq \hat{V}\left(\mu_3\right)\right)\mu_3\left(\overline{C} - \underline{C}\right).$$

Note that $\hat{V}\left(\mu_3\right)$, and thus the probability of a high-price offer, decreases when μ_3 increases. This reflects a *ratchet effect*: the principal is more tempted to insist on a low price ($\underline{C}$ or lower) when the agent is likely to face a low cost; other things being equal, this tends to discourage the agent from investing in cost-reducing activities.

Investing however allows the agent to reduce his cost in the short-term, and raises the probability that future cost will be low as well; this effect is all the more important in the early periods, as the effect will last over a longer horizon. This leads to:

Proposition 8 *The incentives to invest in cost-reducing activities decrease as renewal date approaches.*

Proof: see the Appendix.

Renewal decisions and information decay have thus quite different impacts on quality and cost performance dynamics: incentives to invest in quality appear stronger towards the end of a contract, before renewal, whereas cost-cutting incentives appear to be more important at the beginning of a contract. This is because: (i) the agent benefits more directly from cost reductions than from quality improvements; and (ii) while the principal is more likely to offer a *high* price when she expects a good quality, she is on the contrary likely to insist on *low* price when she expects the agent to face a low cost. Thus, towards the end of a contract, the renewal effect provides an additional incentive to invest a quality but discourages instead cost-cutting activities.

4 Contract duration

We have seen so far that contract renewal can act as an incentive device to induce the agent to provide noncontractible quality. In this section we extend the analysis to consider how the length of the contract affects the incentives provided by renewal decisions. For this purpose we suppose that the principal-agent relationship with productive investment presented in the basic framework is infinitely repeated. In each period, the principal can either delegate the provision of the good or service to the agent, in which case her payoff is of the form $q_t - p_t$, or keep the provision in-house, in which case she obtains V_t, which is uniformly distributed over $\left[\underline{V}, \overline{V}\right]$. In each period t, with probability ρ the agent's initial quality remains the same as in the previous period, and if it is low the agent can upgrade it by investing c. For the sake of exposition, we assume that the principal observes the agent's quality, q_t, even when opting for in-house provision.[24] This simplifies the analysis by making the environment stationary. We will consider two settings, in which contracts last for either one or two periods, and characterize in each case the (stationary) equilibrium levels of investment.

4.1 One period contracts (T=1)

Suppose that contracts only last for one period. At each renewal date t, the principal forms beliefs as to the agent's type based on the agent's past performance. Focusing on productivity investment rather than on quality efforts makes the analysis more tractable, since the last performance observation provides a sufficient statistic for all prior performance. We will moreover focus on stationary Markov equilibria in which renewal decisions only depend on the agent's performance in the previous period. Finally, for simplicity we let $\alpha = 1/2$.

Let μ_t denote the probability that the principal assigns to the agent's quality being high in period t; this probability depends on the agent's type, which in turn is partly determined by the previous quality q_{t-1}, and on the agent's investment in case of low productivity. Since the agent's incentive to invest only depends on the renewal stage, which in turn is driven by current performance, the agent's investment decision does not depend on past performance. For example, upon observing $q_{t-1} = H$, the principal anticipates that the quality will be high with probability:

$$\mu_t = \overline{\mu}_1 \equiv \rho + (1 - \rho)\, i. \tag{10}$$

If instead $q_{t-1} = L$, then the principal anticipates that quality will be high only with probability:

$$\mu_t = \underline{\mu}_1 \equiv 1 - \rho + \rho i. \tag{11}$$

[24]This could for example be the case if the agent is involved in multiple relationships, to which the same productivity and investment patterns apply.

At renewal date t, upon observing the realized value of V_t, the principal has three options: she can either offer $p_t = \overline{C}$ or $p_t = \underline{C}$, or make no acceptable offer. Since her decision has no impact on subsequent performance and renewal stages (in particular, since the principal keeps observing the agent's quality anyway, her renewal decision does not affect future investment), the principal chooses the price p_t so as to maximize her expected payoff for the current period. Offering a high price yields an expected payoff equal to:

$$U_1\left(p_t = \overline{C}\right) = L + \mu_t \Delta - \overline{C},$$

whereas offering a low price yields:

$$U_1\left(p_t = \underline{C}\right) = \frac{L + \mu_t \Delta - \underline{C}}{2} + \frac{V_t}{2}.$$

The principal then offers a high price when $U_1\left(p_t = \overline{C}\right) > U_1\left(p_t = \underline{C}\right)$ (which as before implies $U_1\left(p_t = \overline{C}\right) > V_t$), which under Assumption 1 happens with probability μ_t.

The agent obtains a rent $B = \left(\overline{C} - \underline{C}\right)/2$ only when he is offered a high price. Therefore, his expected payoff is:

$$\Pi_1 = \begin{cases} \overline{\Pi}_1 = \overline{\mu}_1 B - (1-\rho)\, ic + \delta\left(\overline{\mu}_1 \overline{\Pi}_1 + (1-\overline{\mu}_1)\, \underline{\Pi}_1\right) & \text{if} \quad q_{t-1} = H, \\ \underline{\Pi}_1 = \underline{\mu}_1 B - \rho ic + \delta\left(\underline{\mu}_1 \overline{\Pi}_1 + \left(1-\underline{\mu}_1\right) \underline{\Pi}_1\right) & \text{if} \quad q_{t-1} = L. \end{cases}$$

This determines the equilibrium payoffs for the agent, as a function of his investment decision; in particular, we have:

$$\begin{aligned} \overline{\Pi}_1 - \underline{\Pi}_1 &= \left(\overline{\mu}_1 - \underline{\mu}_1\right) B + (2\rho - 1)\, ic + \delta\left(\overline{\mu}_1 - \underline{\mu}_1\right)\left(\overline{\Pi}_1 - \underline{\Pi}_1\right) \\ &= \frac{(2\rho-1)\left((1-i)\, B + ic\right)}{1 - \delta\,(2\rho-1)\,(1-i)}, \end{aligned} \tag{12}$$

Given these continuation equilibrium payoffs, a low type agent's investment decision maximizes:

$$-ic + \delta i \left(\overline{\Pi}_1 - \underline{\Pi}_1\right).$$

This yields:

Proposition 9 *With one-period contracting ($T = 1$), there is no equilibrium in which the low-productivity agent invests with probability* 1*:*

(i) If $\frac{B}{c} \leq \frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}$, the agent never invests;

(ii) If $\frac{B}{c} > \frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}$, the agent invests with probability $i_1^* \in (0,1)$, given by:

$$i_1^* = 1 - \frac{1-\delta\left(2\rho-1\right)}{\delta\left(2\rho-1\right)}\frac{c}{B}. \tag{13}$$

This equilibrium level of investment increases with the relative benefit (B/c), the discount factor (δ) and the degree of information persistence (ρ).

Proof: see the Appendix.

By showing good performance, the agent raises the incentive of the principal to make a high-price offer at renewal stage, which in turn raises the agent's expected rent. But, if the agent were to invest in each period with probability 1, then the expected quality would be independent of past performance, which in turn would nullify the agent's incentive to invest.[25] For this reason no pure strategy equilibrium exists in which the agent invests in each period with probability 1.

In equilibrium, there is however a positive probability of investment if the relative benefit B/c and the weight δ attached to the future are sufficiently important. In this equilibrium, the incentive effect of contract renewal is stronger when past performance provides a good indication about future performance, since this raises the principal's willingness to offer a high price upon observing good performance. The equilibrium value of investment thus decreases with information decay.

4.2 Two-period contracts (T=2)

Suppose now that each contract lasts for two periods: $T = 2$. At renewal date t, the principal looks again at past performance to form her expectation as to the quality that the agent will provide if the contract is renewed. As before, only quality q_{t-1} matters, however. This, in turn, implies that, as in the basic framework, the agent has no incentive to invest in the first execution period of a contract; he may however invest in the second period in order to increase the prospect of being offered a high price at the renewal stage.

Upon observing $q_{t-1} = H$, the principal assigns probability ρ to $\theta_t = H$. Since there will be no investment in period t, if the agent invests with probability i in $t+1$ the

[25] When $i = 1$, the expected rent $\overline{\Pi}_1 - \underline{\Pi}_1$ only comes from the reduction in the likelihood of having to invest in the future. But, as already noted, it is not worth investing for sure in a given period merely to reduce the future probability of investing.

expected qualities will then be $L + \rho\Delta$ in period t and $L + \overline{\mu}_2\Delta$ in period $t+1$, where:

$$\overline{\mu}_2 \equiv 1 - 2\rho(1-\rho)(1-i).$$

Since the principal and the agent are similarly uncertain about the agent's future cost, the relevant prices for two-period contracts are a high price $\overline{p} = \overline{C} + \delta C^e$ and a low price $\underline{p} = \underline{C} + \delta C^e$. When offering $\overline{p}$, the principal anticipates her expected payoff over the two periods to be equal to:

$$\overline{U}_2(p = \overline{p}) = L + \rho\Delta - \overline{C} + \delta(L + \overline{\mu}_2\Delta - C^e). \tag{14}$$

A low price $\underline{p}$ is accepted only with probability 1/2; the principal's expected payoff over the two periods is then equal to:

$$\overline{U}_2(p = \underline{p}) = \frac{L + \rho\Delta - \underline{C} + \delta(L + \overline{\mu}_2\Delta - C^e)}{2} + \frac{V_t + \delta\tilde{V}}{2},$$

where V_t denotes the value of the in-house option in period t and $V^e \equiv \left(\underline{V} + \overline{V}\right)/2$ represents the expected value of the in-house option in period $t+1$. It follows that the principal prefers to make a high-price offer if the realized value of the in-house option, V_t, is such that:

$$V_t \leq \underline{V} + \rho\Delta + \delta(L + \overline{\mu}_2\Delta - C^e - \underline{V} - \frac{\Delta}{2}).$$

Therefore, when $q_{t-1} = H$ the principal offers a high price with probability:

$$\overline{\eta} = \rho + \delta\overline{\mu}_2 + \delta\frac{L - C^e - \underline{V} - \frac{\Delta}{2}}{\overline{V} - \underline{V}}.$$

As in the case of one-period contract, the principal is more willing to offer a high price, the more she expects the agent to provide high quality ($\overline{\mu}_2$ high). The expected quality in turn depends on the agent's expected type and investment behavior. Compared with the case of one-period contracts, however, now the principal anticipates that the agent will not invest in the first period of the contract but only in the second one.

When instead $q_{t-1} = L$, the principal anticipates the expected quality to be $L + (1-\rho)\Delta$ in period t and $L + \underline{\mu}_2\Delta$ in period $t+1$, where:

$$\underline{\mu}_2 = 2\rho(1-\rho) + (1 - 2\rho(1-\rho))\, i = i + 2\rho(1-\rho)(1-i).$$

The probability of a high offer is therefore:

$$\underline{\eta} = 1 - \rho + \delta\underline{\mu}_2 + \delta\frac{L - C^e - \underline{V} - \frac{\Delta}{2}}{\overline{V} - \underline{V}}.$$

Consider the agent's incentive to invest in second period of a contract. Let $\overline{\Pi}_2$ and $\overline{\Pi}_2$ denote the continuation payoffs of the agent in period t given that $q_{t-1} = H$ and $q_{t-1} = L$ were observed, respectively. Following the same reasoning as for a one period contracts, we have:

$$\overline{\Pi}_2 = \overline{\eta}B - \delta 2\rho(1-\rho) ic + \delta^2 \left[\overline{\mu}_2 \overline{\Pi}_2 + (1-\overline{\mu}_2)\underline{\Pi}_2\right], \tag{15}$$

$$\underline{\Pi}_2 = \underline{\eta}B - \delta\left[1 - 2\rho(1-\rho)\right] ic + \delta^2 \left[\underline{\mu}_2 \overline{\Pi}_2 + (1-\underline{\mu}_2)\underline{\Pi}_2\right], \tag{16}$$

and thus:

$$\begin{aligned}\overline{\Pi}_2 - \underline{\Pi}_2 &= \left(\overline{\eta} - \underline{\eta}\right) B + \delta(2\rho-1)^2 ic + \delta^2\left(\overline{\mu}_2 - \underline{\mu}_2\right)\left(\overline{\Pi}_2 - \underline{\Pi}_2\right) \\ &= \frac{(2\rho-1)\left(B + \delta(2\rho-1)\left((1-i)B + ic\right)\right)}{1 - \delta^2(2\rho-1)^2(1-i)}.\end{aligned} \tag{17}$$

As for one period contracts, good performance in period $t-1$ bring three benefits to the agent at renewal stage: it increases the probability of receiving a high price offer by $\left(\overline{\eta} - \underline{\eta}\right)$, reduces the probability to have to invest in the second period of the next contract by $(2\rho-1)^2$ and it raises by $\left(\overline{\mu}_2 - \underline{\mu}_2\right)$ the probability of enjoying $\overline{\Pi}_2$ rather than $\underline{\Pi}_2$ in the next renewal process. As in the case of one-period contracts, the agent never invests systematically with probability 1. He actually never invests in the first period of a contract; he may however invest with probability 1 in the second period of a contract, since the performance in that period affects the principal's belief for the following period and thus the likelihood of a high-price offer.

As before, in the period preceding a renewal stage, the agent will decide whether to invest so as to maximize:

$$-ic + \delta i\left(\overline{\Pi}_2 - \underline{\Pi}_2\right).$$

This leads to:

Proposition 10 *With two-period contracting ($T = 2$), the agent never invests during the first period of a contract; in the second period of a contract:*

(i) If $\frac{B}{c} \leq \frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}$, then the agent also never invests;

(ii) If $\frac{B}{c} \geq \frac{1-\delta^2(2\rho-1)^2}{\delta(2\rho-1)}$, then the agent invests with probability 1;

(iii) If $\frac{1-\delta(2\rho-1)}{\delta(2\rho-1)} < \frac{B}{c} < \frac{1-\delta^2(2\rho-1)^2}{\delta(2\rho-1)}$, then the agent invests with probability $i_2^* \in (0,1)$, given by:

$$i_2^* = \frac{1+\delta(2\rho-1)}{\delta(2\rho-1)} - \frac{1-\delta^2(2\rho-1)^2}{\delta^2(2\rho-1)^2}\frac{c}{B}. \tag{18}$$

This equilibrium level of investment increases with the relative benefit (B/c), the discount factor (δ) and the degree of information persistence (ρ).

Proof: see the Appendix.

As in the basic model, due to information decay the agent does not invest in the first period of a contract. Instead, when the relative benefit of investment is sufficiently high, the effect of past performance on the principal's renewal decision induces a low-productivity agent to invest in the period before renewal. As with one-period contracts, investment incentives increase with the weight δ attached to the future and with the persistence of information, ρ, which enhances the effect of investment on future expected quality.

4.3 Optimal contract duration

A natural question is whether incentives to invest are overall higher under two-period or one-period contracting. Note first that, in both regimes: (i) the agent never invests when $\frac{B}{c} \leq \frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}$; and (ii) the agent invests with positive probability when $\frac{B}{c} > \frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}$. Therefore, for the sake of exposition, we will focus here on the case where $\frac{B}{c} > \frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}$.

Compared with one-period contracting, two-period contracting generates less investment in quality in the first period of the contracts, but more investment in their second period:

Proposition 11 *With two-period contracting, investment in the second period of a contract is at least as large as under one-period contracting: $i_2^* \geq i_1^*$.*

Proof: see the Appendix.

To see why this is the case, let us compare the stakes in continuation values, which under two-period contracting can be expressed as

$$\begin{aligned}
\overline{\Pi}_2 - \underline{\Pi}_2 &= \left(\overline{\eta} - \underline{\eta}\right) B + \delta \left(2\rho - 1\right)^2 ic + \delta^2 \left(\overline{\mu}_2 - \underline{\mu}_2\right) \left(\overline{\Pi}_2 - \underline{\Pi}_2\right) \\
&= \left(2\rho - 1\right) \left(1 + \delta \left(2\rho - 1\right) \left(1 - i\right)\right) B \\
&\quad + \delta \left(2\rho - 1\right)^2 ic \\
&\quad + \delta^2 \left(2\rho - 1\right)^2 \left(1 - i\right) \left(\overline{\Pi}_2 - \underline{\Pi}_2\right),
\end{aligned}$$

and under one-period contracting can be expressed as (decomposing it over two periods, for comparison purposes):

$$\begin{aligned}\overline{\Pi}_1-\underline{\Pi}_1 &= \left(\overline{\mu}_1-\underline{\mu}_1\right)B+(2\rho-1)\,i+\delta\left(\overline{\mu}_1-\underline{\mu}_1\right)\left(\left(\overline{\mu}_1-\underline{\mu}_1\right)B+(2\rho-1)\,i\right)\\ &\quad+\delta^2\left(\overline{\mu}_1-\underline{\mu}_1\right)^2\left(\overline{\Pi}_1-\underline{\Pi}_1\right),\\ &= (2\rho-1)\,(1-i)\,(1+\delta\,(2\rho-1)\,(1-i))\,B\\ &\quad+(2\rho-1)\,(1+\delta\,(2\rho-1)\,(1-i))\,ic\\ &\quad+\delta^2\,(2\rho-1)^2\,(1-i)^2\left(\overline{\Pi}_1-\underline{\Pi}_1\right).\end{aligned}$$

These stakes involve three components. First, a good reputation has a greater impact on the probability of a high price offer under $T=2$ than under $T=1$, due to "crowding out" in the latter case: the principal anticipates that no investment will take place in the first period of the following contract when $T=2$, whilst some investment will take place when $T=1$. Observing high quality is therefore less valuable when $T=1$ than when $T=2$, which is reflected by an additional discount factor $(1-i)$.

The second component refers to the saving in future investment cost; this effect is lower under $T=2$ than under $T=1$, since there is no investment in the following period under $T=2$.

The last component refers to the impact of reputation on future contract negotiations, and it is again reduced under $T=1$ by a discount factor $(1-i)$ due to crowding out. When $i=1$, there is no crowding out and no cost saving. Therefore the benefit from investing coincides in both contracting environments. This explains in particular why the threshold level for positive investment coincides in both regimes. In contracts, it is easily checked that the first effect (resulting from crowding out) already dominates the second effect (cost-saving) whenever the agent invests with positive probability,[26] which implies that the incentive to invest in each period when $T=1$ is lower than the incentive to invest in the second period of a contract when $T=2$.

This leads to:

Proposition 12 *There exists a threshold on the relative benefit B/c, such that two-period contracting generates a greater average quality when the relative benefit lies below this*

[26] The overall impact of the two effects in $\left(\overline{\Pi}_2-\underline{\Pi}_2\right)-\left(\overline{\Pi}_1-\underline{\Pi}_1\right)$ is equal to $(2\rho-1)\left[(1+\delta\,(2\rho-1))\,B-c-i\delta\,(2\rho-1)\right]$, which is positive for any $i\le 1$ when $\frac{B}{c}>1-\delta\,(2\rho-1)$ and investment levels are nonzero only when $\frac{B}{c}>\frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}>1-\delta\,(2\rho-1)$.

threshold, and a lower average quality otherwise. This threshold decreases with information persistence (ρ) *and with the discount factor* (δ).

Proof: see the Appendix.

When the relative benefit B/c is small, investment levels are small under both under one-period and two-period contracting. Two-period contracting, which provides a greater incentive to invest in the period before contract renewal, then tends to generate a greater average quality. When instead the relative benefit is larger, average quality is better under one-period contracting, which induces the agent to invest in all periods rather than in every other period. A greater degree of information persistence (ρ) and a greater discount factor (δ) both enhance investment incentives, and thus strengthen the relative gain of one-period contracting.

5 Conclusion

Underperformance in principal-agent relationships may be a serious problem when verifiability issues impede explit contractual provisions and a limited horizon moreover reduces the scope for implicit contract incentives. However, when quality is (at least partly) the result of the agent's ability, the agent may wish to build a reputation by working harder or by investing to enhance his type; adverse selection and career concerns may then ease the moral hazard problem. We build on this implication to study the performance dynamics that career concerns generate in volatile environments and to derive the implications for contract duration.

The incentive power of career concerns has then been shown to depend on three main factors: (i) the rent that the agent can secure from having the contract renewed; (ii) the information decay generated by the volatility of the environment; and (iii) the duration of the contract. Modelling explicitly the contract renewal decision moreover allows us to relate the expected rent from renewal to past performance, rather than assuming it exogenously as in standard career concerns models, which brings several insights.

First, information decay creates higher incentives for quality provision towards the end of the contract, where performance provides better information about the agent's future ability. As a result, performance improves as the contract expiry date approaches, contrary to the classic career concerns' insight. This incentive effect moreover differs for short-term performance enhancement and for long-term productivity investment, the former being more likely towards the end of the contract, and the latter earlier on.

Second, performance dynamics depend critically on whether performance relates to cost or quality. The same desire to secure a greater rent encourages the agent to deliver quality before renewal, but generates instead a ratchet effect that worsen cost efficiency towards the end of a contract. Finally, our analysis shows that longer contracts may generate greater implicit incentives, opposite to what the current wisdom suggests.

Our results also highlight the importance of granting some discretion to public authorities involved in the selection of contractors for the provision of public services.[27] First, discretion gives the principal the possibility to use past performance to make inference as to the agent's productivity and thus to improve her own choice of whether to take provision in house or contract it out. Second, by making past performance relevant to future contract opportunities, this discretion induces the agent to invest in nonverifiable dimensions. Granting discretion to public authorities is thus particularly important for all public services such as educational services, clinical services and nursing homes, which involve many noncontractible dimensions.[28]

Throughout the paper we have restricted our attention to a single principal-agent relationship. It would be interesting to extend the analysis to allow for the possibility that alternative providers are available at renewal stage. At first sight, given the limited commitment ability of the principal and the incentive properties of the expected rent, our results suggest that restricting participation may facilitate investment (as in Calzolari and Spagnolo, 2009). An in-depth analysis of the effect of potential competition on the incentive power of contract renewal would however constitute an interesting scope for future research.

Finally, while several empirical studies support our predictions (see the introduction), it is clear that more empirical research on the performance dynamics within contracting periods are needed to understand the incentives provided by career concerns in volatile environments.

[27] See Kelman (1990) for an in-depth discussion of granting discretion in public procurement.

[28] An interesting example in this direction is provided by Sweden. During the so-called "light-handed regulation" regime which prevailed until 2003, the Swedish energy regulator was given the discretion to limit the length of local distribution concessions (set otherwise at 25 years) in case of poor performance (it could for example say "I renew for only x years" at the time of the renewal, but also revoke an existing concession before the end of its term).

6 Appendix

Proof of Proposition 2.

Consider first the case $c \leq c^*$. The agent then invests in period 2 whenever $\theta_2 = L$, and then obtains an expected payoff equal to $-c + \rho\delta B$ (when $c = c^*$, the agent is indifferent between investing or not, and either way obtains again $-c+\rho\delta B$). When instead $\theta_2 = H$, the agent obtains an expected payoff equal to $\rho\delta B$; that is, a high productivity has no impact on the future rent but allows the agent to save the investment cost c. Therefore, investing in period 1 when $\theta_1 = L$ is not profitable, since it would cost c only reduce the expected cost of having to invest again in period 2, from $\delta\rho c$ to $\delta(1-\rho)c$ (thus saving $\delta(2\rho-1)c < c$)

When instead $c > c^*$, the agent does not invest in period 2 when $\theta_2 = L$. Therefore, investing in period 1 yields

$$-c + \delta\left[\rho\left(\rho\delta B\right) + (1-\rho)(1-\rho)\delta B\right],$$

whereas in the absence of investment the agent's expected payoff is equal to

$$\delta\left[(1-\rho)\rho\delta B + \rho(1-\rho)\delta B\right].$$

The agent will thus choose again not to invest since the resulting increase in expected rent, $\delta^2(2\rho-1)^2 B$, is even lower than in the second period and thus does not compensate the cost: $\delta^2(2\rho-1)^2 B < \delta(2\rho-1)B = c^* < c$. ■

Proof of Corollary 2. If a low-productivity agent invests in period 2, the expected welfare is $L + \Delta - C^e + \delta E\left[W_3; \overline{q}_3^e\right]$, where $E\left[W_3; q_3^e\right]$ is given by (2); if instead the agent does not invest, the expected welfare is $L - C^e + \delta\left[W_3; \underline{q}_3^e\right]$. Thus, investing in period 2 not only increases current quality by Δ but, by enhancing expected quality in period 3, raises the threshold $\hat{V}$ for a high-price offer $p_3 = \overline{C}$, from $\hat{V}_0 = \underline{V} + (1-\rho)\Delta$ to $\hat{V}_1 = \underline{V} + \rho\Delta$. Therefore, in period 3:

- the agent's expected rent increases by $(2\rho-1)B$, since the principal switches from a low-price to a high-price offer when V lies in the range $\left[\hat{V}_0, \hat{V}_1\right]$.
- the principal's expected payoff also strictly increases, since she faces (weakly) better

options,[29] and moreover opts for different offers in part of the range (e.g., she offers a high price rather than a low one in the range $\left[\hat{V}_0, \hat{V}_1\right]$; she may also switch from no offer to offering a low price).

Letting $\hat{U}$ denote this increase in the principal's expected payoff, the social value of investing in period 2 is thus $-c + \Delta + \delta\left[(2\rho - 1)B + \hat{U}\right]$. ■

Proof of Proposition 3. Suppose that the investment is desirable, but the agent would not invest in the absence of commitment: $c > \delta(2\rho - 1)B$. The principal can then enhance investment incentives by raising the level of her high-price offer $\overline{p}$ above $\overline{C}$. The agent's expected rent is equal to:

$$E\left[\Pi_3; q_3^e, \overline{p}\right] \equiv \int_{\underline{V}}^{\max\left\{\hat{V}\left(q_3^e,\overline{p}\right),\underline{V}\right\}} \left(\overline{p} - C^e\right)\frac{dV}{\Delta},$$

where

$$\hat{V}\left(q_3^e, \overline{p}\right) \equiv q_3^e - \underline{C} - \frac{\overline{p} - \underline{C}}{1 - \alpha}.$$

Investing thus increases this expected rent by:

$$\Delta\Pi_3\left(\overline{p}\right) \equiv \int_{\max\left\{\hat{V}\left(\underline{q}_3^e,\overline{p}\right),\underline{V}\right\}}^{\max\left\{\hat{V}\left(\overline{q}_3^e,\overline{p}\right),\underline{V}\right\}} \left(\overline{p} - C^e\right)\frac{dV}{\Delta}.$$

This expression is continuous in $\overline{p}$; it first increases with $\overline{p}$ as long as $\hat{V}\left(\underline{q}_3^e, \overline{p}\right) \geq \underline{V}$ (the derivative is constant and equal to $2\rho - 1$ in this range), is equal to zero when $\hat{V}\left(\overline{q}_3^e, \overline{p}\right) \leq \underline{V}$, and is concave in $\overline{p}$ in the intermediate range where $\hat{V}\left(\underline{q}_3^e, \overline{p}\right) < \underline{V} < \hat{V}\left(\overline{q}_3^e, \overline{p}\right)$. It thus reaches a maximum value for some price $\hat{p}$ such that $\hat{V}\left(\overline{q}_3^e, \overline{p}\right) > \underline{V}$, and as long as $c \leq \hat{c} \equiv \delta\Delta\Pi_3\left(\hat{p}\right)$, there exists a price $\overline{p} > \overline{C}$ which induces the agent to invest. Adopting such a price however distorts the principal's renewal decision and thus reduces expected welfare, which (using $\hat{V}\left(\overline{q}_3^e, \overline{p}\right) > \underline{V}$) is equal to:

$$E\left[W_3; \overline{p}\right] = \int_{\underline{V}}^{\hat{V}\left(\overline{q}_3^e,\overline{p}\right)} \left(\overline{q}_3^e - C^e\right)\frac{dV}{\Delta} + \int_{\hat{V}\left(\overline{q}_3^e,\overline{p}\right)}^{\min\left\{\tilde{V}\left(\overline{q}_3^e\right),\overline{V}\right\}} \left[\alpha\left(\overline{q}_3^e - \underline{C}\right) + (1 - \alpha)V\right]\frac{dV}{\Delta} + \int_{\min\left\{\tilde{V}\left(\overline{q}_3^e\right),\overline{V}\right\}}^{\overline{V}} V\frac{dV}{\Delta},$$

[29] More precisely, investing strictly increases the value of the high-price and low-price options and has no effect on the value of in-house provision.

with

$$\frac{\partial E\left[W_3,\overline{p}\right]}{\partial \overline{p}}=-\frac{\left(\overline{q}_3^e-\overline{C}\right)-\hat{V}\left(\overline{q}_3^e,\overline{p}\right)}{\Delta}=-\frac{\overline{p}-\overline{C}+\alpha\left(\overline{C}-\underline{C}\right)}{\left(1-\alpha\right)\Delta}<0.$$

Therefore, if the principal chooses to provide additional investment incentives, she will opt for the lowest price $\overline{p}$ satisfying $\delta\Delta\Pi_3\left(\overline{p}\right)\geq c$. Moreover, if $c^s\leq\hat{c}$, it cannot be profitable for the principal to provide these investment incentives, since the cost c^s is already barely compensated by the increase in expected welfare even without the distortion in the principal's renewal decision. Conversely, when c is close to c^*, it is both feasible (since $\hat{V}\left(\overline{q}_3^e,\overline{C}\right)>\underline{V}$, implying that increasing $\overline{p}$ slightly above $\overline{C}$ would indeed increase $\delta\Delta\Pi_3$ and thus compensate the cost c) and desirable (since $c^*<c^s$). Therefore, there exist $\hat{c}^s$, satisfying $\hat{c}^s>c^*$, $\hat{c}^s<c^s$ and $\hat{c}^s\leq\hat{c}$, such that the principal finds it profitable to provide investment incentives if and only if $c<\hat{c}^s$. ■

Proof of Proposition 4.

Let $\sigma\equiv\delta\left(2\rho-1\right)$ and $\beta\equiv B/k$. Using

$$\underline{\Pi}_2=\left(1-\rho\right)\delta\beta+\sigma\beta i_2^*-\frac{\left(i_2^*\right)^2}{2}=\left(1-\rho\right)\delta\beta,$$

maximizing (4) amounts to:

$$\max_{0\leq i_1\leq 1}\sigma\left(\overline{\Pi}_2-\underline{\Pi}_2\right)i_1-\frac{i_1^2}{2}=\sigma\left(\left(1-i_2^*\right)\sigma\beta+\frac{\left(i_2^*\right)^2}{2}\right)i_1-\frac{i_1^2}{2},$$

which leads to:

$$i_1^*=\sigma\left(\left(1-i_2^*\right)\sigma\beta+\frac{\left(i_2^*\right)^2}{2}\right).$$

Since $i_2^*=1$ when $\sigma\beta>1$ and $i_2^*=\sigma\beta$ otherwise, this expression can in both cases be rewritten as:

$$i_1^*=\left(1-\frac{i_2^*}{2}\right)\sigma i_2^*. \tag{19}$$

Therefore, $i_1^*<i_2^*$ (in particular, i_1^* is always lower than 1) and i_1^* increases with σ and β directly (for σ) and indirectly through an increase in i_2^*.[30] Finally, the ratio

$$r\equiv\frac{i_1^*}{i_2^*}=\left(1-\frac{i_2^*}{2}\right)\sigma,$$

[30] Indeed, from (19) we have $\partial i_1^*/\partial i_2^*=\sigma\left(1-i_2^*\right)>0$, and i_2^* (weakly) increases with σ and β.

which characterizes the performance profile over time decreases (through the impact on i_2^*) as β increases; that is, an increase in the relative benefit of the investment has relatively more important impact on productivity and performance towards the end of the contracting period, as the investment expected to be undertaken in the second periods crowds out investment in the first period. By contrast, an increase in σ implies that the investment has a more lasting impact on productivity, which encourages investment in early periods. As a result, despite a similar crowding-out effect through an increase in i_2^*, an increase in σ raises i_1^*/i_2^*: this is obvious when i_2^* is already maximal (i.e., when $\sigma\beta > 1$, and thus $i_2^* = 1$) and, when $i_2^* = \sigma\beta < 1$:

$$\frac{dr}{d\sigma} = \frac{\partial r}{\partial \sigma} + \frac{\partial r}{\partial i_2^*}\frac{\partial i_2^*}{\partial \sigma} = 1 - \frac{i_2^*}{2} - \frac{\sigma}{2}\beta = 1 - i_2^* > 0.$$

Finally, $q_1^e < q_2^e$ since:

$$q_1^e = L + \Pr(q_1 = H) = L + \frac{1+i_1^*}{2}\Delta < L + \frac{1+i_2^*}{2}\Delta,$$

and:

$$\begin{aligned} q_2^e &= L + \Pr(q_2 = H) \\ &= L + \left\{\frac{1+i_1^*}{2}\left[\rho + (1-\rho)\, i_2^*\right] + \frac{1-i_1^*}{2}\left[(1-\rho) + \rho i_2^*\right]\right\}\Delta \\ &= L + \left\{\frac{1+i_2^*}{2} + \frac{i_1^*}{2}(2\rho - 1)(1 - i_2^*)\right\}\Delta > L + \frac{1+i_2^*}{2}\Delta. \end{aligned}$$

■

Proof of Proposition 5. Consider first period 2. When $q_1 = H$, the agent exerts effort if:

$$\gamma \leq G_H \equiv (2\rho - 1)\,\nu_H \delta B, \tag{20}$$

where in equilibrium:

$$\nu_H = \frac{\rho + (1-\rho)\, e_1}{\rho + (1-\rho)\, e_1 + (1 - \rho + \rho e_1)\, e_H},$$

which decreases from 1 to $\frac{\rho+(1-\rho)e_1}{1+e_1}$ as e_H increases from 0 to 1; therefore:

$$\begin{aligned} e_H &= 0 \text{ if } \gamma \geq \delta (2\rho - 1) B, && (21) \\ e_H &= 1 \text{ if } \gamma \leq \frac{\rho + (1-\rho)\, e_1}{1 + e_1}\delta (2\rho - 1) B, && (22) \\ e_H &= \hat{e}_H \equiv \frac{\rho + (1-\rho)\, e_1}{1 - \rho + \rho e_1}\frac{\delta (2\rho - 1) B - \gamma}{\gamma} \text{ otherwise.} && (23) \end{aligned}$$

When instead $q_1 = L$, the agent exerts effort if:

$$\gamma \leq G_L \equiv (2\rho - 1)\,\nu_L \delta B, \tag{24}$$

where in equilibrium:

$$\nu_L = \frac{1-\rho}{1-\rho+\rho e_L},$$

which decreases from 1 to $1-\rho$ as e_L increases from 0 to 1; therefore:

$$\begin{aligned} e_L &= 0 \text{ if } \gamma \geq \delta\,(2\rho-1)\,B; & (25)\\ e_L &= 1 \text{ if } \gamma \leq (1-\rho)\,\delta\,(2\rho-1)\,B & (26)\\ e_L &= \hat{e}_L \equiv \frac{1-\rho}{\rho}\frac{\delta\,(2\rho-1)\,B-\gamma}{\gamma} \text{ otherwise.} & (27)\end{aligned}$$

We thus have $e_H \geq e_L$: since $\frac{\rho+(1-\rho)e_1}{1+e_1}$ lies between $1/2$ and ρ and thus exceeds $1-\rho$, $e_H = 1$ whenever $e_L = 1$; and in the range where $0 < e_L = \hat{e}_L < 1$, e_H is equal to either 1 or $\hat{e}_H > \hat{e}_L$.[31]

Consider now period 1. From (8), the agent invests if:

$$\gamma \leq G_1 \equiv \frac{(1-\rho+\rho e_H)\,\nu_H - (1-\rho+\rho e_L)\,\nu_L}{1+\rho\,(e_H-e_L)\,\delta}\delta^2\,(2\rho-1)\,B.$$

Since $G_1 < G_H$,[32] it follows that $e_H \geq e_1$, with a strict inequality whenever either effort lies between 0 and 1. Since $e_H < 1$ implies $e_1 = 0$, building on the above analysis we have:

- when $\frac{\gamma}{\delta(2\rho-1)B} \geq 1$, $e_H = e_L = e_1 = 0$;
- when $1 > \frac{\gamma}{\delta(2\rho-1)B} > \left[\frac{\rho+(1-\rho)e_1}{1+e_1}\right]_{e_1=0} = \rho$, $1 > e_H = \hat{e}_H > e_L = \hat{e}_L > e_1 = 0$.

Similarly, since $e_1 > 0$ implies $e_H = 1$, we have:

- $e_H = e_L = 1$ when $\frac{\gamma}{\delta(2\rho-1)B} \leq 1-\rho$, in which case

$$\nu_H = \frac{\rho+(1-\rho)\,e_1}{1+e_1}, \nu_L = 1-\rho,$$

[31] It suffices to note that $\frac{\rho+(1-\rho)e_1}{1-\rho+\rho e_1}$ decreases with e_1 and always exceeds $\frac{1-\rho}{\rho}$.

[32] Since $\nu_L > 0$,

$$\frac{G_1}{G_H} = \frac{(1-\rho+\rho e_H)\,\nu_H - (1-\rho+\rho e_L)\,\nu_L}{(1+\rho\,(e_H-e_L)\,\delta)\,\nu_H} < \frac{1-\rho\,(1-e_H)}{1+\rho\,(e_H-e_L)\,\delta} \leq 1,$$

where the last inequality stems from $e_H \geq e_L$.

and thus

$$G_1 = \frac{(2\rho - 1)^2 \delta^2 B}{1 + e_1},$$

which deceases from $(2\rho - 1)^2 \delta^2 B$ to $(2\rho - 1)^2 \delta^2 B/2$ as e_1 increases from 0 to 1. Therefore:

$$\begin{aligned} e_1 &= 0 \text{ if } \gamma \geq (2\rho - 1)^2 \delta^2 B, \\ e_1 &= 1 \text{ if } \gamma < \frac{(2\rho - 1)^2 \delta^2 B}{2}, \\ e_1 &= \hat{e}_1 \equiv \hat{e}_1 = \frac{(2\rho - 1)^2 \delta^2 B}{\gamma} - 1 \text{ otherwise.} \end{aligned}$$

- $e_H = 1 > e_L = \hat{e}_L > 0$ when $\frac{\rho + (1-\rho) e_1}{1 + e_1} \geq \frac{\gamma}{\delta (2\rho - 1) B} > 1 - \rho$, in which case

$$\nu_H = \frac{\rho + (1 - \rho) e_1}{1 + e_1}, \nu_L = \frac{1 - \rho}{1 - \rho + \rho \hat{e}_L},$$

and thus

$$G_1 = \frac{1}{1 + e_1} \frac{(2\rho - 1)^2 \delta^2 B}{1 + \rho (1 - \hat{e}_L) \delta},$$

which decreases again with e_1. Therefore:

$$\begin{aligned} e_1 &= 0 \text{ if } \gamma \geq \frac{(2\rho - 1)^2 \delta^2 B}{1 + \rho (1 - \hat{e}_L) \delta}, \\ e_1 &= 1 \text{ if } \gamma < \frac{1}{2} \frac{(2\rho - 1)^2 \delta^2 B}{1 + \rho (1 - \hat{e}_L) \delta}, \\ e_1 &= \hat{e}_1 \text{ otherwise.} \end{aligned}$$

So, to summarize:

- If $\gamma \leq (1 - \rho) \delta (2\rho - 1) B$, the equilibrium $\{e_1^*, e_H^*, e_L^*\}$ is:

$$\begin{array}{lll} \{1, 1, 1\} & \text{if} & \gamma < \frac{(2\rho-1)^2 \delta^2 B}{2}, \\ \{\hat{e}_1, 1, 1\} & \text{if} & \gamma \in \left[\frac{(2\rho-1)^2 \delta^2 B}{2}, (2\rho - 1)^2 \delta^2 B \right], \\ \{0, 1, 1\} & \text{if} & \gamma > (2\rho - 1)^2 \delta^2 B. \end{array}$$

- If $(1 - \rho) \delta (2\rho - 1) B < \gamma < \rho \delta (2\rho - 1) B$, the equilibrium is:

$$\begin{array}{lll} \{1, 1, \hat{e}_L\} & \text{if} & \gamma < \frac{1}{2} \frac{(2\rho-1)^2 \delta^2 B}{1+\rho(1-\hat{e}_L)\delta}, \\ \{\hat{e}_1, 1, \hat{e}_L\} & \text{if} & \gamma \in \left[\frac{1}{2} \frac{(2\rho-1)^2 \delta^2 B}{1+\rho(1-\hat{e}_L)\delta}, \frac{(2\rho-1)^2 \delta^2 B}{1+\rho(1-\hat{e}_L)\delta} \right], \\ \{0, 1, \hat{e}_L\} & \text{if} & \gamma > \frac{(2\rho-1)^2 \delta^2 B}{1+\rho(1-\hat{e}_L)\delta}. \end{array}$$

- If $\rho\delta(2\rho-1)B < \gamma < \rho\delta(2\rho-1)B$, the equilibrium is $\{0, \hat{e}_H, \hat{e}_L\}$.

- If $\gamma \geq \delta(2\rho-1)B$, the equilibrium is $\{0,0,0\}$.

The comparative statics with respect to B/γ and δ follow by inspection. Finally, the expected qualities in periods 1 and 2 are respectively:

$$
\begin{aligned}
q_1^e &= L + \frac{1}{2}(1+e_1)\Delta, && (28)\\
q_2^e &= L + \frac{1}{2}[\rho + (1-\rho)e_H]\Delta + \frac{1}{2}[e_1(1-\rho+\rho e_H) + (1-e_1)(1-\rho+\rho e_L)]\Delta, && (29)
\end{aligned}
$$

which, after simplification, yields

$$
q_2^e - q_1^e = [e_H - e_1 - \rho(e_H - e_L)(1-e_1)]\frac{\Delta}{2}. \tag{30}
$$

When $e_1 > 0$, $e_H = 1$ and thus $q_2^e - q_1^e = (1-e_1)(1-\rho(1-e_L))\Delta/2 \geq 0$; when instead $e_1 = 0$, then $q_2^e - q_1^e = ((1-\rho)e_H + \rho e_L)\Delta/2 \geq 0$, with a strict inequality whenever $e_L > 0$ or $e_H > 0$. ■

Proof of Proposition 6.

In period 2, the agent never invests since exerting effort is less costly and has the same effect on observed quality, and thus on the expected rent from renewal. Consider now period 1, taking into account that in period 2 the agent may exert effort but not invest. The difference between the expected payoffs from investing (given by (9)) and from exerting effort (given by (7)) is given by

$$
(1-e_H)\nu_H(2\rho-1)^2\delta^2 B + (2\rho-1)e_H\delta\gamma - (c-\gamma),
$$

which is positive for $c \to \gamma$. Thus, in period 1, the agent has more incentive to invest than to exert effort, and is indeed willing to invest whenever:

$$
c \leq [(\rho + (1-\rho)e_H)\nu_H - (1-\rho+\rho e_L)\nu_L]\delta^2(2\rho-1)B + [\rho e_L - (1-\rho)e_H]\delta\gamma. \tag{31}
$$

For example, when γ is small (namely, $\gamma < (2\rho-1)(1-\rho)\delta B$), the agent always exerts effort in period 2 ($e_H = e_L = 1$); in period 1, investment is then more profitable than effort when it reduces the overall cost of delivering high quality ($c < \gamma + (2\rho-1)\gamma$) and is indeed profitable when $c < \delta(2\rho-1)\gamma + \delta^2(2\rho-1)^2 B$. ■

Proof of Proposition 7. If in equilibrium there is no investment in period 1, we are back to the situation analyzed in Section 3.3, where indeed performance increases over time. Consider now an equilibrium in which, when facing a low productivity in period 1 ($\theta_1 = L$), the agent invests (and thus delivers $q_1 = H$); we show that this implies $e_H = 1$.

Since by assumption the agent has invested, q_1 reveals his type in period 1; it is straightforward to show that the agent's effort decisions in period 2 must be such that:

- $e_H = 0$ if $\gamma \geq \delta (2\rho - 1) B$, in which case $e_L = 0$,
- $0 < e_H = \hat{e}_H < 1$ if $\delta (2\rho - 1) B > \gamma > \rho\delta (2\rho - 1) B$, in which case $0 < e_L = \hat{e}_L < 1$,
- and $e_H = 1$ whenever $\gamma \leq \delta (2\rho - 1) B$.

Therefore:

- If $e_H = 0$, then $e_L = 0$ and, from (31), the agent is willing to invest in period 1 if
$$c \leq (\rho\nu_H - (1 - \rho) \nu_L) \delta^2 (2\rho - 1) B.$$
Since $e_H < 1$ implies $\gamma \geq \delta (2\rho - 1) \nu_H B$ (see (5)) and $c \geq \gamma$, we have a contradiction.
- If instead $e_H = \hat{e}_H$, then $e_L = \hat{e}_L$; the agent is therefore indifferent between $e_H = \hat{e}_H$ and $e_H = 0$, and between $e_L = \hat{e}_L$ and $e_L = 0$. Thus the agent's expected payoffs are the same as before, and the same reasoning as for $e_H = 0$ applies.

■

Proof of Proposition 8. Let

$$R \equiv E [\Pi_3 \mid \mu_3 = \rho] - E [\Pi_3 \mid \mu_3 = 1 - \rho]$$

denote the overall value of reputation. When facing a high cost in period 2, the agent will invest if:

$$c < c_2^* \equiv \overline{C} - \underline{C} + \delta R.$$

Consider now the incentives to invest in period 1. If $c > c_2^*$, the agent never invests in period 2; therefore, by not investing in period 1, a high-cost agent obtains an expected payoff equal to:

$$-\overline{C} + \delta \left\{ \rho \left[-\overline{C} + \delta E [\Pi_3 \mid \mu_3 = 1 - \rho] \right] + (1 - \rho) \left[-\underline{C} + \delta E [\Pi_3 \mid \mu_3 = \rho] \right] \right\}.$$

By investing in period 1, the agent obtains instead:

$$-c-\underline{C}+\delta\left\{(1-\rho)\left[-\overline{C}+\delta E\left[\Pi_3 \mid \mu_3=1-\rho\right]\right]+\rho\left[-\underline{C}+\delta E\left[\Pi_3 \mid \mu_3=\rho\right]\right]\right\}.$$

Therefore, the agent invests in period 1 if:

$$c<\overline{c}_1^* \equiv \overline{C}-\underline{C}+\delta(2\rho-1)\left(\overline{C}-\underline{C}+\delta R\right).$$

The conclusion then follows from $\overline{c}_1^* > c_2^*$, which amounts to:

$$\begin{aligned}\overline{C}-\underline{C}+\delta(2\rho-1)\left(\overline{C}-\underline{C}+\delta R\right) &> \overline{C}-\underline{C}+\delta R \\ \iff & \overline{C}-\underline{C}>(1-\delta(2\rho-1))\left(\overline{C}-\underline{C}+\delta R\right) \quad (32)\end{aligned}$$

Note that

$$\begin{aligned} R &= \Pr\left(V \leq \hat{V}(\rho)\right)\rho\left(\overline{C}-\underline{C}\right)-\Pr\left(V \leq \hat{V}(1-\rho)\right)(1-\rho)\left(\overline{C}-\underline{C}\right) \\ &< \Pr\left(V \leq \hat{V}(\rho)\right)\rho\left(\overline{C}-\underline{C}\right)-\Pr\left(V \leq \hat{V}(\rho)\right)(1-\rho)\left(\overline{C}-\underline{C}\right) \\ &= (2\rho-1)\Pr\left(V \leq \hat{V}(\rho)\right)\left(\overline{C}-\underline{C}\right) \\ &< (2\rho-1)\left(\overline{C}-\underline{C}\right). \end{aligned}$$

Therefore, the right-hand side in (32) is lower than:

$$(1-\delta(2\rho-1))(1+\delta(2\rho-1))\left(\overline{C}-\underline{C}\right)=\left(1-\delta^2(2\rho-1)^2\right)\left(\overline{C}-\underline{C}\right),$$

and is thus indeed lower than the left-hand side $\left(\overline{C}-\underline{C}\right)$.

If instead $c<c_2^*$, the agent invests in period 2 whenever needed; therefore, by not investing in period 1, a high-cost agent obtains an expected payoff equal to:

$$-\overline{C}+\delta\left\{-\underline{C}+\delta E\left[\Pi_3 \mid \mu_3=\rho\right]-\rho c\right\},$$

whereas investing in period 1 yields:

$$-c-\underline{C}+\delta\left\{-\underline{C}+\delta E\left[\Pi_3 \mid \mu_3=\rho\right]-(1-\rho)c\right\}.$$

Therefore, the agent invests in period 1 if:

$$-c+\overline{C}-\underline{C}+\delta(2\rho-1)c>0,$$

or:

$$c < \underline{c}_1^* \equiv \frac{\overline{C} - \underline{C}}{1 - \delta(2\rho - 1)}.$$

The inequality $\underline{c}_1^* > c_2^*$ boils down to (32), which concludes the proof. ■

Proof of Proposition 9. From (12), we have:

$$\frac{\partial\left(\overline{\Pi}_1 - \underline{\Pi}_1\right)}{\partial i} = (2\rho - 1)\frac{(1 - \delta(2\rho - 1))c - B}{(1 - \delta(2\rho - 1)(1 - i))^2}.$$

Therefore, if $B/c \leq 1 - \delta(2\rho - 1)$, $\delta\left(\overline{\Pi}_1 - \underline{\Pi}_1\right)$ is maximal for $i = 1$, where it is equal to $\delta(2\rho - 1)c\,(< c)$; thus the agent never invests. If instead $B/c > 1 - \delta(2\rho - 1)$, then $\delta\left(\overline{\Pi}_1 - \underline{\Pi}_1\right)$ increases from $\delta(2\rho - 1)c\,(< c)$ to $\dfrac{\delta(2\rho - 1)B}{1 - \delta(2\rho - 1)}$ as i decreases from $i = 1$ to $i = 0$. Hence: (i) if $B/c \leq \frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}$, then the agent never invests; (ii) if instead $B/c > \frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}$, then the agent invests with probability i_1^*, which is the unique solution to $\delta\left(\overline{\Pi}_1 - \underline{\Pi}_1\right) = c$, **which yields** expression (13). ■

Proof of Proposition 10. From (17), we have:

$$\begin{aligned}
\frac{\partial\left(\overline{\Pi}_2 - \underline{\Pi}_2\right)}{\partial i} &= \frac{\left(1 - \delta^2(2\rho - 1)^2(1 - i)\right)\delta(2\rho - 1)^2(c - B)}{\left(1 - \delta^2(2\rho - 1)^2(1 - i)\right)^2} + \\
&\quad -\frac{\delta^2(2\rho - 1)^3(B + \delta(2\rho - 1)((1 - i)B + ic))}{\left(1 - \delta^2(2\rho - 1)^2(1 - i)\right)^2} \\
&= \frac{\delta(2\rho - 1)^2(1 + \delta(2\rho - 1))}{\left(1 - \delta^2(2\rho - 1)^2(1 - i)\right)^2}((1 - \delta(2\rho - 1))c - B).
\end{aligned}$$

Therefore, if $\frac{B}{c} \leq 1 - \delta(2\rho - 1)$, $\delta\left(\overline{\Pi}_2 - \underline{\Pi}_2\right)$ is maximal for $i = 1$, where it is equal to:

$$\begin{aligned}
\delta(2\rho - 1)(B + \delta(2\rho - 1)c) &\leq \delta(2\rho - 1)((1 - \delta(2\rho - 1))c + \delta(2\rho - 1)c) \\
&= \delta(2\rho - 1)c < c.
\end{aligned}$$

The agent thus never invests. If instead $\frac{B}{c} > 1 - \delta(2\rho - 1)$, then $\delta\left(\overline{\Pi}_2 - \underline{\Pi}_2\right)$ increases from $\delta(2\rho - 1)(B + \delta(2\rho - 1)c)$ to $\dfrac{\delta(2\rho - 1)B}{1 - \delta(2\rho - 1)}$ as i decreases from $i = 1$ to $i = 0$. Hence: (i) If $\frac{B}{c} \leq \frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}$, the agent never invests; (ii) if instead $\frac{B}{c} \geq \frac{1-\delta^2(2\rho-1)^2}{\delta(2\rho-1)}$, then the agent invests with probability 1 in the second period of a contract; (iii) if $\frac{1-\delta(2\rho-1)}{\delta(2\rho-1)} < \frac{B}{c} < \frac{1-\delta^2(2\rho-1)^2}{\delta(2\rho-1)}$, then the agent invests with probability i_2^*, which is the unique solution to $\delta\left(\overline{\Pi}_2 - \underline{\Pi}_2\right) = c$, **which yields** expression (18). ■

Proof of Proposition 11.

When $\frac{B}{c} > \frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}$, under $T = 2$ the investment in the second contracting period satisfies:

$$\begin{aligned} i_2^* &= \max\left\{\frac{1+\delta(2\rho-1)}{\delta(2\rho-1)} - \frac{1-\delta^2(2\rho-1)^2}{\delta^2(2\rho-1)^2}\frac{c}{B}, 1\right\} \\ &= \max\left\{\frac{1+\delta(2\rho-1)}{\delta(2\rho-1)} i_1^*, 1\right\}, \end{aligned}$$

which is strictly greater than i_1^* since $i_1^* < 1$ and $\frac{1+\delta(2\rho-1)}{\delta(2\rho-1)} > 1$. ■

Proof of Proposition 12. Under one-period contracting, the average quality per period is given by:

$$q_1^* = (1-\delta)\frac{\overline{Q}_1 + \underline{Q}_1}{2}, \tag{33}$$

where $\overline{Q}_1$ and $\underline{Q}_1$, which respectively denote the total expected discounted quality when the current quality is either high ($q_t = H$) or low ($q_t = L$), are characterized by:

$$\begin{aligned} \overline{Q}_1 &= H + \delta\left[(\rho + (1-\rho)\, i_1^*)\, \overline{Q}_1 + (1-\rho)(1-i_1^*)\, \underline{Q}_1\right], \\ \underline{Q}_1 &= L + \delta\left[(1-\rho+\rho i_1^*)\, \overline{Q}_1 + \rho(1-i_1^*)\, \underline{Q}_1\right]. \end{aligned}$$

It follows from these conditions that:

$$\frac{\overline{Q}_1 + \underline{Q}_1}{2} = \frac{1}{1-\delta}\left(\frac{H+L}{2} + \frac{\delta}{2}\frac{i_1^*\Delta}{1-\delta(2\rho-1)(1-i_1^*)}\right)$$

Therefore expression (33) becomes

$$q_1^* = \frac{H+L}{2} + \frac{\delta}{2}\frac{i_1^*\Delta}{1-\delta(2\rho-1)(1-i_1^*)}.$$

Under two-period contracting, the average quality is given by:

$$\begin{aligned} q_2^* &= (1-\delta)\left(\frac{H+L}{2} + \delta\left(\frac{1+i_2^*}{2}\overline{Q}_2 + \frac{1-i_2^*}{2}\underline{Q}_2\right)\right) \\ &= (1-\delta)\left(\frac{H+L}{2} + \delta\left(\frac{\overline{Q}_2 + \underline{Q}_2}{2} + \frac{i_2^*\left(\overline{Q}_2 - \underline{Q}_2\right)}{2}\right)\right), \end{aligned} \tag{34}$$

where $\overline{Q}_2$ and $\underline{Q}_2$ now respectively denote the total expected discounted quality, evaluated in the second period of a contract, when the current quality is either high ($q_t = H$) or low

$(q_t = L)$; these expected values are characterized by:

$$\begin{aligned}
\overline{Q}_2 &= H + \delta\rho\left\{H + \delta\left[\left(\rho + (1-\rho)\, i_2^*\right)\overline{Q}_2 + (1-\rho)\left(1 - i_2^*\right)\underline{Q}_2\right]\right\} \\
&\quad + \delta\left(1-\rho\right)\left\{L + \delta\left[\left(1 - \rho + \rho i_2^*\right)\overline{Q}_2 + \rho\left(1 - i_2^*\right)\underline{Q}_2\right]\right\}, \\
\underline{Q}_2 &= L + \delta\left(1-\rho\right)\left\{H + \delta\left[\left(\rho + (1-\rho)\, i_2^*\right)\overline{Q}_2 + (1-\rho)\left(1 - i_2^*\right)\underline{Q}_2\right]\right\} \\
&\quad + \delta\rho\left\{L + \delta\left[\left(1 - \rho + \rho i_2^*\right)\overline{Q}_2 + \rho\left(1 - i_2^*\right)\underline{Q}_2\right]\right\}
\end{aligned}$$

After some simplifications, it follows that:

$$\frac{\overline{Q}_2 + \underline{Q}_2}{2} = \frac{1}{1-\delta}\frac{H+L}{2} + \frac{\delta^2}{1-\delta^2}\frac{i_2^*\left(\overline{Q}_2 - \underline{Q}_2\right)}{2}$$

Therefore expression (34) becomes:

$$q_2^* = \frac{H+L}{2} + \frac{\delta}{2}\frac{1 + \delta\left(2\rho - 1\right)}{1+\delta}\frac{i_2^*\Delta}{1 - \delta^2\left(2\rho-1\right)^2\left(1 - i_2^*\right)}.$$

It is convenient to introduce the notation $\sigma \equiv \delta\left(2\rho - 1\right)$ and $\lambda \equiv c/B$. The investment levels are then:

$$\begin{aligned}
i_1^* &= 1 - \frac{1-\sigma}{\sigma}\lambda, \\
i_2^* &= \max\left\{\frac{1+\sigma}{\sigma} i_1^*, 1\right\} \\
&= \max\left\{\frac{1+\sigma}{\sigma}\left(1 - \frac{1-\sigma}{\sigma}\lambda\right), 1\right\},
\end{aligned}$$

and the average expected qualities are:

$$\begin{aligned}
q_1^* &= \frac{H+L}{2} + \frac{\delta}{2}\frac{i_1^*\Delta}{1 - \sigma\left(1 - i_1^*\right)} \\
&= \frac{H+L}{2} + \frac{\delta\Delta}{2\sigma}\frac{\sigma - \left(1-\sigma\right)\lambda}{1 - \left(1-\sigma\right)\lambda},
\end{aligned}$$

and

$$\begin{aligned}
q_2^* &= \frac{H+L}{2} + \frac{\delta}{2}\frac{1+\sigma}{1+\delta}\frac{i_2^*\Delta}{1 - \sigma^2\left(1 - i_2^*\right)} \\
&= \frac{H+L}{2} + \frac{\delta\Delta}{2\sigma}\frac{1+\sigma}{\left(1+\delta\right)\sigma}\frac{\sigma - \left(1-\sigma\right)\lambda}{1 - \left(1-\sigma\right)\lambda} \text{ if } \lambda > \frac{\sigma}{1-\sigma^2} \text{ (that is, } i_2^* < 1) \\
&= \frac{H+L}{2} + \frac{\delta\Delta}{2}\frac{1+\sigma}{1+\delta} \text{ if } \lambda \leq \frac{\sigma}{1-\sigma^2} \text{ (that is, } i_2^* = 1).
\end{aligned}$$

It follows that, as long as $\lambda > \frac{\sigma}{1-\sigma^2}$ (that is, $i_2^* < 1$), $q_1^* < q_2^*$ (it suffices to note that $\frac{1+\sigma}{(1+\delta)\sigma} > 1$). When instead $\lambda \leq \frac{\sigma}{1-\sigma^2}$, $i_2^* = 1$ and then:

$$\begin{aligned} q_1^* - q_2^* &= \frac{\delta\Delta}{2\sigma}\left(\frac{\sigma-(1-\sigma)\lambda}{1-(1-\sigma)\lambda} - \sigma\frac{1+\sigma}{1+\delta}\right) \\ &= \frac{\delta\Delta}{2\sigma}\left(1 - \frac{1-\sigma}{1-(1-\sigma)\lambda} - \sigma\frac{1+\sigma}{1+\delta}\right). \end{aligned} \tag{35}$$

This quality differential thus decreases with λ, it is moreover positive for $\lambda = 0$:

$$q_1^* - q_2^*|_{\lambda=0} = \frac{\delta\Delta}{2}\left(1 - \frac{1+\sigma}{1+\delta}\right) = \frac{\delta^2\Delta}{2}\frac{2\rho-1}{1+\delta} > 0,$$

but it is negative for $\lambda = \frac{\sigma}{1-\sigma^2}$:

$$q_1^* - q_2^*|_{\lambda=\frac{\sigma}{1-\sigma^2}} = \frac{\delta\Delta(\sigma\delta-1)}{2(1+\delta)} < 0.$$

It follows that there exists $\lambda^* \in \left(0, \frac{\sigma}{1-\sigma^2}\right)$ such that $q_1^* \geq q_2^*$ for $\lambda \leq \lambda^*$ and $q_1^* < q_2^*$ for $\lambda > \lambda^*$.

Finally, differentiating (35) w.r.t. σ , we obtain

$$\begin{aligned} \left.\frac{\partial(q_1^* - q_2^*)}{\partial\sigma}\right|_{\lambda\leq\frac{\sigma}{1-\sigma^2}} &= -\frac{1}{2}\frac{\Delta}{\sigma^2}\frac{\delta\left(\sigma^2+\lambda^2\left(1+2\sigma^2-2\sigma^3+\sigma^4-2\sigma+\delta+\sigma^2\delta-2\sigma\delta\right)\right)}{(\delta+1)(\sigma\lambda-\lambda+1)^2} + \\ &+\frac{1}{2}\frac{\Delta}{\sigma^2}\frac{\delta\lambda\left(1+\delta+\sigma^2-2\sigma^3-\sigma^2\delta\right)}{(\delta+1)(\sigma\lambda-\lambda+1)^2} \end{aligned} \tag{36}$$

which is positive since

$$\left.\frac{\partial(q_1^* - q_2^*)}{\partial\sigma}\right|_{\lambda=0} = \frac{1}{2}\frac{\Delta}{\sigma^2}\delta(1-\sigma)(\sigma+1) > 0,$$

and

$$\left.\frac{\partial^2(q_1^* - q_2^*)}{\partial\sigma\partial\lambda}\right|_{\lambda\leq\frac{\sigma}{1-\sigma^2}} = \frac{1}{2}\frac{\Delta}{\sigma^2}\delta\frac{1-\sigma}{(\sigma\lambda-\lambda+1)^3}\left(\sigma-\lambda+2\sigma\lambda-\sigma^2\lambda+1\right) > 0,$$

The quality differential in (35) (i) increases with σ, which in turn increases with ρ and δ, and (ii) also directly increases with δ; it therefore increases with δ and ρ. Thus λ^* must increase with δ and ρ. ■

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