Optimal Certification Design

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Abstract

This paper analyzes the rating of a product of unknown quality by a certifier who internalizes the buyers' surplus and receives payments from a seller. It shows that contrary to conventional wisdom, a regulation that prohibits contingent payments hinders information revelation and harms social welfare when the contract between seller and certifier is public. If the contract is private (buyers do not observe the payments), contingent payments lead to "rating inflation": high ratings are issued for a wide range of qualities and ratings have limited information value. Mandating flat fees then prevents rating inflation and can increase welfare.

Introduction

The need for external certification arises because sellers such as developers of new financial products, drugs, genetically modified foods and new technologies often lack the expertise, and especially the credibility to evaluate and communicate the quality of their product to potential buyers. A typical business model for certifiers is the seller-pays paradigm, in which buyers access the ratings for free. The role played by credit rating agencies in the recent economic crisis raised concerns over the credibility of this business model. Many scholars have stressed the conflict of interests these agencies face when the fee for a rating

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is proportional to the size of the issue rated,[1] and have argued that certifiers' compensation should not be linked to the ratings and possibly be partly paid by other parties than the seller.[2] This critique resulted in the "Cuomo plan", named after New York State Attorney General Andrew Cuomo, which requires fees of the rating agencies to be independent of ratings and to be paid upfront.

The main contribution of this paper is to show that in a world with rational buyers and transparent contracts between product sellers and their certifiers, rating-contingent payments encourage information production and make all parties better off. Conversely, forcing payments to be noncontingent reduces the amount of information revealed to the public and harms all parties.

We develop a model where a certifier publicly releases his opinion about the quality of a product.[3] We show that under a fixed payment the certifier who internalizes the buyers' interest fails to communicate all the information he has via public ratings. Once the information intended to guide the purchasing decisions of the buyers is released this information is also used by the seller in her pricing decisions which affect the buyers' payoff. Intuitively, a higher rating leads to a higher price and lowers buyers' surplus; this indirect effect makes the certifier cautious and results in partial revelation of information with coarse ratings.[4] On the other hand, if the seller can publicly promise rating-contingent payments she can partially compensate the certifier for the negative externality her pricing has on buyers, which in turn improves information revelation and welfare in equilibrium. As will become clear from the later discussion, transparency (publicity of payments) is crucial for this result.

An important critique of rating agencies relates to the rating inflation that occurred for mortgage-backed securities prior to the crisis of 2007-2008. According to Pagano-Volpin (2009), in 2007 about 60% of structured products had AAA ratings in comparison to less than 1% of corporate issues. Moreover, Griffin and Tang (2009) indicate that just before

[1] According to Partnoy (2006) around 90 percent of rating agencies' revenue is due to the fees paid by the issuers. The fees correspond to 3-4 basis points of the volume of the issue for corporate bonds and go up to 10 basis points for structured finance products.

[2] See for instance Mathis, et al. (2009) and Pagano-Volpin (2009).

[3] Moody's website (www.moodys.com) explains that its ratings are only statements of opinion and should not be viewed as recommendations to purchase or sell securities.

[4] The reluctance of the certifiers to give favorable ratings to sellers is most prominent in standard setting bodies. In a famous case an R&D company Rambus didn't disclose its patent covering DRAM technology when applying for several memory standards governed by the Joint Electron Device Engineering Council (JEDEC). In the subsequent litigations Rambus argued that it didn't disclose the patent because in the past JEDEC unfairly turned down several technologies that it knew were covered by Rambus's patents. For more details see the case 38636 of the European Commission.

2007 the AAA tranches of CDOs were larger than what the rating agency model would deliver suggesting systematic positive adjustments to the rating agency model.

We find that "rating inflation" can happen with fully rational buyers if certifiers do not disclose the payments they receive from the sellers. In this case contingent payments loose part of their attractiveness and may even call for regulation. When the buyers do not see how much the seller is promising to the certifier for high ratings, the seller has an incentive to promise a lot in order to increase the chance of getting a high rating. Consequently high ratings are issued "too often" (rating inflation), and the overall informativeness of ratings decreases. Mandating public disclosure of payments restrains the seller from exaggerating the payments for high ratings, prevents the rating inflation and improves the precision of high ratings.

Our model hinges on the assumption that in the absence of compensation from the seller the certifier is buyer-protective (puts weight $\lambda > 0$ on buyers' welfare); we provide several rationales for this assumption. First, a monopolistic certifier may simply be buyer-protective; moreover if contracts are public and we let the certifier choose the degree of buyer-protectiveness, he would choose some positive degree in order to maximize the expected payment from the issuer.[5] Second, if product examination is costly, some minimum internalization of buyers' surplus is required to induce the certifier to bear the effort cost.[6] Third, if payments from the issuer are private, a positive λ is vital to sustain ratings' credibility. Alternatively parameter λ can be a measure of the extent to which the discontent of the buyers can reduce the certifier's profit, due to potential litigation costs or the costs of additional regulation prompted by poor performance of the certifier.

Literature Review

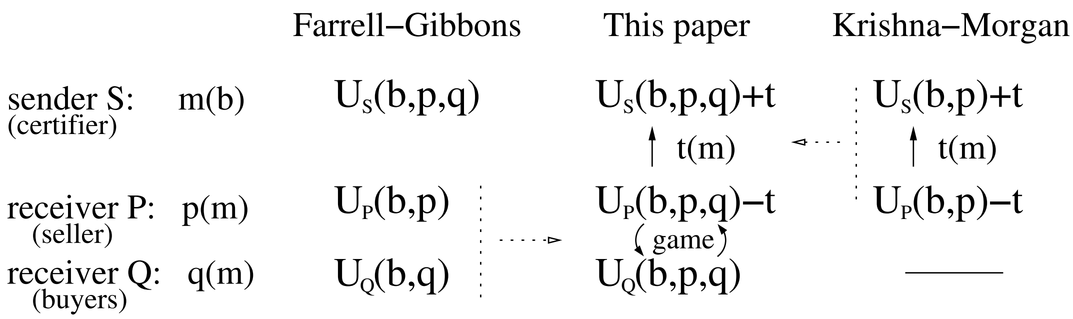
The paper makes two contributions to the literature. First, it advances the literature on public contracting for information by extending it to the games among multiple receivers. To put it simply, in this paper "Farrell-Gibbons meets Krishna-Morgan". First, the certifier's communication with two strategic parties, the buyers and the seller, relates our paper to the literature on cheap talk with multiple audiences. In Farrell-Gibbons (1989) a sender

[5] An example of this is the "chinese wall" in investment banks that separates parts of the bank that have access to insider information from the parts that are supposed to rely solely on public information. For many certifiers inherent reputational concerns lead to some internalization of the welfare of potential buyers. At the same time investment banks after selling shares at initial public offerings, often post a "stabilizing" bid, that is stay ready to buy the shares back at a fixed price, which makes their position closer to that of potential buyers.

[6] In governmental bodies like Food and Drug Administration it is stated clearly that their aim is to serve the interests of consumers. Committees of Standard Setting Organizations that decide on new technological standards, on the other hand, are often composed of potential technology buyers.

S is informed about the true state b and sends a message m to two uninformed receivers: P and Q, who take actions p and q respectively. Parties' payoffs are represented in the Figure 1. Farrell and Gibbons show that the informativeness of communication between the sender S and receiver Q can be limited if receiver P also receives the sender's messages (subversion). In their model receivers do not interact: receiver P's action does not affect receiver Q's payoff and vice versa. We expand Farrell-Gibbons in two ways. First, the two receivers interact strategically. Second, we allow one of the receivers (the seller) to contract with the sender.

Figure 1: Payoff structure



As in Krishna-Morgan (2008), we allow public contingent contracts. Krishna and Morgan show that contingent contracts facilitate communication in the standard, single-receiver Crawford-Sobel model. By contrast, our two receivers play a game after receiving the message. Parties' interdependent payoffs, which are recovered from primitives in a market interaction, do not possess the properties required in Krishna-Morgan; for instance in Krishna-Morgan some results are obtained only for quadratic payoff functions which is not the case in our paper. Thus the analysis of public contracts is a generalization of Krishna-Morgan's results. Furthermore our study of private contracts between the seller and the certifier is also novel as it does not arise with only two parties. The study of private contracts is the paper's second contribution to the literature.

In this it is related to recent paper by Inderst and Ottaviani (2010), which analyzes hidden payments to an information intermediary. Unlike in our model the intermediary does not provide buyers with precise information about the product via ratings but simply recommends it for purchase or not. The intermediary panders to customers and advises them to buy the product they would like even when its production is inefficiently costly from the social welfare perspective. By contrast, in our model the intermediary cares about the net surplus of the buyers and is reluctant to give a high rating which leads to a high price. In Inderst-Ottaviani the kickbacks paid by the sellers can tilt the intermediary's recommendations towards the cost-efficient product and improve social welfare. Authors conclude that caps and disclosure requirements on commissions may lower welfare. Conversely our model with multiple ratings stresses the adverse effect the kickbacks have on information production. To our knowledge our paper is the first to study the endogenous partition of information in multiple ratings under private contracts.

Our paper is also related to other contributions, although more distantly. Most of the literature on certification takes one of two main approaches. Some contributions assume that certifiers can commit to a disclosure rule and ignore the issue of credibility. These papers are silent about how information production, credibility and welfare are affected when the certifier faces a contingent contract. Lizzeri (1999) finds that a monopolistic certifier who charges a fixed fee for ratings discloses minimum information to ensure efficient market exchange. Similarly Farhi et al. (2009) argue that certifiers have no incentive to disclose rejections. They conclude that increasing transparency (requiring certifiers to reveal rejections) benefits sellers. We also advocate for transparency showing that public disclosure of seller's transfers to certifiers is welfare improving. The focus of our study is different, as we look at the certifier's ability to communicate information given the incentives he faces.

Lerner-Tirole (2006)'s approach is closer to ours. They analyze forum shopping when certifiers intrinsically care about product buyers and sellers can make costly concessions. In their main model buyers are homogeneous and the product is profitable only if buyers adopt it, which makes pass/fail examination optimal. They consider certifiers' incentives as given and focus on forum shopping by the sellers, finding that weak applicants go to tougher certifiers and make more concessions. We consider heterogeneous buyers and allow the seller to offer contingent payments to the certifier which enables us to analyze endogenous information production by the certifier. We study public, private and non-contingent contracts and show that pass/fail ratings in general are not optimal. We also shed some light on the issue of forum shopping for ratings which was the main objective of Lerner-Tirole (206) and of Skreta-Veldkamp (2009) and Bolton at al. (2009).

Reputation models show that if the public trusts the certifier he can benefit from private information either by speculating on products he certifies (Benabou-Laroque (1992)), or by issuing high ratings for money (Mathis, et al. (2009)). Provided that reputational concerns fail to restrain the certifier from lying, Mathis et al. propose a flat compensation independent of the rating. Their model exogenously links the certifier's compensation to ratings. We show that the certifier being paid for high ratings arises endogenously as an optimal public contract between the seller and the certifier. Contrary to Mathis et al. this contract does not cause the certifier to overstate the rating. Our analysis of private contracts is more coherent with Mathis et al., advocating that if payments are not transparent then, probably, they should not depend on ratings.

The paper is organized as follows. In section 1 we introduce the general environment. In section 2 we consider public contracts: first we prove a modified revelation principle and then study contingent and noncontingent contracts. Section 3 analyzes private contracts, section 4 is dedicated to welfare analysis, section 5 studies extensions of the basic model: forum shopping, moral hazard, and seller's commitment to a pricing rule. Section 6 concludes.

1 Environment

There is a mass 1 of buyers, each of whom can buy one unit of a product of unknown quality $b \sim U[0,1]$ sold by a monopolistic seller who has zero marginal cost of production. A buyer i receives net utility $b - P$ if he buys a quality b product at price P and enjoys reservation utility $x_i \in [0,1]$ if he decides not to buy; the mass of buyers with reservation utility below x is $F(x)$. We require $F(x)/f(x)$ to be increasing (monotone hazard rate). If the quality b were common knowledge the market demand would be $D(P,b) = F(b-P)$ and the seller would set the monopoly price $P(b) = F(b-P(b))/f(b-P(b))$. However, we assume that only the certifier can learn the true quality (potentially at some cost).

Assumption 1. *The product's quality b is privy to the certifier.*

In the absence of certification about actual quality, b is unknown to all parties, which leads to mispricing and potential profit loss compared to the full information case.[7] The seller can apply to a certifier. The latter then publicly issues some rating m.

The certifier puts weight $\lambda > 0$ on the buyers' surplus $S(b,m)$ and receives monetary payments $t(m)$ for ratings from the seller. The certifier is protected by limited liability $t(m) \geq 0$. He arbitrarily chooses the rating from some set M in order to maximize his objective function U:

Assumption 2. $U(b,m) = \lambda S(b,m) + t(m)$.

The certifier's internalization of the buyers' payoff gives him some credibility in the eyes of the buyers. The parameter λ stands for various reasons why certifiers care about buyers, from buyers' interests being explicitly represented within decision bodies to reputational concerns and explicit monetary incentives. We can easily extend the analysis to the case where the certifier intrinsically cares about the seller as well $U(b,m) = \lambda S(b,m) + \eta\pi(m) +$

[7]It can be shown that the monopoly profit is a convex function of quality, see proposition 3.

$t(m)$. The qualitative results are the same unless η exceeds some threshold η^* (for details see section 2, remark 2).[8]

The seller defines the set of possible ratings M for the certifier and commits to pay corresponding transfers $t(m)$, $m \in M$. However the seller cannot commit to the pricing rule and sets the monopoly price $P(m)$ upon receiving the rating. One might think that there is an uncertainty about the market demand which is resolved only after the rating is issued and the seller needs to adjust her price ex post. Or else the seller might need to set different prices for different market segments.

Assumption 3. *The seller designs the contract* $(M, t(.))$.

In other words the seller acts as an uninformed principal while the certifier plays the role of an informed agent. The seller's role as a principal (assumption 3) is for simplicity, in section 2.3 we let parties negotiate the contract and find similar results.

The assumption that the seller is uninformed when applying for a rating (assumption 1) also simplifies the analysis. If contracts are public, an informed seller as an informed principal could signal quality b through the contract offered $(M, t(.))_b$ to the certifier or by burning money. However if such signals are not available it makes no difference whether the seller is informed or not and our results hold; furthermore the seller may not have the full information acquired by the certifier and could be treated as a less informed party.[9]

Assumption 2 is crucial for ratings' credibility, because a certifier with $\lambda = 0$ does not care about the true quality b and, hence, he has no incentive to reveal it and simply picks the highest transfer. Of course if transfers are zero, he might as well tell the truth; but if investigation were costly or transfers were private, then such a certifier would not be reliable.

2 Public contracts

We start our analysis by considering the case of public contracts, assuming the certifier learns product quality at no cost. The seller publicly announces the set of messages the certifier can use M and corresponding transfers $t(.)$. This contract $(M, t(.))$ induces a game between the buyers, the certifier and the seller. The certifier learns the actual quality b and decides what rating to issue, that is the certifier's strategy $[0,1] \to \Delta(M)$ assigns to each

[8] Introducing η brings little to the model since a general transfer $\tilde{t}(m) = \eta\pi(m) + t(m)$ accounts for it.

[9] Developers of new drugs often have no equipment or legal rights to perform certain tests of their products and have to rely on FDA's assessments. Similarly investment banks might have better information than the issuer about the prospects of the issue because they have more marketing experience.

quality b a probability distribution over possible ratings M with corresponding density function $\sigma(m|b)$, $\int\limits_M \sigma(m|b)dm = 1$. The buyers and the seller form beliefs $M \to \Delta([0,1])$ that specify for each rating m a probability distribution over the possible qualities $b \in [0,1]$ with a density function $\mu(b|m)$, $\int\limits_0^1 \mu(b|m)db = 1$. Having observed the rating m the seller sets the product price, her strategy is $P : M \to R^+$. Buyers decide whether to buy the product, their decisions generate a market demand $D : R^+ \times M \to R^+$ which defines the quantity the seller can sell at price P given the rating m. Finally payoffs are realized.

A subgame perfect Bayesian-Nash equilibrium of this game is a quadruple (D, σ, P, μ) that ensures that buyers buy their preferred quantity of product given the price and their realized beliefs; the certifier maximizes his payoff given the seller's pricing function and the buyers' beliefs function; the seller maximizes profit given the buyers' realized beliefs; buyers' realized beliefs are consistent with Bayes' rule. More formally consider a continuation equilibrium $(D^*, \sigma^*, P^*, \mu^*)$ induced by some contract $(M, t(.))$. Referring to equilibrium beliefs μ^* we express buyers' quality expectation for each rating m as $\widehat{b}^*(m) = E(b|m, \mu^*) = \int\limits_0^1 b\mu^*(b|m)dx$.

Suppose a credible rating m is issued and price P is set, then only buyers who *perceive* their utility from the purchase $E[b - P|m] = \widehat{b}^*(m) - P$ to be greater than their outside option x_i agree to buy the product and the market demand is $D^*(P, m) = F(\widehat{b}^*(m) - P)$. If we denote by $E[V(b, D)|m] = V(\widehat{b}^*(m), D)$ the aggregate expected utility of the buyers then $D^*(P, m) = \underset{D \geq 0}{argmax}[V(\widehat{b}^*(m), D) - PD]$. Anticipating this reaction the seller sets profit maximizing price $P^*(m) = F(\widehat{b}^*(m) - P^*(m))/f(\widehat{b}^*(m) - P^*(m))$. Observe that $P^*(m) = P^*(\widehat{b}^*(m))$ and we can denote $D^*(m) = D^*(\widehat{b}^*(m)) = D^*(P(\widehat{b}^*(m)), \widehat{b}^*(m))$.

Suppose actual quality is b, then buyer i *receives* the surplus $b - P^*(m) - x_i$ if he buys the product. [10] Total buyers' surplus for quality b and rating m is computed as $S(b, m) = \int\limits_0^{F^{-1}(D^*(m))} [b - P^*(m) - x]\, dF(x)$, seller's profit is $\pi(m) = P^*(m)D^*(m)$ and we have

Remark 1. Given quality b the expectation of product's quality $\widehat{b}^*(m)$ defines the payoffs of the buyers and the seller: $S(b, \widehat{b}^*(m))$ and $\pi(\widehat{b}^*(m))$.

Now let's go back one stage, where the certifier has just learned quality b and decides on the rating. The equilibrium reporting strategy for the certifier is $\sigma^*(m'|b) \geq 0$ for $m' \in \underset{m \in M}{\arg\max}\ \lambda S(b, m) + t(m)$ and $\sigma^*(m'|b) = 0$ otherwise. Bayes' rule, when applicable,

[10] If the rating m is imprecise actual quality b might happen to be lower than the expected one, that is some buyers might buy the product and experience negative surplus afterwards.

requires $\mu^*(b'|m)\int\limits_0^1 \sigma^*(m|b)db = \sigma^*(m|b')$.

Monotonicity lemma

For any actual quality b, equilibrium beliefs $\mu^*(b|m)$ and certifier's reporting strategy $\sigma^*(m,b)$ define a set of expected qualities $\widetilde{B}^*(b) = \{\widehat{b}^*(m) : \sigma^*(m|b) > 0\}$ communicated by the certifier. It turns out that these expected qualities increase with quality.

Lemma 1. *In equilibrium expected qualities inferred from the certifier's rating are weakly increasing in actual quality, that is* $\forall b_2, b_1 \in [0,1]$ *from* $b_2 > b_1$ *follows* $\inf \widetilde{B}^*(b_2) \geq \sup \widetilde{B}^*(b_1)$.

Proof. For $b_2 > b_1$ take any $m_2 : \widehat{b}^*(m_2) \in \widetilde{B}^*(b_2)$ and $m_1 : \widehat{b}^*(m_1) \in \widetilde{B}^*(b_1)$. Revealed preference writes $\lambda S(b_2, m_2) - \lambda S(b_2, m_1) \geq \lambda S(b_1, m_2) - \lambda S(b_1, m_1)$. Substituting for S we get $\lambda \left[F\left(\widehat{b}^*(m_2) - P^*(\widehat{b}^*(m_2))\right) - F\left(\widehat{b}^*(m_1) - P^*(\widehat{b}^*(m_1))\right)\right](b_2 - b_1) \geq 0$. Given that $\frac{F(.)}{f(.)}$ is increasing (monotone hazard rate property) we have $\frac{\partial F(\widehat{b}^* - P^*(\widehat{b}^*))}{\partial \widehat{b}^*} = \frac{f^3}{2f^2 - Ff'} > 0$, hence we must have $\widehat{b}^*(m_2) \geq \widehat{b}^*(m_1)$, which implies $\inf \widetilde{B}^*(b_2) \geq \sup \widetilde{B}^*(b_1)$ QED.

It follows that the set of qualities associated with each rating is convex. For instance if each rating corresponds to several qualities then ratings define a partition $\{b_i\}_{i=1,\ldots,N}$ of the interval $[0,1]$.

Modified revelation principle

The above analysis implies that all the information needed for the market is embedded in quality expectations $\widehat{b}^*(m)$ that in turn determine prices and parties' payoffs. This suggests that a direct contract where the certifier uses expected quality as rating can account for all possible outcomes. We extend the modified revelation principle established in Krishna-Morgan (2008) and Bester-Strausz (2001) to our framework.

Proposition 1. *For any equilibrium under a contract* $(M, t(.))$ *there exists an outcome-equivalent equilibrium in pure strategies under some direct contract* $(\widehat{B}, t(.))$ *in which the certifier truthfully announces expected quality* $\widehat{b} \in \widehat{B} \subseteq [0,1]$.

Proof. Take any subgame perfect Bayesian-Nash equilibrium $(D^*, \sigma^*, P^*, \mu^*)$ induced by a contract $(M, t(.))$. In this equilibrium for a given b any message m corresponds to a quality expectation $\widehat{b}^*(m) = \int\limits_0^1 b\mu^*(b|m)db$, in other words beliefs define a correspondence $M \to \widehat{B}$, where $\widehat{B} = \bigcup\limits_{m \in M} \widehat{b}^*(m) \subseteq [0,1]$. Therefore certifier's reporting strategy $[0,1] \to \Delta(M)$ for every b induces a probability distribution over expected qualities $[0,1] \to \Delta(\widehat{B})$.

By remark 1 for every b parties' payoffs are pinned down by the expected quality $\widehat{b}^*(m)$. Thus for any equilibrium under $(M, t(.))$ we can find an equivalent equilibrium under contract $(\widehat{B}, t(.))$ with $t(\widehat{b}) = \sup\{t(m) : \widehat{b}^*(m) = \widehat{b}\}$ and certifier's reporting strategy $[0,1] \to \Delta(\widehat{B})$. Indeed if a message m, resulting in $\widehat{b}^*(m)$ and $t(m)$, is sent by the certifier under $(M, t(.))$ then $\widehat{b} = \widehat{b}^*(m)$ resulting in $t(\widehat{b}) = t(m)$ must be chosen under $(\widehat{B}, t(.))$.[11] Finally considering pure strategies $[0,1] \to \widehat{B}$ is without loss of generality because in any equilibrium for almost every $b \in [0,1]$ the set of expected qualities communicated with positive probability $\widehat{b}(b) = \{\widehat{b} : \sigma^*(m,b) > 0\}$ is single valued, that is randomization could happen at points that have measure zero. Indeed from lemma 1 follows that $\bar{b}(b) = \sup\{\widehat{b}(b)\}$ is monotone. At any point b where $\bar{b}(b) > \inf\{\widehat{b}(b)\}$ function $\bar{b}(b)$ is discontinuous (for any $b' < b$ and $b'' > b$ by lemma 1 we have $\bar{b}(b') \leq \inf\{\widehat{b}(b)\} < \bar{b}(b) \leq \inf\{\widehat{b}(b'')\}$. Since $\bar{b}(b)$ is monotone it can be discontinuous only at a countable number of points, therefore for almost every point $\sup\{\widehat{b}(b)\} = \inf\{\widehat{b}(b)\}$ and $\widehat{b}(b)$ is single valued QED.

The modified revelation principle greatly simplifies the analysis allowing us to consider only direct announcements of expected qualities by the certifier $\widehat{b} \in \widehat{B}$. One might wonder what happens if the certifier instead of reporting from the set $\widehat{B}$ would report something from outside this set $a \notin \widehat{B}$ or would simply refuse to report at all. We assume that for these announcements transfers are zero $t(a) = 0$ and buyers' beliefs are such that $E(b|a) = \widehat{b}'$ for some $\widehat{b}' \in \widehat{B}$. Then for any $b \in [0,1]$ rating $\widehat{b}' \in \widehat{B}$, $t(\widehat{b}') \geq 0$ dominates any $a \notin \widehat{B}$ because $\lambda S(b, \widehat{b}') + t(\widehat{b}') \geq \lambda S(b, \widehat{b}')$. This assumption allows us to characterize the upper bound on seller welfare. Alternatively, we could dispose with the limited liability assumption and assume that the certifier is very risk averse, his utility from money is t for $t \geq 0$ and $-\infty$ for $t < 0$. Then it suffices for the seller to specify $t(a) < 0$ for $a \notin \widehat{B}$.

2.1 Noncontingent contracts

Before solving for the optimal contingent contract we check what happens if payments t to the certifier are required to be independent of ratings he issues. We assume that the equilibrium that is preferred by the seller obtains and demonstrate later on that this equilibrium is preferred by the buyers as well. Since the uninformative babbling equilibrium always exists $\widehat{B} = \{1/2\}$ the seller cannot do worse with certification than without it.

The game proceeds as follows: first, the seller defines the set of ratings $\widehat{B}$ and noncontingent payment t, second, the certifier learns quality b and issues a rating $\widehat{b} \in \widehat{B}$, finally, the market outcome obtains.

[11] Note that a message m' such that $\widehat{b}^*(m') = \widehat{b}$ and $t(m') < \sup\{t(m) : \widehat{b}^*(m) = \widehat{b}\}$ is never sent under $(M, t(.))$ because there exists m such that $\widehat{b}^*(m) = \widehat{b}$ and $t(m) > t(m')$.

Suppose a credible rating $\widehat{b}$ has been published by the certifier. Buyers buy the quantity which maximizes their expected surplus $D(\widehat{b},P) = \underset{D\geq 0}{argmax}[V(\widehat{b},D) - PD]$, the seller sets the price $P(\widehat{b}) = \underset{P\geq 0}{argmax}[D(P,\widehat{b})P]$. For given quality b and rating $\widehat{b}$ the market outcome results in buyers' surplus $S(b,\widehat{b}) = V(b,D(\widehat{b})) - P(\widehat{b})D(\widehat{b})$ and we can proceed to the communication stage.

Problem 1. $\max_{\widehat{b}\in\widehat{B}} \lambda S(b,\widehat{b}) + t$

Observe that a fixed payment t has no influence on certifier's reports and certifier's preferences are aligned with the interests of potential buyers, in this situation one might expect the certifier to reveal all the information to buyers $\widehat{b} = b$. However, when ratings are public the certifier fails to reveal all the information he has.

Proposition 2. *If buyers took an announcement at face value then with noncontingent contracts the certifier would underreport the quality:* $\widehat{b}(b) < b$ *for any* $b > 0$.

Proof. Buyers' surplus $S(b,\widehat{b}) = V(b,D(\widehat{b})) - P(\widehat{b})D(\widehat{b})$ is affected by the rating $\widehat{b}$ in two ways: directly due to the buyers revising their purchasing decisions $D(\widehat{b})$ and indirectly through the seller's pricing $P(\widehat{b})$. Suppose the certifier reports truthfully $\widehat{b} = b$, then by the envelope theorem $\frac{\partial(V(b,D(.))-P(.)D(.))}{\partial D} = \partial(\underset{D\geq 0}{max}[V(\widehat{b},D) - PD])/\partial D = 0$ and the direct effect of the rating vanishes. At the same time for any $b > 0$ the indirect effect is negative $\frac{\partial S(b,\widehat{b})}{\partial \widehat{b}}|_{\widehat{b}=b} = \left(\frac{\partial(V(b,D(.))-P(.)D(.))}{\partial P}\frac{\partial P(.)}{\partial \widehat{b}}\right)|_{\widehat{b}=b} = \left(-D(.)\frac{\partial P(.)}{\partial \widehat{b}}\right)|_{\widehat{b}=b} < 0$ because the price increases with the rating $\frac{\partial P(.)}{\partial \widehat{b}} = \frac{f^2(.)-F(.)f'(.)}{2f^2(.)-F(.)f'(.)} > 0$ (guaranteed by the monotone hazard rate property). Hence if buyers take ratings at face value the certifier underreports the quality $\widehat{b}(b) < b$ for any $b > 0$, substituting for $D(\widehat{b})$ and $P(\widehat{b})$ we get $\widehat{b}(b) = b - \frac{F(.)}{f(.)}\left(1 - \frac{F(.)f'(.)}{f^2(.)}\right) < b$ QED.

This result stems from the fact that the price set by the seller contains a mark-up which causes a loss of buyers' welfare. Buyers' holding unreasonably low quality expectation lowers buyers' aspirations to buy and, as a result, lowers the equilibrium price. For moderate quality understatements the price reduction effect exceeds the loss due to some buyers wrongly abstaining from the purchase. Therefore the buyer-protective certifier underreports quality. It turns out that buyers' surplus would be higher if the buyers were to hold these downward biased expectations.[12]

Remark 2. If the certifier were to internalize profits $U(b,\widehat{b}) = \lambda S(b,\widehat{b}) + \eta\pi(\widehat{b}) + t$ he would underreport only if $\eta < \eta^*$, for some $\eta^* > 0$.

[12]It implies that it might be beneficial for the buyers to be naive and not to use Bayesian updating.

In this case the certifier would set $\widehat{b} = b - \frac{F(.)}{f(.)}\left(1 - \frac{F(.)f'(.)}{f^2(.)}\right) + \frac{\eta}{\lambda}\frac{F(.)}{f(.)}\left(2 - \frac{F(.)f'(.)}{f^2(.)}\right)$ which is smaller than b if $\eta < \eta^* = \lambda\frac{1-F(.)f'(.)/f^2(.)}{2-F(.)f'(.)/f^2(.)}$. Thus our analysis can be easily extended to the cases with $\eta \in (0, \eta^*)$.

While naive buyers make the certifier understate the quality, we proceed by requiring buyers to be Bayesian, that is ratings must verify $\widehat{b} = E(b|\widehat{b})$. By lemma 1 in a pure strategy equilibrium the certifier's reports are given by a monotone function $\widehat{b}(b)$. Thus we say that reports $\widehat{b}(b)$ are incentive compatible if they constitute the certifier's equilibrium strategy given $\widehat{B}$ and t.

$$\widehat{b}(b) \in \underset{\widehat{b}\in\widehat{B}}{argmax}\left[\lambda S(b, \widehat{b}) + t\right], \forall b \in [0, 1] \tag{1}$$

Corollary 1. *If buyers are Bayesian then full revelation $\widehat{b} = b$ is not incentive compatible with noncontingent contracts.*

Suppose Bayesian buyers believe $\widehat{b} = b$ and expect $E(b|\widehat{b}) = \widehat{b}$ then by proposition 2 the certifier would underreport quality $\widehat{b}(b) < b$, a contradiction. It follows that only imprecise public ratings are feasible. The publicity of ratings is crucial for the result of proposition 2. As in Farrell-Gibbons (1989), the private communication with one of the audiences (buyers) might be more informative than the public one.[13]

Every equilibrium induced by the rating set $\widehat{B}$ and the transfer t is unambiguously characterized by the certifier's equilibrium reporting strategy $\widehat{b}(b)$ and transfer t. As we mentioned before we let the seller choose the equilibrium she prefers in case of multiplicity, which means that we let her choose her preferred reporting $\widehat{b}(b)$ among those that are incentive compatible. Besides incentive compatibility the seller must satisfy the limited liability constraint $t \geq 0$ and Bayesian updating or rational expectations constraint $\widehat{b} = E(b|\widehat{b}(b) = \widehat{b})$, her maximization problem becomes:

Problem 2. $\underset{\{\widehat{b}(.),t\}}{max} \int_0^1 \pi(\widehat{b}(b))db - t$

s.t

IC: $\widehat{b}(b) \in \underset{\widehat{b}\in\widehat{B}}{argmax}\left[\lambda S(b, \widehat{b}) + t\right], \forall b \in [0, 1]$;

LL: $t \geq 0$;

RE: $\widehat{b} = E(b|\widehat{b}(b) = \widehat{b}), \forall\widehat{b} \in \widehat{B}$.

For constant t the set of incentive compatible reporting strategies that uphold rational expectations is the set of reporting strategies supported as equilibria of the communication

[13] When communication is private the certifier's report does not affect the price set by the seller $\frac{\partial P}{\partial \widehat{b}} = 0$ which implies $\frac{\partial S(b,\widehat{b})}{\partial \widehat{b}}|_{\widehat{b}=b} = 0$ and makes reportings $\widehat{b}(b) = b$ incentive compatible (for details see the proof of proposition 2).

game. It turns out that for any $F(x)$ satisfying monotone hazard rate property seller's profit is a convex function of $\widehat{b}$.

Proposition 3. *Seller's profit given the rating $\widehat{b}$ is a convex function of $\widehat{b}$.*

Proof. Seller's profit conditional on receiving rating $\widehat{b}$ is $\pi(\widehat{b}) = P(\widehat{b})F(\widehat{b} - P(\widehat{b}))$, by the envelope theorem we obtain $\pi'(\widehat{b}) = P(\widehat{b})f(\widehat{b} - P(\widehat{b}))$. Substituting $P(\widehat{b}) = F(\widehat{b} - P(\widehat{b}))/f(\widehat{b} - P(\widehat{b}))$ we get $\pi'(\widehat{b}) = F(\widehat{b} - P(\widehat{b}))$. Monotone hazard rate property ($F(.)f'(.) < f(.)^2$) implies $\frac{\partial P(\widehat{b})}{\partial \widehat{b}} = \frac{f(.)^2 - F(.)f'(.)}{2f(.)^2 - F(.)f'(.)} \in (0,1)$, therefore $\pi'(\widehat{b})$ is increasing in $\widehat{b}$ and $\pi(\widehat{b})$ is convex QED.

It follows that the seller is information loving and prefers the most informative equilibrium. The seller chooses her preferred equilibrium by defining the rating set $\widehat{B}$. From now on we make an additional assumption to simplify the demand function and obtain an explicit solution.

Assumption 4. $F(x) = x^{\alpha}$, $\alpha > 0$

Given the rating $\widehat{b}$ the demand is $D(\widehat{b}, P) = (\widehat{b} - P)^{\alpha}$ and the price is $P(\widehat{b}) = \widehat{b}/(1+\alpha)$.

Proposition 4. *An optimal noncontingent contract $(\widehat{B}, t)$ has $t = 0$ and results in a partition equilibrium $\{b_i\}_{i=0,...,N(\alpha)}$ with ratings $\widehat{b}_i = (b_{i-1} + b_i)/2$, $i = 1, ..., N(\alpha)$; high ratings are more precise than low ratings : $b_i - b_{i-1} < b_{i-1} - b_{i-2}$.*

The detailed proof can be found in the appendix. It is clear that $t = 0$ is optimal. Second, observe that α, which measures the demand elasticity, determines the mark-up (equal to the price $P = \widehat{b}/(1+\alpha)$ since costs are zero). When demand is inelastic ($\alpha < \alpha^* \approx 1.34$) the mark-up is high and the certifier fails to communicate product quality because he is reluctant to issue high ratings that result in high prices. In this case an optimal contract commands a single rating $\widehat{B} = \{1/2\}$ which results in a babbling equilibrium with reporting $\widehat{b}(b) = 1/2$ for $b \in [0; 1]$. The mark-up is less of a problem when demand is elastic ($\alpha \geq \alpha^*$) and the certifier can credibly issue high ratings. In this case $N(\alpha) \geq 2$ informative ratings can be sustained in equilibrium.[14] By proposition 3 the seller's expected profit is higher whenever $\widehat{b}(b)$ reveals more information, hence she implements a partition equilibrium with the maximum number of ratings possible. The remark below implies that the buyers and the certifier (if t is noncontingent) favor the equilibrium chosen by the seller.

[14]The mark-up charged by the seller plays a role similar to that of the sender's "bias" in the standard Crawford-Sobel model, where it was shown that in a cheap talk game the babbling equilibrium prevails when the divergence of interest between the sender and the receiver exceeds a threshold and, a partition equilibrium can emerge if the divergence is moderate.

Remark 3. The expectation of the buyers' surplus given the rating $\widehat{b}$ is a fraction of the seller's profit: $S_E(\widehat{b}) = \frac{\alpha}{1+\alpha}\pi(\widehat{b})$.

Proof. For a given $\widehat{b}$ we can express $\pi(\widehat{b}) = \frac{\gamma}{1+\alpha}\widehat{b}^{1+\alpha}$, $\gamma = \left(\frac{\alpha}{1+\alpha}\right)^{\alpha}$. The buyers' expected surplus conditional on rating $\widehat{b}$ is $S_E(\widehat{b}) = E[\int\limits_0^{\widehat{b}-P(\widehat{b})}(b - P(\widehat{b}) - x)dF(x) \mid \widehat{b}(b) = \widehat{b}]$. Bayes' rule $E\left[b|\widehat{b}(b) = \widehat{b}\right] = \widehat{b}$ implies $S_E(\widehat{b}) = \int\limits_0^{\widehat{b}-P(\widehat{b})} F(x)dx$. Substituting $F(x) = x^a$ and $P(\widehat{b}) = \widehat{b}/(1+\alpha)$ we obtain $S_E(\widehat{b}) = \frac{\alpha\gamma}{(1+\alpha)^2}\widehat{b}^{1+\alpha} = \frac{\alpha}{1+\alpha}\pi(\widehat{b})$ QED.

This convenient property stems from the assumption $F(x) = x^{\alpha}$. For more general demand functions the buyers and the certifier favor the revelation of information if the following condition holds $f'' \geq \frac{Ff'(2Ff'-f^2)-f^4}{F^2 f}$, otherwise a conflict between the information loving seller and the information averse certifier (and the buyers) arises.

Practices of many real certifiers seem to comply with the finding of proposition 4. Indeed the certifiers that charge fixed fees often issue few coarse ratings. For instance the Food and Drug Administration and many Standard Setting Organizations set standard application fees and issue pass/fail ratings. Similarly certifiers of consumer goods such as the European New Car Assessment Programme or the Michelin Guide typically charge fixed fees and evaluate products on five- and three-star scales, respectively. This is in stark contrast to the business model of credit rating agencies and investment banks whose fees are explicitly linked to the volume of the deal. Before moving to the analysis of contingent fees we illustrate the intuition behind proposition 4 with a two quality model.

Example with two equally likely quality levels ($b \in \{b_L, b_H\}$) and uniform distribution ($F(x) = x$).

In this case the seller chooses between the babbling equilibrium $\widehat{B}_b = \{\frac{b_L+b_H}{2}\}$ and an informative equilibrium with two ratings $\widehat{B}_i = \{\widehat{b}_L, \widehat{b}_H\}$ if the latter exists.[15] In a babbling equilibrium the only message available to certifier is $\frac{b_L+b_H}{2}$ which he sends for any quality level $\widehat{b}(b) = \frac{b_L+b_H}{2}$, $\forall b$. Consider now a potential informative equilibrium with $\widehat{B}_i = \{\widehat{b}_L, \widehat{b}_H\}$. Due to certifier's bias we expect him to mix between $\widehat{b}_L$ and $\widehat{b}_H$ when the quality is high (b_H) and to report $\widehat{b}_L$ when the quality is low (b_L). We denote by p the probability of reporting $\widehat{b}_L$ when actual quality is b_H. Certifier's incentive compatibility for b_H writes $\frac{\lambda}{2}(b_H - \widehat{b}_L)\widehat{b}_L + \frac{\lambda}{8}\widehat{b}_L^2 \leq \frac{\lambda}{2}(b_H - \widehat{b}_H)\widehat{b}_H + \frac{\lambda}{8}\widehat{b}_H^2$. Bayesian updating delivers $\widehat{b}_H = b_H$ (only the high type sends this message) and $\widehat{b}_L = \frac{b_L+pb_H}{1+p}$. Substituting for $\widehat{b}_H$ in (IC) we get $\widehat{b}_L \leq b_H/3$, hence the high type could play mixed strategy $p \in (0,1)$ if $\widehat{b}_L = b_H/3$, prefers high rating $p = 0$ if $\widehat{b}_L < b_H/3$ and prefers low rating $p = 1$ if $\widehat{b}_L > b_H/3$. Suppose $b_L > b_H/3$ then $\widehat{b}_L = \frac{b_L+pb_H}{1+p} > b_H/3$ and the babbling equilibrium occurs, that is $p_b^* = 1$

[15]Here we allow for partial revelation hence we can have $\{\widehat{b}_L, \widehat{b}_H\} \neq \{b_L, b_H\}$.

and $\widehat{b}_L = \frac{b_L+b_H}{2}$. On the other hand if $b_L \leq b_H/3$ we can have fully informative equilibrium were $p_i^* = 0$, $\widehat{b}_L = b_L$ and $\widehat{b}_H = b_H$. Moreover, for $b_L < b_H/3$, a "mixed" equilibrium is feasible, indeed $\widehat{b}_L = b_H/3$, $\widehat{b}_H = b_H$ and $p_m^* = \frac{b_H-3b_L}{2b_H}$ satisfy all equilibrium conditions. Now let's turn to the seller's choice of equilibrium. First, it is clear that for $b_L > b_H/3$ the babbling equilibrium is unavoidable and the seller gets expected profit $\pi_b = (b_L+b_H)^2/16$. Second, for a given rating $\widehat{b}$, the seller's profit is a convex function $\pi(\widehat{b}) = \widehat{b}^2/4$, therefore she prefers as much information to be revealed as possible. As a result for $b_L \leq b_H/3$ she always picks the fully informative equilibrium that delivers expected profit $\pi_i = (b_L^2+b_H^2)/8 > \pi_b$.

The example illustrates that the seller favors information production by the certifier. However, when qualities are close, $b_L > b_H/3$, and contingent transfers are not allowed the certifier cannot restrain from issuing a low rating, which in turn causes the equilibrium to be uninformative. In the next section we show how the certifier's fear of high ratings can be alleviated if the seller promises to compensate him by monetary transfers. It turns out that the certifier being paid for issuing high ratings restores the credibility of these ratings and resurrects informative equilibria.

2.2 Contingent contract

Now we solve for an optimal contract assuming that the seller can commit to a transfer schedule that specifies a nonnegative payment to the certifier $t(\widehat{b})$ for every rating $\widehat{b} \in \widehat{B}$ he issues. As before any equilibrium induced by the contract $(\widehat{B}, t(.))$ is unambiguously characterized by equilibrium reports $\widehat{b}(b)$ and payments $t(\widehat{b})$. We formulate a new incentive compatibility constraint for the certifier's reports to be implementable.

$$\widehat{b}(b) \in \arg\max_{\widehat{b}\in\widehat{B}} \left[\lambda S(b,\widehat{b}) + t(\widehat{b})\right], \forall b \in [0,1] \tag{2}$$

Using the incentive compatibility, limited liability and Bayesian updating constraints we write down the seller's maximization problem:

Problem 3. $\underset{\{\widehat{b}(.),t(.)\}}{max} \int_0^1 \left[\pi(\widehat{b}(b)) - t(\widehat{b}(b))\right] db$

s.t

IC: $\widehat{b}(b) \in \underset{\widehat{b}\in\widehat{B}}{argmax} \left[\lambda S(b,\widehat{b}) + t(\widehat{b})\right]$, $\forall b \in [0,1]$;

LL: $t(\widehat{b}) \geq 0$;

RE: $\widehat{b} = E(b|\widehat{b}(b) = \widehat{b})$, $\forall \widehat{b} \in \widehat{B}$.

To characterize the optimal contract we extend the insights from Krishna-Morgan (2008) to our environment.

Lemma 2. *An optimal contract results in full revelation for high qualities and pooling over intervals for low qualities:* $\widehat{b}(b) = b$ *for quality interval* $[b^*; 1]$ *and on the interval* $[0; b^*)$ *qualities are pooled in intervals* $[b_{k-1}; b_k) \subseteq [0; b^*)$, $k = 1, ..., K$.

The proof can be found in the appendix. Intuitively the seller favors information production, but perfect revelation is costly because the certifier demands high compensation for revealing high quality. To economize on transfers the seller accepts some pooling for low qualities and lowers the transfers for low ratings, this in turn lowers the transfers necessary to induce the certifier to issue a high rating.

Proposition 5. *If* $\alpha \leq 1$ *then the optimal contract commands a single rating with zero transfer for low qualities* $b < b^*(\alpha, \lambda)$ *and perfectly revealing ratings with positive transfers for high qualities* $b \geq b^*(\alpha, \lambda)$; $b^*(\alpha, \lambda) = min\left[\frac{(1+\alpha)^2}{(1+\alpha)/\lambda + 1 + 3\alpha + \alpha^2} \frac{2^{1+\alpha} - 2 - \alpha + \frac{1}{1+\alpha}}{2^{1+\alpha} - 2 - \alpha}, 1\right]$.

Proof. Without loss of generality we can set $t_1 = 0$ and drop the limited liability constraint for the rest of the ratings. Consider the less constrained seller's problem using "instantaneous" profit adjusted for transfers $\pi_A(b) = \gamma\beta b^{1+\alpha} - \gamma\lambda b^\alpha$, $\beta = \frac{1}{1+\alpha} + \lambda\frac{1+3\alpha+\alpha^2}{(1+\alpha)^2}$ (this profit function incorporates (IC), for details see the proof of lemma 2).

Problem 4. $\max\limits_{\{b_i, t_i\}_{i=1,...,N}} \sum\limits_{i=1}^{N} \pi_A(\frac{b_i + b_{i-1}}{2})(b_i - b_{i-1}) + \int\limits_{b_N}^{1} \pi_A(b)db + \frac{\gamma\lambda\kappa}{2^{1+\alpha}} b_1^{1+\alpha} - t_1$

s.t.

IC: $t_{i+1} - t_i + \lambda S(b_i, \frac{b_i + b_{i+1}}{2}) - \lambda S(b_i, \frac{b_i + b_{i-1}}{2}) = 0$, $i = 1, ..., N$;

LL: $t_1 = 0$;

Given that $\alpha \leq 1$ the "instantaneous" profit adjusted for transfers $\pi_A(b)$ is a convex function for all $b \in [0, 1]$, indeed $\pi_A'' = \gamma\beta(1+\alpha)\alpha b^{\alpha-1} + \gamma\lambda\alpha(1-\alpha)b^{\alpha-2} \geq 0$. Suppose the solution has $N \geq 2$ coarse ratings, then a contract which induces perfect revelation over the interval $[b_{N-1}, b_N)$ would increase profit, a contradiction. Hence there is at most one interval of pooling with a border point b^*. Optimizing $(\gamma\beta\left(\frac{b^*}{2}\right)^{1+\alpha} - \gamma\lambda\left(\frac{b^*}{2}\right)^\alpha)b^* + \int\limits_{b^*}^{1}(\gamma\beta b^{1+\alpha} - \gamma\lambda b^\alpha)db + \frac{\gamma\lambda\kappa}{2^{1+\alpha}} {b^*}^{1+\alpha}$ over b^* we obtain $b^* = min\left[\frac{\lambda}{\beta}\frac{2^{1+\alpha} - 2 - \alpha + \frac{1}{1+\alpha}}{2^{1+\alpha} - 2 - \alpha}, 1\right] > 0$ for $\alpha > 0$, $\lambda > 0$.

The solution to the above problem delivers the optimal contract if it verifies limited liability constraint for all ratings. Given that $t_1 = 0$ for $b \in [0, b^*)$ and $t(\widehat{b})$ is strictly increasing for $b \in [b^*, 1]$ we only need to check $t(b^*) \geq 0$. Incentive compatibility gives $t(b^*) = \lambda S(b^*, \widehat{b}_1) - \lambda S(b^*, b^*) = \frac{\lambda}{2^{1+\alpha}} {b^*}^{1+\alpha}\left(1 - \frac{\alpha(2^{1+\alpha} - 1)}{(1+\alpha)^2}\right) \geq 0$, since for $\alpha \leq 1$ we have $\alpha(2^{1+\alpha} - 1) \leq \alpha(2 + \alpha + (1+\alpha)\alpha/2) \leq 1 + 2\alpha + \alpha^2$. Thus the solution involved delivers the optimal contract QED.

Numerical solutions for a wide range of parameters show that a property similar to the one stated in proposition 5 holds for $\alpha > 1$.

The interpretation of the bang-bang solution is that low quality applicants with $b < b^*(\alpha,\lambda)$ receive low imprecise quality assessments (rejections) and pay nothing, while applicants with $b \geq b^*(\alpha,\lambda)$ receive fully revealing ratings $\widehat{b} = b$ and pay increasingly high transfers for these ratings. The nature of the optimal contract is intuitive. In the absence of a limited liability constraint $t(.) \geq 0$, an optimal contract would lead to perfect revelation since both parties benefit from more information. However, with limited liability such a contract implies a very high expected payment to the certifier and is too expensive for the seller. Given that transfers increase additively $t_m = t_1 + f(\widehat{b}(.))$ the best way for the seller to economize on the expected transfers is to pay nothing for the lowest rating $t_1 = 0$, and increase the probability b_1 of this ratings being issued.

Theorem 1. *For $\alpha \leq 1$ the optimal contract leads to less information production when the certifier is more protective of the buyer: the threshold $b^*(\alpha,\lambda)$ weakly increases with λ.*

Proof. If $b^*(\alpha,\lambda) < 1$ then $b^*(\alpha,\lambda) \sim \frac{(1+\alpha)^2}{(1+\alpha)/\lambda+1+3\alpha+\alpha^2}$ which is an increasing function of λ, otherwise $b^*(\alpha,\lambda) = 1$ and $\frac{\partial b^*(\alpha,\lambda)}{\partial\lambda} = 0$ QED.

When λ increases the certifier becomes tougher with the seller and demands higher compensation for precise ratings. Consequently the seller prefers a contract where a large set of qualities is associated with the cheap imprecise rating.

Example with $b \in \{b_L, b_H\}$ and $F(x) = x$.

From the previous section we know that if $b_L \leq b_H/3$ perfect revelation prevails without any transfers, so the seller can't do better with transfers. Suppose $b_L > b_H/3$ and consider as before an informative equilibrium $\widehat{B}_i = \{\widehat{b}_L, \widehat{b}_H\}$ with certifier issuing rating $\widehat{b}_L$ with probability p when actual quality is b_H. (IC) for b_H writes $\frac{\lambda}{2}(b_H - \widehat{b}_L)\widehat{b}_L + \frac{\lambda}{8}\widehat{b}_L^2 + t_L \leq \frac{\lambda}{2}(b_H - \widehat{b}_H)\widehat{b}_H + \frac{\lambda}{8}\widehat{b}_H^2 + t_H$. As long as the certifier is downward biased, (IC) for the low type has no bite and it is optimal to set $t_L = 0$. Bayes' rule states $\widehat{b}_H = b_H$, $\widehat{b}_L = \frac{b_L+pb_H}{1+p}$ and rewriting (IC) for the high type we get $t_H = \frac{\lambda}{8}(b_H - \widehat{b}_L)(3\widehat{b}_L - b_H)$. Given that p unambiguously determines equilibrium and we let the seller choose an equilibrium, we can write her maximization problem $\max\limits_{p\in[0,1]} \pi_i(p) = \frac{1+p}{2}\frac{\widehat{b}_L^2}{4} + \frac{1-p}{2}(\frac{\widehat{b}_H^2}{4} - t_H)$, s.t. IC and Bayes' rule. Substituting for t_H and p we reformulate the problem $\max\limits_{\widehat{b}_L\in[b_L,(b_L+b_H)/2]} \pi_i(\widehat{b}_L) = \frac{1}{8}(\widehat{b}_L^2 + b_H^2 - (b_H + \widehat{b}_L)(\widehat{b}_L - b_L) - \frac{\lambda}{2}(b_H + b_L - 2\widehat{b}_L)(3\widehat{b}_L - b_H))$. We obtain FOC $\frac{\partial\pi_i}{\partial\widehat{b}_L} = \frac{1}{16}\left(2b_L - 2b_H + \lambda(12\widehat{b}_L - 5b_H - 3b_L)\right) \gtrless 0$ and SOC $\frac{\partial^2\pi_i}{\partial\widehat{b}_L^2} = \frac{3\lambda}{4} > 0$, hence the solution is bang-bang. The seller prefers the informative equilibrium $\widehat{b}_L = b_L$ over the babbling one $\widehat{b}_L = \frac{b_L+b_H}{2}$ when $\pi_i = \frac{1}{2}\frac{b_L^2}{4} + \frac{1}{2}\left(\frac{b_H^2}{4} - \frac{\lambda}{8}(b_H - b_L)(3b_L - b_H)\right) \geq \pi_b = \frac{1}{4}\left(\frac{b_L+b_H}{2}\right)^2$ which is equivalent to $b_L \leq \frac{1+\lambda}{1+3\lambda} b_H$. When contingent contracts are allowed the seller incentivizes the certifier to reveal all the information for $b_L \leq \frac{1+\lambda}{1+3\lambda} b_H$ at the same time noncontingent contracts lead to full revelation only if $b_L \leq b_H/3 < \frac{1+\lambda}{1+3\lambda} b_H$, $\forall \lambda > 0$, thus contingent contacts facilitate information production. The buyers' expected surplus is convex in $\widehat{b}_i$, therefore the buyers prefer the informative equilibrium $ES_i = \frac{1}{2}\frac{b_L^2}{8} + \frac{1}{2}\frac{b_H^2}{8} > EU_b = \frac{1}{8}\left(\frac{b_L+b_H}{2}\right)^2$ and, hence, they are better off with contingent contracts. The certifier also favors contingent contracts since in this case on top of the improved buyers' welfare he benefits from the positive payments. In other words we have illustrated that contingent public contracts facilitate information production and increase welfare of all parties.

In the next section we perform a robustness check allowing the initial contract to be negotiated by parties.

2.3 Contract negotiations

Suppose that the certifier and the seller negotiate the initial contract $\{\widehat{b}(.), t(.)\}$ before the certifier learns the quality (both parties are uninformed). The expected payoffs are $U_E = \lambda \int_0^1 S(b, \widehat{b})db + T_E$ for the certifier and $\pi_E = \int_0^1 \pi(\widehat{b}(b))db - T_E$ for the seller, here $T_E = \int_0^1 t(\widehat{b}(b))db$ is the expected payment. Ex ante the parties have congruent intrinsic interests $\int_0^1 S(b, \widehat{b})db = \frac{\alpha}{1+\alpha} \int_0^1 \pi(\widehat{b}(b))db$ (see remark 3), therefore the negotiation is trivial once the size of T_E has been decided. Indeed given the constraint $\int_0^1 t(\widehat{b}(b))db = T_E$ the same contract $\{\widehat{b}(.), t(.)\}_{T_E}$ maximizes both U_E and π_E. Thus to accommodate for negotiation it suffices to maximize the seller's profit (problem (3)) with an additional constraint on the expected payment $\int_0^1 t(\widehat{b}(b))db \geq T$, with parameter $T \geq 0$ being a measure of the certifier's bargaining power.

Problem 5. $\max\limits_{\{\widehat{b}(.), t(.)\}} \int_0^1 \left[\pi(\widehat{b}(b)) - t(\widehat{b}(b))\right] db$

s.t

IC: $\widehat{b}(b) \in \arg\max\limits_{\widehat{b} \in \widehat{B}} \left[\lambda S(b, \widehat{b}) + t(\widehat{b})\right]$, $\forall b \in [0, 1]$;

LL: $t(\widehat{b}) \geq 0$;

RE: $\widehat{b} = E(b|\widehat{b}(b) = \widehat{b})$, $\forall \widehat{b} \in \widehat{B}$;

Negotiation: $\int_0^1 t(\widehat{b}(b))db \geq T$.

The following theorem establishes a link between the optimal negotiated contract $\{\widehat{b}(.), t(.)\}^{**}$ and the contract unilaterally designed by the seller studied in the previous section $\{\widehat{b}(.), t(.)\}^*$.

Theorem 2. *For $\alpha \leq 1$ the optimal negotiated contract $\{\widehat{b}(.), t(.)\}^{**}$ corresponds to an optimal contract unilaterally proposed by the seller $\{\widehat{b}(.), t(.)\}^{*}$ to a certifier with $\lambda' \leq \lambda$: $\{\widehat{b}(b)\}^{**}_{\lambda} \equiv \{\widehat{b}(b)\}^{*}_{\lambda'}$.*

The proof can be found in the appendix. Intuitively the information revelation is costly and it is always the seller who pays for it. She purchases a lot of information either when she is compelled to do so because of certifier's high bargaining power, or when she voluntary decides to do so because the certifier is not very buyer-protective and it is inexpensive to compensate him. Indeed if the seller were to design the contract unilaterally and the certifier were less buyer-protective $\lambda' < \lambda$, the seller would induce more information revelation according to the theorem 1. Alternatively a certifier with a high bargaining power T requires some extra compensation and both parties agree to use this compensation to elicit more information since this augments their payoffs.

The results obtained in section 2 hinge on the assumption that contracts are public and so the buyers observe the incentives the certifier is facing. In section 3 we explain what happens if it is not the case.

3 Private contracts

In this section we assume that the transfer $t(.)$ proposed to the certifier is not observed by the buyers. In order to interpret the ratings properly the buyers must form rational expectations about the equilibrium contract $t^e(.)$. This restricts the set of contracts that can be supported in equilibrium because the contract must fulfill buyers' expectations ex post $t(.) = t^e(.)$. One can easily verify that for this environment the modified revelation principle holds and considering direct mechanisms without loss of generality.[16]

As an introduction for this section let us consider an example with two quality levels.

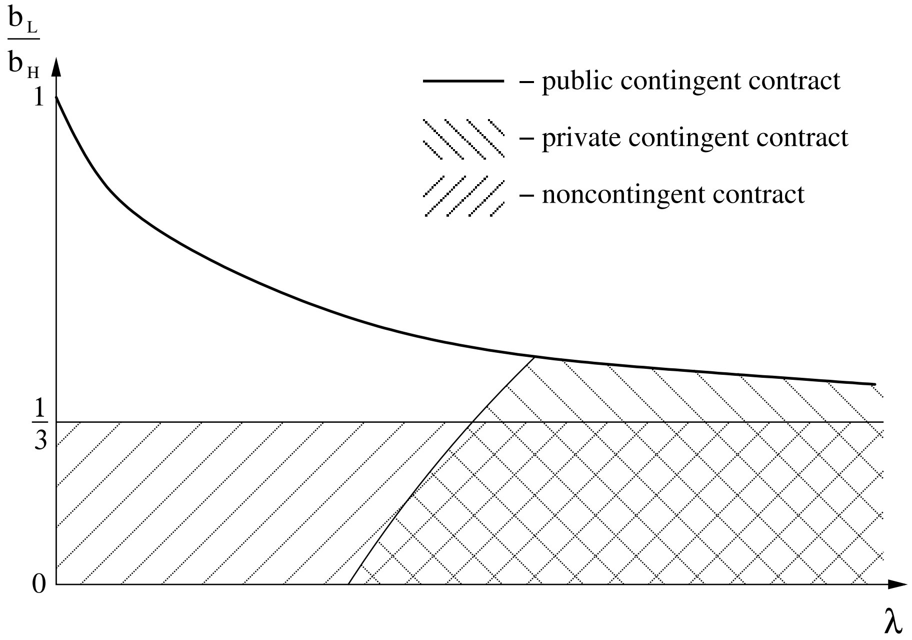
3.1 Example with $b \in \{b_L, b_H\}$ and $F(x) = x$.

Consider a fully informative equilibrium $\widehat{B}_i = \{b_L, b_H\}$, and let's check under what conditions it can occur.[17] Denote by t^e_H an equilibrium expectation of a transfer offered for the high rating. Since in the fully informative equilibrium Bayesian beliefs are $\widehat{b}_L = b_L$ and $\widehat{b}_H = b_H$ incentive compatibility constraints become $t^e_H \geq \frac{\lambda}{8}(b_H - b_L)(3b_L - b_H)$ for the high type and $t^e_H \leq \frac{\lambda}{8}(b_H - b_L)(3b_H - b_L)$ for the low type. The seller's expected profit is given by $\pi = \frac{1}{2}\frac{\widehat{b}_L^2}{4} + \frac{1}{2}(\frac{\widehat{b}_H^2}{4} - t_H)$. Observe that the seller's choice of t_H does not affect $\widehat{b}_L$ and

[16]Indeed, remark 1 is still valid: all the relevant information is embedded in the quality expectation $\widehat{b}$.
[17]Later on we show that partially informative equilibria are never implemented.

$\widehat{b}_H$ that depend only on t^e_H, hence in equilibrium it must be the case that the seller does not want to deviate and increase t_H in order to enhance the chances of getting the high rating. The high type already issues a high rating and we need to check the low type. If the certifier has observed b_L, he agrees to issue a high rating if $t'_H > \frac{\lambda}{8}(b_H - b_L)(3b_H - b_L)$; the seller would promise such a transfer if her profit would increase $t'_H < \frac{b_H^2}{4} - \frac{b_L^2}{4}$. Combining these inequalities we get a necessary condition for the fully informative equilibrium to be implementable with private contracts: $b_L \leq \frac{3\lambda-2}{2+\lambda} b_H$. As was shown in the previous section the seller prefers this equilibrium over the babbling one if $b_L < \frac{1+\lambda}{1+3\lambda} b_H$. In principle other equilibria are potentially feasible with the low (high) type mixing between the ratings, however these equilibria require the seller getting the same profit for any rating $\pi(\widehat{b}_H) = \frac{\widehat{b}_H^2}{4} - t_H = \pi(\widehat{b}_L) = \frac{\widehat{b}_L^2}{4}$, because otherwise she would increase (decrease) t_H to increase (decrease) the probability of getting the high rating. This in turn implies that in these equilibria the seller gets less in terms of expected profit than in the babbling equilibrium $\pi = \frac{1}{2}\pi(\widehat{b}_H) + \frac{1}{2}\pi(\widehat{b}_L) = \widehat{b}_L^2/4 < \pi_b = \frac{1}{4}\left(\frac{b_L+b_H}{2}\right)^2$ because $\widehat{b}_L = b_L < \frac{b_L+b_H}{2}$. Thus she never implements these equilibria.

Figure 2: Informative equilibrium under different contracts



With public contracts the informative equilibrium is implemented if $b_L < \frac{1+\lambda}{1+3\lambda} b_H$ (area below the bold line on figure 1), this is also the case with private contracts if $\frac{1+\lambda}{1+3\lambda} b_H \leq \frac{3\lambda-2}{2+\lambda} b_H \Leftrightarrow \lambda \geq \frac{3+\sqrt{41}}{8}$. However, if $\lambda < \frac{3+\sqrt{41}}{8}$ the informative equilibrium under private contacts is implemented less often $b_L \leq \frac{3\lambda-2}{2+\lambda} b_H < \frac{1+\lambda}{1+3\lambda} b_H$. This is so because a certifier with a low λ does not care about buyers that much and can be easily convinced to issue the high rating even when actual quality is low; buyers anticipate this and do not trust certifier's ratings, as a result the informative equilibrium ceases to exist. Figure 2 illustrates when the informative equilibrium is implemented for different types of contracts available depending on λ. On the vertical axis we have the ratio b_L/b_H and λ on the horizontal one.

3.2 General case

The seller's ability to induce a babbling equilibrium by walking away from the certifier is crucial for the subsequent analysis. This follows from our assumption that the seller is not informed when she applies for certification. If this is not the case, then a seller who didn't apply may be believed to be of a low quality as in Lizzeri (1999). In such an environment the seller can be worse off from having a certifier around.

We proceed with a general analysis of private contracts. The set of ratings $\widehat{B}$ is public. However the buyers do not observe the proposed $t(\widehat{b})$ directly but form expectations $t^e(\widehat{b})$ that ought to be fulfilled in equilibrium. Suppose buyers anticipate the equilibrium reporting strategy, which maps actual qualities $b \in [0,1]$ into the set of ratings $\widehat{B}$, to be $\hat{b}^e(b)$. In a perfect Bayesian equilibrium, it must be in the best interest of the seller to propose transfers $t(\widehat{b}) = t^e(\widehat{b})$ to the certifier that induce reportings $\widehat{b}(b) = \widehat{b}^e(b)$. In other words expectations $\widehat{b}^e(b)$ must be robust to possible hidden contract manipulations by the seller. Formally, let transfers $t^*(\widehat{b})$ and associated reportings $\widehat{b}^*(b)$ solve

Problem 6. $\max\limits_{\{\widehat{b}(.),t(.)\}} \; E\left[\pi(\widehat{b}(b)) - t(\widehat{b}(b))\right]$

s.t.

IC: $\widehat{b}(b) \in \underset{\widehat{b}\in\widehat{B}}{argmax}\left[\lambda S(b,\widehat{b}) + t(\widehat{b})\right], \forall b \in [0,1];$

LL: $t(\widehat{b}) \geq 0.$

In equilibrium, the solution to problem 6 must satisfy $(\widehat{b}^*(.), t^*(.)) = (\widehat{b}^e(.), t^e(.))$. Moreover the equilibrium reporting strategy $\widehat{b}^e(b)$ in itself must be consistent with rational expectations $\widehat{b} = E(b|\widehat{b}^e(b) = \widehat{b})$ for any $\widehat{b} \in \widehat{B}$.[18] We say that the pair $(\widehat{b}^e(b), t^e(\widehat{b}))$ satisfies the non-manipulability constraint (NM) if it solves problem 6, satisfies $(\widehat{b}^*(b), t^*(\widehat{b})) = (\widehat{b}^e(b), t^e(\widehat{b}))$ and $\widehat{b} = E(b|\widehat{b}^e(b) = \widehat{b})$ for any $\widehat{b} \in \widehat{B}$. We denote the set of all such pairs by $\widehat{\mathcal{B}}$, thus a reporting $\widehat{b}(b)$ respects the NM constraint iff $(\widehat{b}(b), t(\widehat{b})) \in \widehat{\mathcal{B}}$. The set $\widehat{\mathcal{B}}$ is non empty because the reporting $\widehat{b}(b) = 1/2$, $\forall b \in [0,1]$ and transfer $t = 0$ that lead to the uninformative equilibrium belong to it. Indeed the seller cannot benefit from bribing the certifier if ratings $\widehat{b} \neq 1/2$ generate expectations at most equal to 1/2.

[18] To see why the latter constraint is important suppose that the buyers expect $\widehat{b}^e(b) = 1$, $b \in [0,1]$ and the solution to problem 6 delivers $\widehat{b}^*(b) = 1$, $b \in [0,1]$. It is clear that such a reporting strategy is not compatible with rational expectations because $E(b|\widehat{b}^e(b) = 1) = E(b) < 1$.

Proposition 6. *The non-manipulability constraint prohibits intervals of full revelation if* $\lambda \neq \frac{1+\alpha}{1-\alpha-\alpha^2}$.

Proof. The non-manipulability constraint can be reformulated as

$\widehat{b}^e(b) \in \underset{\hat{b}(.)}{argmax} \left[\int_0^1 \left(\pi(\hat{b}) - (U(b) - \lambda S(b, \hat{b})) \right) db \right]$

s.t.

IC: $\widehat{b} \in \widehat{B}$; $\dot{U} = \lambda S_b(b, \hat{b})$;

LL: $U(b) \geq \lambda S(b, \hat{b})$.

Ratings respect rational expectations: $\widehat{b} = E(b|\widehat{b}^e(b) = \widehat{b})$, $\forall \widehat{b} \in \widehat{B}$. Here $U(b) = \underset{\widehat{b} \in \widehat{B}^e}{max}[\lambda S(b, \widehat{b}) + t(\widehat{b})]$ is the certifier's utility. Define the corresponding functional $J = \int_0^1 \left(\pi(\hat{b}) - (U(b) - \lambda S) + \phi(b)(U(b) - \lambda S) + p(b)(\lambda S_b - \dot{U}) \right) db$. Suppose buyers expect perfect revelation $\widehat{b}(b) = b$ for some interval $[\underline{b}, \overline{b}]$, then for this interval reports $\widehat{b} \in [\underline{b}, \overline{b}]$ must maximize the "instantaneous" profit $\pi(\hat{b}) - (U(b) - \lambda S) + \phi(b)(U(b) - \lambda S) + p(b)\lambda S_b$. From the (IC) $t(.)$ is strictly increasing over this interval, hence, $t(.) > 0$ and $\phi(b) = 0$ for any $b \in (\underline{b}, \overline{b})$. Deriving FOC and substituting $\pi(\widehat{b}) = \frac{\gamma}{1+\alpha}\widehat{b}^{1+\alpha}$, $S(b, \widehat{b}) = \gamma \widehat{b}^{\alpha}(b - \kappa \widehat{b})$ we get a necessary condition $b + \lambda(\alpha b - (1+\alpha)\kappa b) + \lambda \alpha p(b) = 0$ for $b \in (\underline{b}, \overline{b})$, which in turn requires $1 + \lambda(\alpha - \frac{1+\alpha+\alpha^2}{1+\alpha}) + \lambda \alpha \dot{p} = 0$. Given that the co-state variable has to satisfy $\dot{p} = -\partial H / \partial U = 1 - \phi(b)$ we get $\dot{p} = 1$ for $b \in (\underline{b}, \overline{b})$, hence the necessary condition is violated if $\lambda \neq \frac{1+\alpha}{1-\alpha-\alpha^2}$ QED.

If buyers were to expect perfect revelation for some qualities, then the seller would elicit inflated or deflated ratings depending on the demand elasticity and the buyer-protectiveness of the certifier.

Now consider a reporting strategy $\widehat{b}^e(b)$ that has no intervals of perfect revelation and satisfies RE: $\widehat{b} = E(b|\widehat{b}^e(b) = \widehat{b})$, $\forall \widehat{b} \in \widehat{B}$. Such a strategy is fully characterized by border points $\{b_i^e\}_{i=0,\ldots,N}$ and we obtain

Proposition 7. *If the certifier is lax* $\lambda < \frac{1+\alpha}{1+\alpha+\alpha^2}$ *the highest rating is issued in more than 50% of cases* ($b_{N-1}^e < 1/2$).

The proof is in the appendix. This observation is intuitive, the seller wants to increase the probability of getting the highest rating and offers a generous payment for it. A lax certifier puts little weight on the buyers' welfare, therefore he issues the highest rating for a wide range of qualities $(b_{N-1}, 1]$. In other words the highest rating is "inflated" which is in a sharp contrast with the public contracts case where we had perfect revelation on the top and pooling at the bottom.

The key difference with public contracts is that the reporting being implemented $\widehat{b}(b)$ has no direct effect on Bayesian updating $\widehat{b} = E(b|\widehat{b}^e(b) = \widehat{b})$. Instead the updating is done using the anticipated reporting $\widehat{b}^e(b)$. In case of public contracts the choice of $\widehat{b}(b)$ had an immediate effect on interpretation of expected qualities $\widehat{b} = E(b|\widehat{b}(b) = \widehat{b})$ and the seller had no possibility to "manipulate" the contract, hence there was no (NM) constraint. Taking into account the (NM) constraint, the seller's maximization problem becomes:

Problem 7. $\underset{(\widehat{b}(.),t(.))}{max}\ E\left[\pi(\widehat{b}(b)) - t(\widehat{b}(b))\right]$
s.t. NM: $(\widehat{b}(.), t(.)) \in \widehat{\mathcal{B}}$.

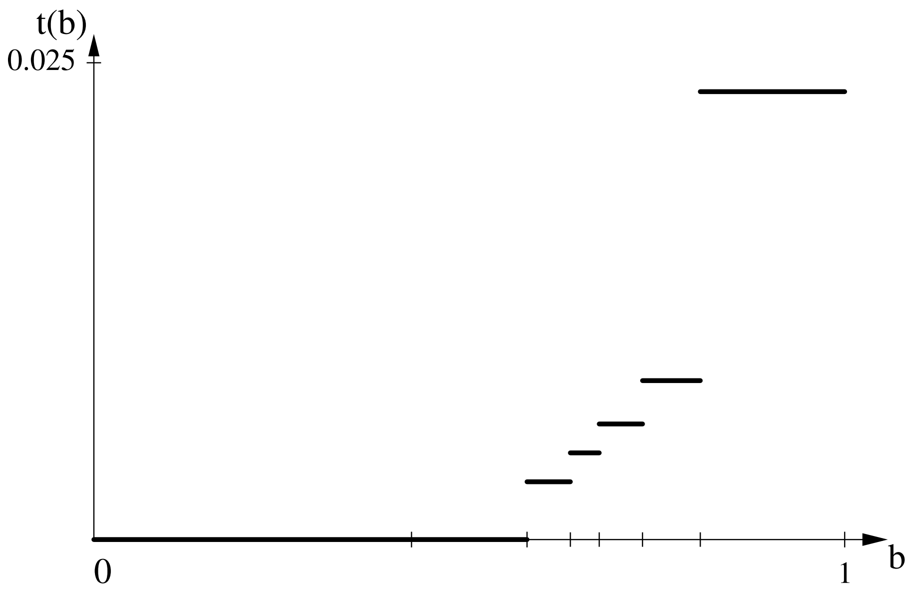
3.3 Numerical illustration for $\alpha \in [1.5, 3.5]$ and $\lambda \in [0.5, 2.5]$

Here we numerically solve for optimal private contract and obtain the following preliminary results.

Conjecture: In the optimal private contract the lowest and the highest ratings are the least "precise".

Figure 3 illustrates this result. Intuitively imprecise low ratings allow to economize on transfers as in the case of public contracts. The highest rating is imprecise because of "bribery". The seller wants the certifier to issue high ratings and is ready to pay for that. Paying the certifier more for intermediate ratings makes the certifier prefer intermediate ratings over both low and high ratings. Because of this competition effect from higher ratings the seller does not increase payments for intermediate ratings as much as she does for the highest rating. As a result the highest rating is issued too often and is imprecise.

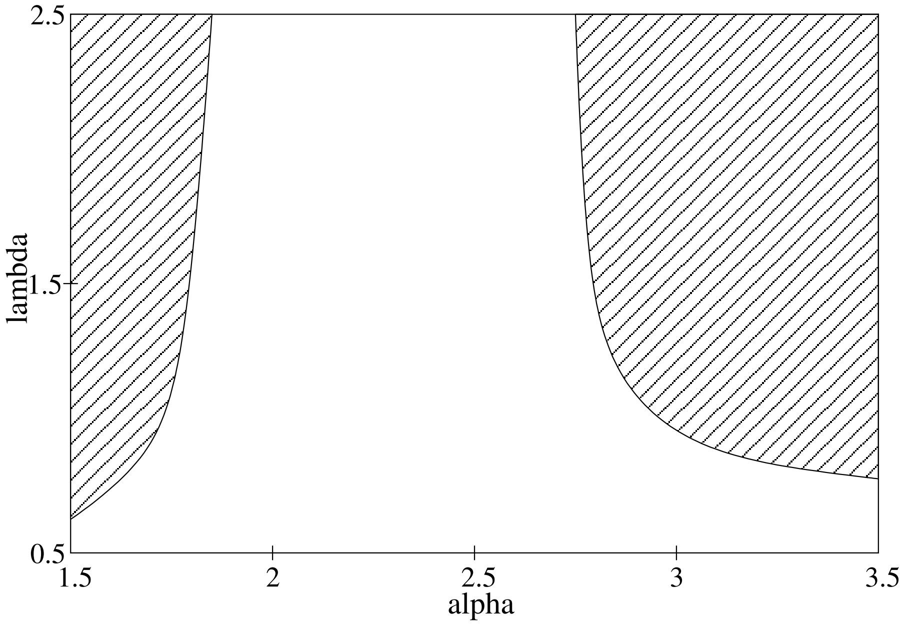
Figure 3: Optimal private contract for $\alpha = 2$, $\lambda = 1$.



Conjecture: The more buyer-protective is the certifier (higher λ) the more attractive is *the private contract for the seller compared to the noncontingent contract.*

We compare the seller's expected profit under the private and under the noncontingent contract. We find that private contract delivers higher profit than the noncontingent one when the certifier is tough (λ is high) and when demand is either inelastic (α is low) or very elastic (α is high); this corresponds to the striped area on figure 4.

Figure 4: Relative profit under private and noncontingent contract.



The intuition is the following. Once the seller has an option to make side payments to the certifier it is profitable for him to do so. However, buyers anticipate this and reinterpret the ratings accordingly. Depending on the parameters, the seller's profit net of transfers to the certifier can be higher or lower under the private contract than the profit under the noncontingent contract. Private contracts are preferred when the certifier is tough (high λ) which moderates the seller's desire to bribe the certifier.

4 Welfare analysis

The social welfare computed as a sum of the expected buyers' surplus, seller's profit and certifier's payoff is directly linked to the amount of information produced in equilibrium. This is so because all parties in our framework are information loving as follows from proposition 3 and remark 3. For a given reporting rule $\widehat{b}(.)$ the expected social welfare is $W(\widehat{b}(.)) = \int_0^1 \left[S(\widehat{b}(x)) + \pi(\widehat{b}(x)) + \lambda S(\widehat{b}(x))\right] dx = (1 + \frac{(1+\lambda)\alpha}{1+\alpha}) \int_0^1 \pi(\widehat{b}(x))dx$ and we immediately obtain

Proposition 8. *The utilitarian social planner is information loving.*

This proposition has several implications.

Corollary 2. *For $\alpha \leq 1$ in case of public contracts the social welfare increases when 1) the certifier becomes less buyer protective (λ decreases), 2) parties negotiate the contract and the certifier has high bargaining power.*

Proof. From theorems 1 and 2 follows that low λ and high bargaining power of the certifier improve information revelation, hence by proposition 8 welfare increases QED.

Corollary 3. *Public contingent contracts dominate noncontingent contracts from the social welfare perspective.*

Proof. The noncontingent contract is a special case of a public contingent contract. If an information loving seller prefers the contingent contract over the noncontingent one even though the former requires positive payment from her, then the expected social welfare, which is proportional to the gross profit of the seller, is the highest under the public contingent contract QED.

Numerical solutions and the example with two quality levels indicate that the private contract leads to a lower welfare than the public contract. Yet one might think of circumstances when the private contract outperforms the public one, for instance when the certifier is very buyer-protective (highλ) the seller prefers not to pay for any information under the public contract but might not resist the temptation to get a high rating under the private contract. Unfortunately we are not able to resolve this question decisively due to our limited knowledge of the outcomes under the private contract. Relative performance of the private contract and the noncontingent contract is also ambiguous. If the certifier is sufficiently tough with the seller (high λ) then the private contract might outperform the noncontingent contract. In the striped area on figure 4 the seller's net profit for the private contract is higher than for the noncontingent contract, social welfare is proportional to the gross seller's profit and, hence, is also higher for the private contract. In the example with two qualities this case corresponds to the striped area above the horizontal line on figure 2. If, on the other hand, the certifier is lax (low λ) the noncontingent contract might be preferred, this corresponds to the striped area on the left below the horizontal line on figure 2.

Corollary 4. *If the social planner with the utilitarian objective function $W(\widehat{b}(.))$ were to choose the contract and faced no shadow cost of public funds, she would induce full information revelation $\widehat{b}(b) = b$.*

Proof. Indeed transfers paid to the certifier play a purely redistributive role and do not affect total welfare, at the same time $W(\widehat{b}(.))$ is maximized when $\widehat{b}(b)$ is fully revealing QED.

In the general case, the optimal contract proposed by the seller does not lead to the full revelation and the information is under-provided compared with the first best. This is so because the seller bears all the costs of information production but does not internalize a fraction of the surplus which accrues to the buyers. One way to get closer to the first best is to ask buyers to pay the certifier for his services, which would require buyers to commit to contingent payments. Given the free rider problem and lack of commitment on the buyers' side such a scheme is unlikely to succeed. An alternative regulation which might appear controversial at a first glance calls for subsidizing the seller's certification expenses. Indeed if contracts are public and the seller receives a subsidy for each dollar spent on certification, she induces more information production in equilibrium and social welfare improves.

5 Extensions

Thus far we assumed that the certifier is monopolistic and learns the product's quality at no cost. In this section we relax these assumptions. First, we let the seller choose among multiple certifiers, then we study the moral hazard on the part of the certifier.

5.1 Forum shopping

Suppose there is a continuum of certifiers with different degrees of buyer-protectiveness $\lambda \in [0, \infty)$ that are known to all parties. Now the seller can choose the certifier she prefers prior to signing the contract. After the seller has picked the certifier the game proceeds as before. We assume that each certifier issues public ratings observed by all potential buyers. Consider the noncontingent contract first.

Proposition 9. *If payments cannot be conditioned on ratings the seller is indifferent among certifiers.*

Proposition 9 follows directly from proposition 4. If transfers are not contingent the seller cannot affect the certifier's incentives and she pays zero for all ratings. When transfers are zero certifier's ability to reveal information is determined by his intrinsic bias. This bias is pinned down by the monopolistic mark-up $P = b/(1+\alpha)$ which depends on the elasticity parameter α only and, hence, is the same for all certifiers independently of λ. As a result the certifier's reportings and the seller's expected payoff are independent of the certifier's buyer-protectiveness λ. Put differently, from the seller's point of view it does not matter how much the certifier values buyers' surplus compared to transfers, if the transfers are required not to intervene with the certifier's reporting incentives.

Proposition 10. *Under a public contingent contract the seller hires the least buyer-protective certifier ($\lambda_{PB} = 0$).*

Proof. Consider two certifiers $\lambda_1 < \lambda_2$, suppose the seller contracts with the more buyer-protective certifier and designs an optimal contingent contract $\{\widehat{b}(.), t(.)\}^*$. Then the seller can get higher profits by approaching the less buyer-protective certifier with a contract that specifies $\widehat{b}(.) = \widehat{b}(.)^*$ and $t(.) = \frac{\lambda_1}{\lambda_2} t(.)^*$. Since $\{\widehat{b}(.), t(.)\}^*$ satisfies incentive compatibility for λ_2 certifier $\widehat{b}(b) \in \underset{\widehat{b} \in \widehat{B}}{\arg\max} \left[\lambda_2 S(b, \widehat{b}) + t(\widehat{b})^*\right]$, $\forall b \in [0, 1]$ the modified contract satisfies incentive compatibility for λ_1 certifier $\widehat{b}(b) \in \underset{\widehat{b} \in \widehat{B}}{\arg\max} \left[\lambda_1 S(b, \widehat{b}) + \frac{\lambda_1}{\lambda_2} t(\widehat{b})^*\right]$, $\forall b \in [0, 1]$. But the modified contract requires lower expected payment from the seller $\frac{\lambda_1}{\lambda_2} \int_0^1 t(\widehat{b}(b))db \leq \int_0^1 t(\widehat{b}(b))db$, therefore the seller favors the certifier who is less buyer-protective. For small but positive λ_2 expected payment $\int_0^1 t(\widehat{b}(b))db$ is positive and the inequality is strict, thus the seller strictly prefers to contract with a certifier with $\lambda = 0$ QED.

Intuitively for the seller it is cheap to induce information revelation when the certifier cares little about the buyers. In this case the certifier's intrinsic bias due to monopoly pricing can be easily compensated by small payments. As a result in the absence of other concerns, the seller picks the certifier who cares about the buyers the least. Expectedly, this reasoning is not valid when the contract is private.

Lemma 3. *If the contract is private a certifier with $\lambda = 0$ cannot make any credible quality announcement except the prior, the babbling equilibrium prevails.*

Proof. Suppose the certifier issues several ratings $\widehat{b}_i$, $i = 1, ..., N$. Given that he only cares about the payment we must have $t(\widehat{b}_i) = t$ for $i = 1, ..., N$. Pick the rating with the highest net profit to the seller $\widehat{b}_{i^*} = \underset{\widehat{b}_i}{argmax}[\pi(\widehat{b}_i) - t]$. If $\pi(\widehat{b}_{i^*}) > \pi(\widehat{b}_j)$, for some j then increasing $t(\widehat{b}_{i^*})$ by an infinitely small amount delivers $\pi(\widehat{b}_{i^*}) - t$ for sure to the seller, a profitable deviation. Therefore $\pi(\widehat{b}_{i^*}) = \pi(\widehat{b}_i)$ for $i = 1, ..., N$ which implies $\widehat{b}_i = \widehat{b}_j$, for any i, j. But then $E(b|\widehat{b}_i) = 1/2$, $i = 1, ..., N$, that is all ratings are uninformative QED.

Corollary 5. *In case of private contracts the seller either contracts with a certifier who is buyer-protective ($\lambda_{PR} > 0$) or does not hire a certifier at all.*

This result follows immediately from the lemma. A lax certifier ($\lambda = 0$) produces no information to the market, hence the seller may as well sell her product unrated. A buyer-protective certifier ($\lambda > 0$), who is able to issue credible ratings can be of positive value for the seller as the analysis below illustrates.

Example with $b \in \{b_L, b_H\}$ and $F(x) = x$.

In section 3.1 it was shown that if $b_L > \frac{3\lambda - 2}{2+\lambda} b_H$ the bubbling equilibrium prevails, hence certifiers with $\lambda < \frac{2(b_H + b_L)}{3b_H - b_L}$ are of no interest to the seller and her expected profit is $\pi_b = \frac{1}{4}\left(\frac{b_L + b_H}{2}\right)^2$. On the other hand if the seller picks a certifier with $\lambda \geq \frac{2(b_H+b_L)}{3b_H - b_L} > 0$ she can implement the informative equilibrium. From the previous analysis we know that whenever $b_L \leq b_H/3$ the informative equilibrium is implemented without payments and expected profit is $\pi_i = \frac{1}{2}\frac{b_L^2 + b_H^2}{4} > \pi_b$, hence the seller is happy with any $\lambda_{PR} \geq \frac{2(b_H+b_L)}{3b_H - b_L}$. When $b_L > b_H/3$ the seller promises a transfer $t_H = \frac{\lambda}{8}(b_H - b_L)(3b_L - b_H)$ and gets expected profit $\pi_i = \frac{1}{2}\frac{b_L^2+b_H^2}{4} - \frac{1}{2}\left(\frac{\lambda}{8}(b_H - b_L)(3b_L - b_H)\right)$ which decreases with λ and is higher than π_b only if $\lambda \leq \frac{b_H - b_L}{3b_L - b_H}$. Consequently the seller prefers to contract with a certifier whose buyer-protectiveness is just enough to implement the informative equilibrium $\lambda_{PR} = \frac{2(b_H+b_L)}{3b_H - b_L}$ if this equilibrium delivers higher profit than the babbling one $\lambda_{PR} \leq \frac{b_H - b_L}{3b_L - b_H} \Leftrightarrow b_L \leq \frac{10}{7+\sqrt{149}} b_H$. Conversely when $b_L > \frac{10}{7+\sqrt{149}} b_H$ the highest profit the seller can attain with any certifier is the bubbling equilibrium profit, hence, she does not hire a certifier at all (one might think that there is a small cost of applying for certification). To sum up, the seller's preferences over certifiers are as follows: $\lambda_{PR} \in [\frac{2(b_H+b_L)}{3b_H - b_L}, \infty)$ if $b_L \in [0, \frac{1}{3}b_H]$; $\lambda_{PR} = \frac{2(b_H+b_L)}{3b_H - b_L}$, if $b_L \in (0, \frac{10}{7+\sqrt{149}} b_H]$; $\lambda_{PR} = \emptyset$ if $b_L \in (\frac{10}{7+\sqrt{149}} b_H, b_H)$.

If the certifier cares little about the buyers and the contract is private, the seller can not resist the temptation to offer very generous payment to the certifier, hoping to receive a high rating with high probability. However, with rational buyers this temptation leads to imprecise ratings and the seller gets a low expected profit from dealing with a certifier who is not tough enough. In order to commit not to offer generous payments for high ratings the seller contracts with a buyer-protective certifier $\lambda_{PR} > 0$. However, the tougher the certifier the more expensive it is to compensate him for his intrinsic bias (see the proof of previous proposition). It might happen that the certifier who is tough enough to reduce the seller's temptation to inflate ratings is so expensive to compensate that the seller prefers to sell the product unrated. In this case no certifier with $\lambda \in [0, \infty)$ is adequate to the seller's needs.

5.2 Moral hazard

Here we take our simplified set-up with $b \in \{b_L, b_H\}$, $F(x) = x$ and consider public contingent contracts. Throughout the analysis we assumed that the certifier learns the actual quality of the product b at no cost. Suppose now that after the certifier and the seller have signed the contract the certifier decides whether to acquire a perfect signal about the actual quality $b \in \{b_L, b_H\}$ at a cost $c > 0$, or shirk and remain uninformed. We restrict the analysis to public contingent contracts.

Proposition 11. *In the presence of moral hazard the optimal contract induces the informative equilibrium and commands* $t_H = t_H^*$, $t_L = 0$ *if* $t_H^* \leq \frac{\lambda}{8}(b_H^2 - b_L^2)$ *and* $t_H^* < \frac{1}{16}(b_H - b_L)^2$ *, and induces the babbling equilibrium with* $t_H = t_L = 0$ *otherwise, here* $t_H^* = max[2c + \frac{\lambda}{8}(b_H - b_L)(3b_L - b_H), 0]$.

Proof. Consider the informative equilibrium where the certifier acquires the signal and reports truthfully. For reports to be truthful the transfer for the high rating must satisfy (IC) $\frac{\lambda}{8}(b_H - b_L)(3b_L - b_H) \leq t_H \leq \frac{\lambda}{8}(b_H - b_L)(3b_H - b_L)$. The certifier's ex ante expected payoff can be computed as $U(c, t_H) = \frac{1}{2}\frac{\lambda}{8}b_L^2 + \frac{1}{2}(\frac{\lambda}{8}b_H^2 + t_H) - c$. Suppose the certifier decides to shirk and acquires no information. When he issues a rating his quality expectation is equal to the prior $E(b) = \frac{b_L + b_H}{2}$, hence, he reports b_L whenever $\frac{\lambda}{2}(E(b) - b_L) \cdot b_L + \frac{\lambda}{8}b_L^2 \geq \frac{\lambda}{2}(E(b) - b_H) \cdot b_H + \frac{\lambda}{8}b_H^2 + t_H$(equivalent to $t_H \leq \frac{\lambda}{8}(b_H^2 - b_L^2)$) and he reports b_H otherwise. The certifier's ex ante expected payoff from shirking is $U(0, t_H) = \frac{\lambda}{8}(2b_H - b_L) \cdot b_L + max[t_H - \frac{\lambda}{8}(b_H^2 - b_L^2), 0]$. In equilibrium the certifier must choose to acquire information, hence the following inequality must hold $c \leq \frac{t_H}{2} - \frac{\lambda}{16}(b_H - b_L)(3b_L - b_H) - max[t_H - \frac{\lambda}{8}(b_H^2 - b_L^2), 0]$. The seller's expected profit $\pi_i = \frac{1}{2}\frac{b_L^2}{4} + \frac{1}{2}(\frac{b_H^2}{4} - t_H)$ decreases with t_H, for this reason she sets high t_H only if this is necessary to incentivize the certifier either to acquire information (moral hazard) or to reveal it truthfully (IC). It is easy to see that the seller would never set $t_H > \frac{\lambda}{8}(b_H^2 - b_L^2)$ because of the moral hazard prime only. At the same time (IC) commands at most $t_H = \frac{\lambda}{8}(b_H - b_L)(3b_L - b_H) < \frac{\lambda}{8}(b_H^2 - b_L^2)$, therefore in equilibrium $t_H \leq \frac{\lambda}{8}(b_H^2 - b_L^2)$. The moral hazard constraint implies (IC) $t_H \geq 2c + \frac{\lambda}{8}(b_H - b_L)(3b_L - b_H) > \frac{\lambda}{8}(b_H - b_L)(3b_L - b_H)$. Denote $t_H^* = max[2c + \frac{\lambda}{8}(b_H - b_L)(3b_L - b_H), 0]$, in equilibrium the seller sets $t_H = t_H^*$, $t_L = 0$ provided that $t_H^* \leq \frac{\lambda}{8}(b_H^2 - b_L^2)$ and her expected profit is higher than in the bubbling equilibrium $\pi_i > \pi_b = \frac{1}{4}\left(\frac{b_L + b_H}{2}\right)^2 \Leftrightarrow t_H^* < \frac{1}{16}(b_H - b_L)^2$ QED.

The necessity to compensate the certifier for costly product examination on top of ensuring that he has adequate reporting incentives requires a high payment from the seller. Consequently moral hazard makes the seller more likely to opt out from certification and the uninformative equilibrium is implemented more often compared to the no moral hazard case.

The finding that the moral hazard constraint implies the incentive compatibility constraint seems to be an artifact of the model with two qualities, which precludes semi-separating equilibria.[19] When product quality is a continuous variable the incentive compatibility constraint is not implied by the moral hazard constraint in general. The analysis of this case is quite involved and deserves an independent investigation.

Observe that moral hazard constraint requires $t_H^* \leq \frac{\lambda}{8}(b_H^2 - b_L^2)$, which is hard to satisfy for small λ. Hence, one might expect that the seller would prefer to contract with a certifier who is buyer-protective $\lambda > 0$. Suppose the seller can choose among many certifiers with different $\lambda \in [0, \infty)$.

Corollary 6. *In the presence of moral hazard the seller either hires a buyer-protective certifier ($\lambda_{PR} > 0$) or does not hire a certifier at all.*

Proof. The seller's profit decreases with t_H hence the seller would like to minimize $t_H^*(\lambda) = 2c + \frac{\lambda}{8}(b_H - b_L)(3b_L - b_H)$ subject to $t_H^*(\lambda) \leq \frac{\lambda}{8}(b_H^2 - b_L^2)$, $t_H^*(\lambda) < \frac{1}{16}(b_H - b_L)^2$. If $b_L < b_H/3$, then $\partial t_H^*(\lambda)/\partial\lambda < 0$, the seller sets $t_H^* = 0$ and hires a certifier with $\lambda_{MH} \in [\frac{16c}{(b_H-b_L)(3b_L-b_H)}, \infty)$. If $b_L = b_H/3$ optimal transfer is $t_H^* = 2c$ and given that $c < \frac{1}{32}(b_H - b_L)^2$ the seller is indifferent among certifiers with $\lambda_{MH} \in [\frac{16c}{(b_H^2-b_L^2)}, 2]$; if $c \geq \frac{1}{32}(b_H - b_L)^2$ the seller prefers to sell the product unrated. Finally, if $b_L > b_H/3$, then $\partial t_H^*(\lambda)/\partial\lambda > 0$ and the seller tries to hire a certifier with the lowest λ that ensures $t_H^*(\lambda) \leq \frac{\lambda}{8}(b_H^2 - b_L^2)$ and $t_H^*(\lambda) < \frac{1}{16}(b_H - b_L)^2$. Consequently she picks $\lambda_{MH} = \frac{8c}{(b_H-b_L)^2}$ if $t_H^*(\lambda^*) = \frac{b_H+b_L}{b_H-b_L}c < \frac{1}{16}(b_H - b_L)^2$. On the other hand when $c \geq \frac{(b_H-b_L)^3}{16(b_H+b_L)}$ the seller can not gain from hiring any certifier and sells the product unrated. The seller's preferences over certifiers in the presence of moral hazard are as follows: if $b_L \in [0, \frac{1}{3}b_H)$ then $\lambda_{MH} \in [\frac{16c}{(b_H-b_L)(3b_L-b_H)}, \infty)$; if $b_L = \frac{1}{3}b_H$ and $c < \frac{1}{32}(b_H - b_L)^2$ then $\lambda_{MH} \in [\frac{16c}{(b_H^2-b_L^2)}, 2]$, if $c \geq \frac{1}{32}(b_H - b_L)^2$ then $\lambda_{MH} = \emptyset$; if $b_L \in (\frac{1}{3}b_H, b_H)$ and $c < \frac{(b_H-b_L)^3}{16(b_H+b_L)}$ then $\lambda_{MH} = \frac{8c}{(b_H-b_L)^2}$, if $c \geq \frac{(b_H-b_L)^3}{16(b_H+b_L)}$ then $\lambda_{MH} = \emptyset$ QED.

It is challenging to incentivize a certifier who cares little about buyers ($\lambda \approx 0$) to investigate the product. First, noncontingent payments provide no incentives to investigate, because the certifier can always report at random and get paid. Second, incentive compatible contingent payments call for high payments for high ratings. If moral hazard is extreme and very high transfers ought to be offered, a certifier who is not buyer-protective enough cannot resist the temptation to shirk and issue the highest rating. To circumvent this the seller deals with a certifier which is sufficiently buyer-protective.

[19]Suppose the certifer were indifferent between issuing ratings $\widehat{b}_H$ and $\widehat{b}_L$ when $b = b_L$ and prefered rating $\widehat{b}_H$ when $b = b_H$, then issuing rating $\widehat{b}_H$ would be his dominant strategy and he would not bother to acquire information.

5.3 Commitment on price

Throughout the analysis we assumed that the seller cannot commit to a pricing rule and sets the ex post optimal price. This creates a sort of hold-up problem and makes a buyer-protective certifier reluctant to give high ratings to the seller. Intuitively the seller, and possibly the buyers, can gain if the seller can commit to a pricing rule and circumvent the hold-up problem. This possibility is particularly appealing for Standard Setting Organizations (SSOs) where the hold-up problem is aggravated by the network externalities among manufacturers, which enhance the market power of the technology owner once the technology is endorsed (see Farrell et al.).[20]

In this section we, first, allow the seller to commit to a pricing rule $P(.)$ ex ante. Then, we study commitment to a price cap and to a price floor. We keep the simple framework with two equally likely quality levels and the uniform distribution $b \in \{b_L, b_H\}$, $F(x) = x$. Moreover we require a fixed payment to the certifier $t = const$. Studying a general model with continuous quality and contingent transfers is an exciting task, which we leave for future research.

We study deterministic pricing (one can show that the seller does not want to randomize). We analyze a fully informative equilibrium and later prove that this equilibrium is always preferred to the uninformative equilibrium by the seller. In a fully informative equilibrium the certifier's reports are $\widehat{b}(b_L) = \widehat{b}_L = b_L$ and $\widehat{b}(b_H) = \widehat{b}_H = b_H$, and the seller's prices are $P(\widehat{b}_L) = P_L$ and $P(\widehat{b}_H) = P_H$. Without loss of generality we impose $P_j \leq \widehat{b}_j$. Then given quality b_i and report $\widehat{b}_j$ the buyers' demand is $D(\widehat{b}_j) = \widehat{b}_j - P_j$ and the buyers' surplus can be expressed as

$$S(b_i, \widehat{b}_j) = (\widehat{b}_j - P_j)(b_i - \widehat{b}_j + (\widehat{b}_j - P_j)/2)$$

The seller chooses the pricing rule and the transfer to maximize his expected profit

Problem 8. $\underset{\{P(.),t\}}{max} \ \frac{1}{2}P(\widehat{b}_L)(\widehat{b}_L - P(\widehat{b}_L)) + \frac{1}{2}P(\widehat{b}_H)(\widehat{b}_H - P(\widehat{b}_H)) - t$

s.t. IC: $\widehat{b}_i = \underset{\widehat{b} \in \{\widehat{b}_L, \widehat{b}_H\}}{\arg\max}[\lambda S(b_i, \widehat{b}_j) + t]$, $i = L, H$; *RE:* $E(b|\widehat{b}_i) = \widehat{b}_i$, $i = L, H$; *LL:* $t \geq 0$.

If the seller can only commit to a price cap or a price floor she solves an analogous problem. The following theorem describes the optimal contract.

Theorem 3. *If the seller can commit to a pricing rule then under a fixed fee she sets* $t = 0$ *and implements an informative equilibrium:* $\widehat{b}_L = b_L$, $\widehat{b}_H = b_H$;

[20]Most of the SSOs require FRAND (fair, reasonable, and non-discriminatory) licensing of patented technologies, which still leaves the technology owner a lot of freedom with pricing. Some SSOs, VMEbus and International Trade Association (VITA) for instance, require the owner of the technology to commit to the royalty cap before the technology is endorsed.

if $b_L \in [0, \frac{1}{3}b_H]$ *the optimal pricing rule commands monopoly prices:* $P_L = b_L/2$, $P_H = b_H/2$;

if $b_L \in (\frac{1}{3}b_H, b_H)$ *the optimal pricing rule commands a price above the monopoly price for low rating and a price below the monopoly price for high rating:* $P_L > b_L/2$ *and* $P_H < b_H/2$.

Proof. First, $t = 0$. In an informative equilibrium the seller sets monopoly price $P_i = b_i/2$, $i = L, H$ if this pricing satisfies the certifier's IC. According to the results of section 2.1 this happens when $b_L \leq b_H/3$ and the seller prefers the informative equilibrium over the uninformative one. When $b_L > b_H/3$ the monopoly pricing is not incentive compatible. The IC for the high type binds $(b_H - P_H)^2/2 = (b_L - P_L)(b_H - b_L + (b_L - P_L)/2)$, from the necessary condition the associated Lagrange multiplier is $\eta = \frac{2P_L - b_L}{b_H - P_L} = \frac{b_H - 2P_H}{b_H - P_H} > 0$, which implies $P_L > b_L/2$ and $P_H < b_H/2$. It remains to show that the seller prefers the informative equilibrium. In the uninformative equilibrium $\widehat{b} = (b_L + b_H)/2$, $P = (b_L + b_H)/4$ and the seller gets $\pi_b = \frac{(b_L+b_H)^2}{16}$. In the informative equilibrium the seller's profit is at least $\pi_i = \frac{3(b_L+b_H)^2}{32} > \pi_b$. Indeed, if the seller sets $P_L = P_H = (b_L + b_H)/4$ then the certifier reports truthfully $\widehat{b}_L = b_L$, $\widehat{b}_H = b_H$ and the seller gets $\pi_i = \frac{b_L+b_H}{4}(\frac{1}{2}(b_L - \frac{b_L+b_H}{4}) + \frac{1}{2}(b_H - \frac{b_L+b_H}{4})) = \frac{3(b_L+b_H)^2}{32}$. Hence, the seller implements the informative equilibrium QED.

Intuitively, the seller prefers ex post monopoly pricing and implements it whenever it is incentive compatible with fully informative ratings. If the monopoly pricing is not incentive compatible, that is the certifier is tempted to issue a low rating when the quality is high, the seller chooses the pricing that discourages the certifier from underreporting. The seller increases the price in case of low rating above the monopoly level and lowers the price in case of high rating below the monopoly level so that the certifier, who cares about the buyers' surplus, prefers to issue high rating when actual quality is high.

Corollary 7. *When the seller commits to a price cap under a fixed fee she sets* $t = 0$;

if $b_L \in [0, \frac{1}{3}b_H]$ *she implements an informative equilibrium, optimal prices coincide with monopoly prices:* $P_L = b_L/2$, $P_H = b_H/2$;

if $b_L \in (\frac{1}{3}b_H, x^* b_H]$ *she implements an informative equilibrium, for a low rating the optimal price is the monopoly price, for a high rating the price is capped below the monopoly level:* $P_L = b_L/2$ *and* $P_H = \overline{P} < b_H/2$;

if $b_L \in (\overline{x} b_H, b_H)$ *she implements the uninformative equilibrium and sets the monopoly price* $P = (b_L + b_H)/4$;

here $\overline{x} \approx 0.83$ *solves* $49x^4 - 140x^3 + 134x^2 - 44x + 1 = 0$.

Proof. When $b_L \leq b_H/3$ the seller implements the informative equilibrium and sets the monopoly price $P_i = b_i/2$, $i = L, H$. If $b_L > b_H/3$ consider the informative equilibrium. In case of low rating the seller sets the monopoly price $P_L = b_L/2$. We express P_H from the

IC for the high type $P_H = b_H - \sqrt{b_L(b_H - \frac{3}{4}b_L)} < b_H/2$ for $b_L \in (\frac{1}{3}b_H, b_H)$. The profit the seller gets is $\pi_i = \frac{1}{2}(b_L/2)^2 + \frac{1}{2}(b_H - \sqrt{b_L(b_H - \frac{3}{4}b_L)})\sqrt{b_L(b_H - \frac{3}{4}b_L)}$ which must be higher than the profit she gets in the uninformative equilibrium $\pi_b = \frac{(b_L+b_H)^2}{16}$. Denoting $b_L = xb_H$, with $x \in (\frac{1}{3}, 1)$ we express $(\pi_i - \pi_b)/b_H^2 = \frac{1}{8}x^2 + \frac{1}{2}\sqrt{x(1-\frac{3}{4}x)} - \frac{1}{2}x(1-\frac{3}{4}x) - \frac{1}{16}(1+2x+x^2) = \frac{7x^2-10x-1}{16} + \frac{1}{2}\sqrt{x(1-\frac{3}{4}x)}$. Solving we obtain $\pi_i > \pi_b$ whenever $x < \overline{x} \approx 0.83$, $\overline{x}$ solves $49x^4 - 140x^3 + 134x^2 - 44x + 1 = 0$ QED.

Corollary 8. *When the seller commits to a price floor under a fixed fee she sets* $t = 0$*;*

if $b_L \in [0, \frac{1}{3}b_H]$ *she implements an informative equilibrium; optimal prices coincide with monopoly prices:* $P_L = b_L/2$, $P_H = b_H/2$*;*

if $b_L \in (\frac{1}{3}b_H, x^*b_H]$ *she implements an informative equilibrium, the optimal pricing rule commands a price above the monopoly level for low rating and the monopoly price for high rating, that is* $P_L = \underline{P} > b_L/2$ *and* $P_H = b_H/2$*;*

if $b_L \in (\underline{x}b_H, b_H)$ *she implements the uninformative equilibrium and sets the monopoly price* $P = (b_L + b_H)/4$*;*

here $\underline{x} \approx 0.85$ *solves* $17x^4 - 12x^3 - 58x^2 + 84x - 31 = 0$.

The proof is analogous to that of corollary 7. Both corollaries are intuitive. If the monopoly pricing is not incentive compatible with fully informative ratings, that is it prompts the certifier to issue a low rating when quality is high, the seller tries to encourage the certifier to issue a high rating by changing the pricing. She does so either by promising a price below the monopoly level in case of high rating if she can commit to a price cap, or by promising a price above the monopoly price in case of low rating if she can commit to a price floor.

Now we turn to the welfare analysis under different regimes: full commitment on price, a price cap, a price floor, and no commitment on price. The expected social welfare is the expectation of the seller's profit, the buyers' surplus and the certifier's payoff $W = E(\pi + S + \lambda S) = \frac{1}{2}\sum_{i=L,H}(P_i(\widehat{b}_i - P_i) + (1+\lambda)(\widehat{b}_i - P_i)(b_i - \widehat{b}_i + (\widehat{b}_i - P_i)/2))$. It might appear that full commitment on price leads to the highest welfare, however, this is not always the case as the theorem below states.

Theorem 4. *The utilitarian social planner's preferences over commitment regimes (full commitment on price, price cap, price floor, and no commitment on price) are as follows:*

if $b_L \in [0, b_H/3]$ *he is indifferent between full commitment on price, price cap, price floor, and no commitment on price;*

if $b_L \in (\frac{1}{3}b_H, \overline{x}b_H]$ *he prefers a price cap;*

if $b_L \in (\overline{x}b_H, 1)$ *he prefers full commitment on price;*

here $\overline{x} \approx 0.83$ *solves* $49x^4 - 140x^3 + 134x^2 - 44x + 1 = 0$.

The detailed proof is in the appendix. When $b_L \leq b_H/3$ any regime leads to the monopoly pricing and the social planner is indifferent among regimes. If $b_L \in (\frac{1}{3}b_H, \overline{x}b_H]$ then by theorem 3 and corollaries 7,8 the regime with a price cap leads to the lowest prices and results in informative ratings, thus social welfare is the highest. If $b_L > \overline{x}b_H$ only full commitment on price and price floor can lead to informative ratings. The prices are the lowest under full commitment, hence the social planner prefers this regime over a price floor.

All in all the results obtained here support the idea of letting sellers to commit to a price cap when applying for certification. As theorem 4 shows price cap helps to restore informative certification when only a fixed payment to the certifier is possible. Recent proposals to introduce price cap commitments by Standard Setting Organizations are in line with these findings and, as our results suggest, can alleviate SSO's biases and improve social welfare. On the other hand letting sellers to commit to an arbitrary pricing rule is not necessarily a good policy. According to our findings this might lead to strategic "overpricing" of bad products. In this case the seller commits to a price above the monopoly level in case of low rating in order to encourage the certifier to issue a high rating. Such a practice can be detrimental for the welfare, hence commitment to a price cap might be preferred over commitment to an arbitrary pricing rule.

6 Conclusion

This paper takes an original stand on certification and sheds some light on possible improvements in regulation. The current debate about the role of credit rating agencies (CRA) stresses conflicts of interest. Scholars argued that issuer-compensated CRAs inflate ratings and mislead the investors, and so issuers must be restricted to pay non-contingent fees. This argument is certainly correct if investors are naive and take ratings at face value, otherwise there is no clear reason why rating inflation is a concern and why a fixed fee should help. We develop a rational expectations model where the seller and the certifier contract for information and show that restricting their contracting possibilities to a fixed fee can be detrimental for information production and welfare.

Fundamentally there is no harm from contingent contracts as long as contracts are public. If the certifier receives a high payment for a high rating, then a rational buyer who *observes* the payment should comprehend that the rating might have been issued for money and interpret the rating accordingly. This in turn reduces the seller's temptation to increase the payment for a high rating.

By contrast, "rating inflation" can occur when the contract is private. If rational buyers *do not observe* the payment going to the certifier their perception of the ratings does not depend on the actual payments. This creates an incentive for the seller to elicit the high rating by offering a generous compensation. As a result the highest rating is issued "too" often and is imprecise.

With rational expectations, contingent private contracts create rating inflation and can be detrimental for welfare, not because investors are cheated but because ratings are coarse and information is lost. A desirable regulation is, therefore to promote transparency, that is, to mandate public contracts between the seller and the certifier. A fixed fee, on the other hand, prevents rating inflation but it may also decrease the overall precision of ratings.

7 Appendix

Proof of proposition 4. An optimal noncontingent contract $(\widehat{B}, t)$ has $t = 0$ and results in a partition equilibrium $\{b_i\}_{i=0,...,N(\alpha)}$ with ratings $\widehat{b}_i = (b_{i-1} + b_i)/2$, $i = 1, ..., N(\alpha)$; high ratings are more precise than low ratings : $b_i - b_{i-1} < b_{i-1} - b_{i-2}$.

The transfer has no effect on the certifier's decision, hence $\widehat{t} = 0$. By corollary 1 the intervals of full revelation $\widehat{b} = b$ are not incentive compatible, hence the reporting strategy is fully described by a sequence of boundary points $\{b_i\}_{i=0,...,N}$, that satisfy $b_0 = 0$, $b_N = 1$, $b_i \geq b_{i-1}$and correspond to ratings $\widehat{b}_i = \frac{b_{i-1}+b_i}{2}$, $i = 1, ..., N$. IC writes $\widehat{b}_i \in \arg\max_{\widehat{b} \in \widehat{B}} \left[\lambda S(b, \widehat{b}) + t\right]$ for $b \in [b_{i-1}, b_i]$. As in the Crawford-Sobel the necessary condition for (IC) to hold is that the certifier is indifferent between ratings $\widehat{b}_{i+1}$ and $\widehat{b}_i$ when $b = b_i$. Given that $S(b, \widehat{b}) = \gamma \widehat{b}^\alpha (b - \kappa \widehat{b})$, $\gamma = \left(\frac{\alpha}{1+\alpha}\right)^\alpha$, $\kappa = \frac{1+\alpha+\alpha^2}{(1+\alpha)^2}$ the sequence $\{b_i\}$ must solve the following system

$$\gamma \widehat{b}_{i+1}^\alpha (b_i - \kappa \widehat{b}_{i+1}) = \gamma \widehat{b}_i^\alpha (b - \kappa \widehat{b}_i),\ i = 1, ..., N-1 \tag{3}$$

with boundaries $b_0 = 0$, $b_N = 1$ and restrictions $b_i \geq b_{i-1}$. The indifference condition is also sufficient for (IC) because of the single crossing property $\frac{\partial^2 S(\widehat{b}, b)}{\partial \widehat{b} \partial b} > 0$ which implies $S(b_i, \widehat{b}_{i+1}) \gtreqless S(b_i, \widehat{b}_i)$ for $b \gtreqless b_i$. In principle $\{b_i\}_{i=0,...,N}$ can contain $b_j = b_{j-1}$ for some j if ratings happen to correspond to elements like (b_{j-2}, b_{j-1}), $\{b_j\}$ and (b_j, b_{j+1}) with $b_j = b_{j-1}$. If this is the case we can always merge all the $b_j = b_{j-1}$ and obtain a sequence that satisfies $b_i > b_{i-1}$ and solves the system (3). The corresponding reporting strategy satisfies (IC) and delivers the same outcome as $\widehat{b}(b)$ for almost every point. Indeed by lemma 1 $\widehat{b}(b)$ is monotone, hence it has at most countable number of discontinuity points. That it is all boundary points b_i, $i = 1, ..., N-1$ and, hence, the points $b_j = b_{j-1}$ have measure zero. We say the sequence $\{b_i\}_{i=0,...,N}$ satisfies (IC) or is incentive compatible if it solves system (3) with $b_0 = 0$, $b_N = 1$ and $b_i > b_{i-1}$ for $i = 1, ..., N$.

Observe that the sequence with $N = 1$, $b_0 = 0$, $b_1 = 1$ always satisfies (IC), it corresponds to the babbling equilibrium. It is easy to see that if a sequence $\{b_i\}_{i=0,...,N}$, $N \geq 2$ respects (IC) then the sequence $\{x_i\}_{i=0,...,N-1}$ such that $x_i = b_i/b_{N-1}$ also respects (IC). The rest of the proof follows from lemmas 4-5 (see below). Lemma 4 proves that for a given α there exists a maximum number of ratings $N(\alpha) < \infty$ and a corresponding partition which is incentive compatible. Lemma 5 shows that the seller always prefers the partition with the highest number of ratings.

Lemma 4. *For each $\alpha > 0$ only reporting strategies with a number of ratings $N \leq N(\alpha) < \infty$ are incentive compatible.*

Proof. Define a function $g(\widehat{b}) = S(1, \widehat{b})$, take an incentive compatible sequence $\{b_i\}_{i=0,...,N}$ and consider the equation

$$g(y) = g(\widehat{b}_N) \tag{4}$$

This equation has a solution $y_1 = \widehat{b}_N < 1$. The function $g(y)$ is monotone for $y \geq 1$: $g'(y) = \gamma y^{\alpha-1}(\alpha - \kappa(1+\alpha)y)) < 0$ for any $y \geq 1$ because $(1+\alpha)\kappa = \frac{1+\alpha+\alpha^2}{1+\alpha} > \alpha$, thus the equation can have at most one solution $y_2 \geq 1$. Suppose such a solution exists, then take $M = N + 1$, $A = 2y_2 - 1 \geq 1$ and construct a sequence $\{x_i\}_{i=0,...,M}$ such that $x_i = b_i/A$, $i = 0, ..., N$ and $x_M = 1$. If $A = 1$ this sequence does not satisfy $x_M > x_{M-1} = 1$ and is not incentive compatible according to our definition (in fact it is outcome equivalent to the initial sequence $\{b_i\}_{i=0,...,N}$). If on the other hand, $A > 1$ this sequence is incentive compatible and corresponds to a reporting strategy with $M = N + 1$ ratings. Indeed x_i, $i = 0, ..., N$ solve the system (3) and satisfy $x_i > x_{i-1}$ because the sequence $\{b_i\}_{i=0,...,N}$ is incentive compatible. From (4) follows $\gamma\left(\frac{y_2}{A}\right)^\alpha\left(\frac{1}{A} - \kappa\frac{y_2}{A}\right) = \gamma\left(\frac{(1+b_{N-1})/2}{A}\right)^\alpha\left(\frac{1}{A} - \kappa\frac{(1+b_{N-1})/2}{A}\right) \Leftrightarrow \gamma\left(\frac{x_M+x_{M-1}}{2}\right)^\alpha\left(x_{M-1} - \kappa\frac{x_M+x_{M-1}}{2}\right) = \gamma\left(\frac{x_{M-1}+x_{M-2}}{2}\right)^\alpha\left(x_{M-1} - \kappa\frac{x_{M-1}+x_{M-2}}{2}\right)$ thus $\{x_i\}_{i=0..M}$ solves the system (3). Given that $x_{M-1} = 1/A$, for $A > 1$ we have $x_i > x_{i-1}$, $i = 1, ..., M$, hence $\{x_i\}_{i=0,...,M}$ is incentive compatible.

Given that $g'(y) < 0$ for $y > y^* = \frac{\alpha(1+\alpha)}{1+\alpha+\alpha^2}$ and $g(y) \to -\infty$ when $y \to \infty$, the solution $y_2 \geq 1$ to (4) exists iff $g(\widehat{b}_N) \leq g(1)$. This in turn requires $\widehat{b}_N \leq y^{**} < y^*$. Indeed, if $\widehat{b}_N \geq y^*$ then $g(\widehat{b}_N) > g(1)$ because $\widehat{b}_N < 1$ and $g'(y) < 0$ for $y > y^*$. Moreover, $g'(y) > 0$ for $y < y^*$, $g(0) = 0$ and $g(y^*) > g(1) > 0$ thus there exists y^{**}such that $g(y^{**}) = g(1)$ and $g(\widehat{b}_N) \leq g(1)$ for $\widehat{b}_N \leq y^{**}$. Observe that $\widehat{b}_N = y^{**}$ implies $y_2 = 1$ and $A = 1$, while $\widehat{b}_N < y^{**}$ results in $A = 2y_2 - 1 > 1$ and allows to add one more rating. Hence for an incentive compatible sequence $\{b_i\}_{i=0,...,N}$ with N ratings we can construct an incentive compatible sequence $\{x_i\}_{i=0,...,M}$ with $M = N + 1$ ratings iff $\widehat{b}_N < y^{**} < y^*$, $g(y^{**}) = g(1)$.

Suppose $\widehat{b}_N < y^{**}$ and an additional rating can be introduced, then from $x_M = 1/A$ we get $\widehat{x}_M = \frac{1+x_M}{2} = \frac{1+A}{2A}$. Let's show that $\widehat{x}_M > \widehat{b}_N$, that is the highest rating in an incentive compatible sequence is higher whenever the total number of ratings in this sequence is higher. It is sufficient to prove $g(\widehat{x}_M) > g(\widehat{b}_N)$, because $g'(y) > 0$ for $y < \widehat{b}_N < y^*$ implies $g(y) \leq g(\widehat{b}_N)$ for any $y \leq \widehat{b}_N$ and $\widehat{x}_M \leq \widehat{b}_N$ is not possible. Substituting $y_2 = \frac{1+A}{2}$ into (4) we get $g(\widehat{b}_N) = g(\frac{1+A}{2})$, at the same time $g(\widehat{x}_M) = g(\frac{1+A}{2A})$. Therefore we need to show $g(\frac{1+A}{2A}) > g(\frac{1+A}{2})$ for $A > 1$, which is equivalent to $(\frac{1+A}{2A})^\alpha(1-\kappa\frac{1+A}{2A}) > (\frac{1+A}{2})^\alpha(1-\kappa\frac{1+A}{2})$. Given that $g(\widehat{b}_N) = g(\frac{1+A}{2}) > 0$ rearranging we obtain a condition: $\chi(A) = -\kappa + (2-\kappa)A - (2-\kappa)A^{1+\alpha} + \kappa A^{2+\alpha} > 0$. Substituting $\kappa = \frac{1+\alpha+\alpha^2}{(1+\alpha)^2}$ we get $\chi''(A) = \frac{A^{\alpha-1}}{1+\alpha}(A(2+3\alpha+3\alpha^2+\alpha^3) - (\alpha+3\alpha^2+\alpha^3)) > 0$ for any $A \geq 1$, thus $\chi'(A) \geq \chi(1) = -\alpha(2-\kappa) + \kappa(2+\alpha) = \frac{2}{1+\alpha} > 0$ which implies $\chi(A) > \chi(1) = 0$ for $A > 1$. Thus we have proved $\widehat{x}_M > \widehat{b}_N$. It immediately follows that any incentive compatible sequence $\{b_i\}_{i=0,\ldots,N}$ satisfies $b_i - b_{i-1} > b_{i-1} - b_{i-2}$.

All in all we have shown that from an incentive compatible sequence $\{b_i\}_{i=0,\ldots,N}$ with N ratings and $\widehat{b}_N < y^{**}$ we can construct an incentive compatible sequence $\{x_i\}_{i=0,\ldots,M}$ with $M = N+1$ ratings and $\widehat{x}_M > \widehat{b}_N \Leftrightarrow x_{M-1} > b_{N-1}$. If the new sequence satisfies $\widehat{x}_M < y^{**}$ we can repeat the manipulations and obtain a sequence with $N+2$ ratings and so on. Now we show that if we start adding ratings in the manner described above we will arrive to a point where $\widehat{b}_{N(\alpha)} > y^{**}$ and no partition with $N(\alpha)+1$ ratings exists. This would give us the maximum number of ratings achievable for a given α.

Suppose that the iterative process described above never stops and for any number of ratings N we can construct an incentive compatible sequence $\{b_i\}_{i=0,\ldots,N}$. For each sequence take the highest rating $\widehat{b}_N$ and form an increasing sequence $\{\widehat{d}_N\}_{N\in\mathbb{N}}$ of such ratings. If $\widehat{d}_N \geq y^{**}$ for some N then no partition with $N+1$ ratings exists, because the iterative process cannot be continued, a contradiction. Thus $\widehat{d}_N < y^{**}$ for any N and there must exist some $\widehat{d}_\infty \leq y^{**}$ such that $\widehat{d}_N < d_\infty$ for any $N \geq 1$ and $\lim_{N\to\infty} \widehat{d}_N = \widehat{d}_\infty$. For any $\varepsilon > 0$ we can find $N(\varepsilon)$ such that $\widehat{d}_{N(\varepsilon)} > \widehat{d}_\infty - \varepsilon$, then for $N(\varepsilon)+1$ the iterative process delivers $\widehat{d}_{N(\varepsilon)+1} = \frac{1+A}{2A}$, here A solves $g(\frac{1+A}{2}) = g(\widehat{d}_{N(\varepsilon)})$. As has been shown before $g(\frac{1+A}{2A}) - g(\frac{1+A}{2}) \sim \chi(A) \geq \frac{2(A-1)}{1+\alpha}$, hence $g(\widehat{d}_{N(\varepsilon)+1}) - g(\widehat{d}_{N(\varepsilon)}) \sim \frac{1-\widehat{d}_{N(\varepsilon)+1}}{\widehat{d}_{N(\varepsilon)+1}-1/2} > c(1-\widehat{d}_\infty)$ for some $c > 0$. Given that $\widehat{d}_\infty \in [\frac{1}{2}, y^{**})$ and $g'(y)$ is positive and bounded for $y \in [\frac{1}{2}, y^{**})$ it must be that $\widehat{d}_{N(\varepsilon)+1} - \widehat{d}_{N(\varepsilon)} > C(1-\widehat{d}_\infty)$ for some $C > 0$. Then $\widehat{d}_\infty > \widehat{d}_{N(\varepsilon)+1} > \widehat{d}_{N(\varepsilon)} + C(1-\widehat{d}_\infty) > \widehat{d}_\infty - \varepsilon + C(1-\widehat{d}_\infty)$ an combining with $\widehat{d}_\infty \leq y^{**} < y^* = \frac{\alpha(1+\alpha)}{1+\alpha+\alpha^2} < 1$ for ε small enough we get a contradiction $-\varepsilon + C(1-\widehat{d}_\infty) < 0$.

Thus the iterative process stops for some $N(\alpha)$ because the sequence $\{\widehat{d}_N\}_{N=1,\ldots,K}$ exceeds the threshold y^{**} and no incentive compatible sequence $\{b_i\}_{i=0,\ldots,N(\alpha)+1}$ with $N(\alpha)+1$ exists. Given that from any incentive compatible sequence with $M \geq 2$ ratings we can construct an incentive compatible sequence with $M-1$ ratings we conclude that no incentive compatible sequence with $M>N(\alpha)$ ratings exists. Therefore any incentive compatible sequence is finite and defines a partition of the interval $[0,1]$.

Lemma 5. *If partitions with N and $N-1$ ratings are incentive compatible the seller chooses the partition with N ratings.*

Proof. Consider the seller's gross expected profit $E\left[\pi|\{b_i\}\right]=\frac{\gamma}{1+\alpha}\sum\limits_{i=1}^{N}\left(\frac{b_{i-1}+b_i}{2}\right)^{1+\alpha}(b_i-b_{i-1})$. Rewriting and substituting $b_0=0$, $b_N=1$ we obtain the expression for the sum $\Sigma=\sum\limits_{i=1}^{N-1}\left((b_{i-1}+b_i)^{1+\alpha}-(b_{i+1}+b_i)^{1+\alpha}\right)b_i+b_N(b_N+b_{N-1})^{1+\alpha}=\sum\limits_{i=1}^{N-1}\left((b_{i-1}+b_i)^{\alpha}-(b_{i+1}+b_i)^{\alpha}\right)b_i^2+b_N^2\left(b_N+b_{N-1}\right)^{\alpha}$. Incentive compatibility can be written as $(b_{i-1}+b_i)^{1+\alpha}-(b_{i+1}+b_i)^{1+\alpha}=\frac{2}{\kappa}\left((b_{i-1}+b_i)^{\alpha}-(b_i+b_{i+1})^{\alpha}\right)b_i$ for $i=1,...,N-1$. Substituting incentive compatibility in Σ we derive $\sum\limits_{i=1}^{N-1}\left((b_{i-1}+b_i)^{\alpha}-(b_i+b_{i+1})^{\alpha}\right)b_i^2=-\frac{\kappa}{2-\kappa}b_N\left(b_N+b_{N-1}\right)^{\alpha}b_{N-1}$ and obtain $\Sigma=b_N\left(b_N+b_{N-1}\right)^{\alpha}\left(b_N-\frac{\kappa}{2-\kappa}b_{N-1}\right)$.

Substitute $b_N=1$. For incentive compatible partitions $\{a_i\}_{i=1,...,N-1}$ and $\{b_i\}_{i=1,...,N}$ $E\left[\pi|\{b_i\}\right]\geq E\left[\pi|\{a_i\}\right]$ whenever $(1+b_{N-1})^{\alpha}\left(1-\frac{\kappa}{2-\kappa}b_{N-1}\right)\geq(1+a_{M-1})^{\alpha}\left(1-\frac{\kappa}{2-\kappa}a_{M-1}\right)$ for $M=N-1$. Take the function $g(y)=y^{\alpha}(1-\kappa y)$ from the proof of lemma 4 and note that $\widehat{b}_N=\frac{1+b_{N-1}}{2}$, then $E\left[\pi|\{b_i\}\right]=\frac{2^{1+\alpha}}{2-\kappa}g(\widehat{b}_N)$ and we only need to prove $g(\widehat{b}_N)\geq g(\widehat{a}_M)$. But in the course of proof of lemma 4 it was shown that if $\{b_i\}_{i=1,...,N}$ and $\{a_i\}_{i=1,...,N-1}$ are incentive compatible partitions with N and $M=N-1$ ratings correspondingly, then $\widehat{b}_N>\widehat{a}_M$ and $g(\widehat{b}_N)>g(\widehat{a}_M)$ which implies $E\left[\pi|\{b_i\}\right]>E\left[\pi|\{a_i\}\right]$.

Proof of lemma 2. An optimal contract implies pooling for low qualities and separation for high qualities.

Optimal contract solves

Problem 9. $\underset{\{\widehat{b}(.),t(.)\}}{max}\int\limits_0^1\left[\pi(\widehat{b}(b))-t(\widehat{b}(b))\right]db$

s.t

IC: $\widehat{b}(b)\in\arg\max\limits_{\widehat{b}\in\widehat{B}}\left[\lambda S(b,\widehat{b})+t(\widehat{b})\right]$, $\forall b\in[0,1]$;

LL: $t(\widehat{b})\geq 0$;

RE: $\widehat{b}=E(b|\widehat{b}(b)=\widehat{b})$, $\forall\widehat{b}\in\widehat{B}$.

Here $\pi(\hat{b})=\frac{\gamma}{1+\alpha}\hat{b}^{1+\alpha}$is the profit given rating $\widehat{b}$, $S(b,\hat{b})=\gamma\hat{b}^{\alpha}(b-\kappa\hat{b})$ is the buyers' surplus given actual quality b and rating $\widehat{b}$. For a reporting strategy $\hat{b}(b)$ define certifier's utility $U(b)=\underset{\widehat{b}\in\widehat{B}}{max}[\lambda S(b,\widehat{b})+t(\widehat{b})]$ and transfers $t(\hat{b}(b))=U(b)-\lambda S(b,\hat{b}(b))$. By the envelope theorem $U(b)$ is continuous, differentiable almost everywhere and can be expressed as $U(b)=\int\limits_0^b\lambda S_b(b,\hat{b})db+U(0)$, we get $t(\hat{b}(b))=\int\limits_0^b\lambda S_b(b,\hat{b})db+U(0)-$

$\lambda S(b,\hat{b}(b))$. Using $\int\limits_0^1 db \int\limits_0^b \lambda S_b(b,\hat{b})db = \int\limits_0^1 (1-b)\lambda S_b(b,\hat{b})db$ we rewrite seller's expected profit as $\pi = \int\limits_0^1 \left(\pi(\hat{b}) + \lambda S(b,\widehat{b}) - (1-b)\lambda S_b(b,\widehat{b})\right) db - U(0)$. Provided that $S(b,\widehat{b})$ is linear in b and expectations are rational for a partition $\{b_i\}_{i=1,\ldots,N}$, $b_0 = 0$, $b_N = 1$ we get the expression $\pi = \int\limits_0^1 \left(\pi(\hat{b}) + \lambda S(\widehat{b},\widehat{b}) - (1-\widehat{b})\lambda S_b(b,\widehat{b})\right) db - t(0) + \lambda S(0,\widehat{b}(0))$ which after substitutions boils down to

$$\pi = \sum_{i=1}^{N} \int\limits_{b_{i-1}}^{b_i} [\gamma\beta\widehat{b}^{1+\alpha} - \gamma\lambda\widehat{b}^{\alpha}]db + \frac{\gamma\lambda\kappa}{2^{1+\alpha}} b_1^{1+\alpha} - t_1$$

Here $\kappa = \frac{1+\alpha+\alpha^2}{(1+\alpha)^2}$ and $\beta = \frac{1}{1+\alpha} + \lambda\frac{1+3\alpha+\alpha^2}{(1+\alpha)^2}$. For every $b > b_1$ the "instantaneous" profit function is $\pi(b) = \gamma\beta b^{1+\alpha} - \gamma\lambda b^{\alpha}$. Compute $\pi''(b) = \gamma\alpha b^{\alpha-2}((1+\alpha)\beta b - \lambda(\alpha-1))$ and observe that $\pi''(b) > 0$ for $b > b^* = \frac{\lambda}{\beta}\frac{\alpha-1}{\alpha+1}$ and $\pi''(b) \leq 0$ otherwise. [21]

Let's now show that if an optimal contract has perfect revelation for some interval (b_L, b_H) then it has perfect revelation for $(b_L, 1]$. First, it must be the case that $b_L \geq b^*$ otherwise introducing some pooling in the interval (b_L, b^*) would increase profits since $\pi''(b) < 0$ for $b < b^*$. Note that for small $\varepsilon > 0$, such that $b^* - \varepsilon > b_L$ an incarnation of pooling in the interval $(b^* - \varepsilon, b^*)$ would not violate the constraint $t(b) \geq 0$ because $t(b_L) \geq 0$ and $t'(b) > 0$ for perfect revelation. Second, perfect revelation requires $t'(b) > 0$ hence it is possible to induce it over the whole interval $(b_L, 1]$, without violating the constraint $t(b) \geq 0$. It is also optimal to do so for any $b > b_L \geq b^*$ because $\pi''(b) > 0$.[22] We conclude that in an optimal contract pooling can happen for low qualities $b \leq b_L$ and perfect revelation takes place for high qualities $b > b_L$.

Proof of theorem 2. An optimal negotiated contract $\{\widehat{b}(.), t(.)\}^{**}$ corresponds to an optimal contract unilaterally proposed by the seller $\{\widehat{b}(.), t(.)\}^{*}$ to a certifier with $\lambda' \leq \lambda$: $\{\widehat{b}(b)\}^{**}_{\lambda} \equiv \{\widehat{b}(b)\}^{*}_{\lambda'}$.

Take the contract that would prevail if the seller were to make a take it or leave it offer $\{\widehat{b}(.), t(.)\}^{*}$ and denote the corresponding expected transfer $T^* \geq 0$. It immediately follows that whenever $T \leq T^*$, the constraint $\int\limits_0^1 t(\widehat{b}(b))db \geq T$ in problem 5 is not binding and the negotiated contract coincides with $\{\widehat{b}(.), t(.)\}^{*}$, that is $\lambda' = \lambda$. On the other hand for $T > T^*$ the constraint $\int\limits_0^1 t(\widehat{b}(b))db \geq T$ must be binding. Consider a modification of problem 5 where seller's profit is augmented by a factor $A \geq 1$, that is her objective

[21] The "instantaneous" profit is the contingent profit the seller would get if she were to induce perfect revelation for all b.

[22] One can easily check that IC prohibits perfect revelation for a single point surrounded by intervals of pooling and that it is not optimal to have perfect revelation for a single point $b = 0$

is $\int_0^1 \left[A\pi(\widehat{b}(b)) - t(\widehat{b}(b))\right] db$, to find an optimal contract $\{\widehat{b}(.), t(.)\}^A$ and the associated expected transfer T_A.

Suppose that the negotiated contract is not fully revealing, then it is easy to see that there is a threshold A^* such that for all $A \leq A^*$ the optimal modified contract is the same $\{\widehat{b}(.), t(.)\}^A = \{\widehat{b}(.), t(.)\}^{**}$ and $T_A = T$, while for $A > A^*$ we get $T_A > T$. At the same time the modified problem for $A = A^*$ has a solution $\{\widehat{b}(.), t(.)\}^{A^*}$ which coincides with the solution of this same problem without the constraint $\int_0^1 t(\widehat{b}(b))db \geq T$, because for any $A > A^*$ the constraint $\int_0^1 t(\widehat{b}(b))db \geq T$ is not binding. But a modified problem without the constraint $\int_0^1 t(\widehat{b}(b))db \geq T$ is equivalent to the seller's maximization problem from the previous section with $\lambda' = \lambda/A^* \leq \lambda$ and all transfers divided by A^* which results in the contract $\{\widehat{b}(.), t(.)\}^*_{\lambda'}$. Therefore we obtain the following relation between optimal contracts $\{\widehat{b}(b)\}^{**}_\lambda \equiv \{\widehat{b}(b)\}^{A^*}_\lambda \equiv \{\widehat{b}(b)\}^*_{\lambda'}$, $\{t(\widehat{b})\}^{**}_\lambda \equiv \{t(\widehat{b})\}^{A^*}_\lambda \equiv \{\frac{\lambda}{\lambda'}t(\widehat{b})\}^*_{\lambda'}$.

Suppose the negotiated contract $\{\widehat{b}(.), t(.)\}^{**}$ is fully revealing $\widehat{b}(b) = b$, $\forall b \in [0,1]$. Define λ_{max} the maximal λ that results in full revelation when the seller makes a take it or leave it offer. Let $\lambda' = min[\lambda_{max}, \lambda]$ and obtain $\{\widehat{b}(.), t(.)\}^*_{\lambda'}$ with the expected transfer $T^*_{\lambda'}$, given that $\lambda' \leq \lambda_{max}$ theorem 1 implies that $\{\widehat{b}(b)\}^*_{\lambda'}$ is fully revealing. Now take $A^* = \lambda/\lambda' \geq 1$ and solve for the optimal negotiated contract with the augmented profit as before. The resulting contract $\{\widehat{b}(.), t(.)\}^{A^*}$ coincides with the initial optimal negotiated contract $\{\widehat{b}(.), t(.)\}^{**}$ because the latter is already fully revealing and the seller cannot benefit by changing the contract, hence $\{\widehat{b}(b)\}^{**}_\lambda \equiv \{\widehat{b}(b)\}^{A^*}_\lambda \equiv \{\widehat{b}(b)\}^*_{\lambda'}$. But at the same time the modified problem without the constraint $\int_0^1 t(\widehat{b}(b))db \geq T$ is equivalent to the seller's problem with λ', hence we must have $\{t(\widehat{b})\}^{**}_\lambda \equiv \{t(\widehat{b})\}^{A^*}_\lambda \equiv \{\frac{\lambda}{\lambda'}t(\widehat{b}) + t_0\}^*_{\lambda'}$ where $t_0 = T - \frac{\lambda}{\lambda'}T^*_{\lambda'} \geq 0$.

Proof of proposition 7. If the certifier is lax $\lambda < \frac{1+\alpha}{1+\alpha+\alpha^2}$ the highest rating is issued in more than 50% of cases ($b^e_{N-1} < 1/2$).

The non-manipulability constraint can be written as

$$\{b^*_i, t^*_i\}_{i=1,...,N} = \underset{\{b_i,t_i\}_{i=1,...,N}}{argmax} \sum_{i=1}^{N} (\pi(\widehat{b}_i) - t_i)(b_i - b_{i-1})$$

s.t.

IC: $\lambda S(b_{i-1}, \widehat{b}_{i-1}) + t_{i-1} = \lambda S(b_{i-1}, \widehat{b}_i) + t_i$, $i = 2, ..., N$;

LL: $t_i \geq 0$, $i = 1, ..., N$.

Rational expectations: $\{b^*_i, t^*_i\} = \{b^e_i, t^e_i\}$, $i = 1, ..., N$; $\widehat{b}_i = \frac{b^e_i + b^e_{i-1}}{2}$, $\forall \widehat{b}_i \in \widehat{B}$; $b^e_0 = 0$, $b^e_N = 1$.

Denoting $\tau_1 = t_1$ and $\tau_i = t_i - t_{i-1}$, $i = 2, ..., N$ we can write down the Lagrangian

$L = \sum_{i=1}^{N}(\pi(\widehat{b}_i) - \sum_{j=1}^{i}\tau_j)(b_i - b_{i-1}) + \sum_{i=2}^{N}\mu_i(\tau_i + \lambda S(b_{i-1}, \widehat{b}_i) - \lambda S(b_{i-1}, \widehat{b}_{i-1})) + \sum_{i=1}^{N}\eta_i \sum_{j=1}^{i}\tau_j$.

FOC requires: $-1 + \sum_{j=1}^{N}\eta_j = 0$ (τ_1); $-(1 - b_{i-1}) + \mu_i + \sum_{j=i}^{N}\eta_j = 0$ (τ_i), $i = 2, ..., N$; $\pi(\widehat{b}_{i-1}) - \pi(\widehat{b}_i) + \tau_i + \mu_i\lambda(S_b(\widehat{b}_i) - S_b(\widehat{b}_{i-1})) = 0$ (b_{i-1}), $i = 2, ..., N$. Taking into account $\pi(\widehat{b}) = \frac{\gamma}{1+\alpha}\widehat{b}^{1+\alpha}$ and $S(b, \widehat{b}) = \gamma\widehat{b}^{\alpha}(b - \kappa\widehat{b}) = S_b(\widehat{b})b - \kappa(1 + \alpha)\pi(\widehat{b})$ the IC gives $\tau_i = \lambda S(b_{i-1}, \widehat{b}_{i-1}) - \lambda S(b_{i-1}, \widehat{b}_i) = \lambda b_{i-1}(S_b(\widehat{b}_{i-1}) - S_b(\widehat{b}_i)) - \lambda\kappa(1 + \alpha)(\pi(\widehat{b}_{i-1}) - \pi(\widehat{b}_i))$ and we express $\mu_i = b_{i-1} + \frac{1-\lambda\kappa(1+\alpha)}{\lambda}\frac{\pi(\widehat{b}_i)-\pi(\widehat{b}_{i-1})}{S_b(\widehat{b}_i)-S_b(\widehat{b}_{i-1})}$ which delivers $2b_{i-1} - 1 + \frac{1-\lambda\kappa(1+\alpha)}{\lambda}\frac{\pi(\widehat{b}_i)-\pi(\widehat{b}_{i-1})}{S_b(\widehat{b}_i)-S_b(\widehat{b}_{i-1})} + \sum_{j=i}^{N}\eta_j = 0$, $i = 2, ..., N$. Given that $\pi(\widehat{b}_i) > \pi(\widehat{b}_{i-1})$, $S_b(\widehat{b}_i) > S_b(\widehat{b}_{i-1})$ and $\eta_i \geq 0$ we must have $2b_{N-1} - 1 < 0$ for $\lambda < \frac{1+\alpha}{1+\alpha+\alpha^2}$.

Proof of theorem 4. The utilitarian social planner's preferences over commitment regimes are as follows:

if $b_L \in [0, \frac{1}{3}b_H]$ he is indifferent between full commitment on price, a price cap, a price floor, and no commitment on price;

if $b_L \in (\frac{1}{3}b_H, \overline{x}b_H]$ he prefers a price cap;

if $b_L \in (\overline{x}b_H, b_H)$ he prefers full commitment on price.

First, the social planner never prefers a regime that leads to the uninformative equilibrium with $W_b = \pi_b + (1 + \lambda)S_b$, here π_b is the seller's profit, S_b is the buyers' surplus. Indeed, in an informative equilibrium with full commitment on price social welfare $W_i = \pi_i + (1 + \lambda)S_i$, hence under the commitment regime which is preferred by the social planner the welfare is at least W_i. We need to show $W_i \geq W_b$. By theorem 3 $\pi_i \geq \pi_b$, it remains to prove $S_i \geq S_b$. If $b_L \leq b_H/3$ then by remark 3 $S \sim \pi$, hence $\pi_i \geq \pi_b$ implies $W_i \geq W_b$. If $b_L > b_H/3$ then under the full commitment regime prices satisfy $\frac{b_H}{2} - P_H = (P_L - \frac{b_L}{2})\frac{b_H-P_H}{b_H-P_L} \geq 0$ (see the proof of theorem 3). The buyers' surplus is $S_i = \frac{1}{2}(\frac{(b_L-P_L)^2}{2} + \frac{(b_H-P_H)^2}{2}) \geq \frac{1}{4}(\frac{b_L^2}{4} + b_L(\frac{b_L}{2} - P_L) + \frac{b_H^2}{4} + b_H(\frac{b_H}{2} - P_H) = \frac{b_L^2+b_H^2}{16} + \frac{P_L-\frac{b_L}{2}}{b_H-P_L}(b_H(b_H - P_H) - b_L(b_H - P_L))$. $S_i \geq \frac{b_L^2+b_H^2}{16}$ because $P_L \geq \frac{b_L}{2}$, $P_H \leq \frac{b_H}{2}$ and $b_H^2 - b_HP_H - b_Lb_H + b_LP_L \geq \frac{b_H^2}{2} - b_Lb_H + \frac{b_L^2}{2} \geq 0$. Under the uninformative equilibrium the buyers' surplus is $S_b = \frac{(b_L+b_H)^2}{32} \leq \frac{b_L^2+b_H^2}{16} \leq S_i$, thus $W_i \geq W_b$.

When $b_L \leq b_H/3$ any regime leads to monopoly pricing and the social planner is indifferent. If $b_L \in (\frac{1}{3}b_H, \overline{x}b_H]$ then by theorem 3 and corollaries 7,8 the regime with a price cap leads to the lowest prices and results in informative ratings, thus the social welfare is the highest with a price cap. If $b_L \in (\overline{x}b_H, 1)$ only full commitment on price and a price floor can lead to informative ratings. Under full commitment prices satisfy $P_L \geq \frac{b_L}{2}$, $P_H \leq \frac{b_H}{2}$ and $\frac{(b_H-P_H)^2}{2} = (b_L - P_L)(b_H - b_L) + \frac{(b_L-P_L)^2}{2}$, under a price floor they respect $P_L = \underline{P} \geq \frac{b_L}{2}$, $P_H = \frac{b_H}{2}$ and $\frac{(b_H/2)^2}{2} = (b_L - \underline{P})(b_H - b_L) + \frac{(b_L-\underline{P})^2}{2}$. $P_H \leq \frac{b_H}{2}$ implies $(\underline{P} - P_L)(b_H - b_L) \geq \frac{(b_L-\underline{P})^2}{2} - \frac{(b_L-P_L)^2}{2}$ or $(\underline{P} - P_L)(b_H - \frac{\underline{P}+P_L}{2}) \geq 0$. Given that $b_H > \frac{\underline{P}+P_L}{2}$

we have $\underline{P} \geq P_L$, hence both prices are lower under full commitment on price and the social planner prefers this regime over a price floor.

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