A Multisector Equilibrium Search Model of Labor Reallocation*

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Abstract

How much unemployment can be attributed to intersectoral mobility frictions? If sector-specific shocks induce workers to move and reallocation is associated with longer unemployment spells, unemployment increases. In addition, slow labor reallocation leads to a worse allocation of labor further increasing unemployment. I develop a multisector search model of labor reallocation to structurally estimate the importance of these two forces. In a two-sector calibration of the model to construction and non-construction, barriers to intersectoral labor mobility generate ten percent of observed aggregate unemployment. I then examine how the dispersion of sectoral shocks during the Great Recession has contributed to unemployment due to sectoral reallocation. Contrary to a long-standing argument articulated in Lilien (1982), the dispersion of shocks hardly increased unemployment due to sectoral reallocation. While the outflow of labor from construction increased, the inflow decreased relative to the benchmark in which shocks are symmetric.

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1 Introduction

The 2008 recession and its linkages to the housing boom and bust have brought renewed attention to the importance of intersectoral mobility in explaining aggregate unemployment. Some have argued that the increased need for reallocation of construction workers to other sectors of the economy has contributed to the high and persistent level of unemployment in the United States. When workers need to reallocate themselves to other sectors and this process is time-consuming, unemployment might rise as workers carry out this slow transition. This hypothesis was first articulated in Lilien (1982) and is the basis for theories of the natural rate of unemployment such as Lucas and Prescott (1974).

Theoretically, whether or not intersectoral mobility increases unemployment is unclear. If sector-specific shocks induce workers to move and reallocation is associated with longer unemployment spells, unemployment increases. Conversely, labor reallocation, albeit slow, might lead to a better allocation of labor thus reducing unemployment. I develop a multisector search model of labor reallocation to structurally estimate the importance of labor reallocation frictions in explaining aggregate unemployment. Unemployed workers move in response to stochastic sectoral productivity shocks as well as idiosyncratic taste shocks so that gross flows of unemployed workers exceed net flows. In a two-sector calibration of the model to construction and non-construction, barriers to intersectoral labor mobility generate ten percent of aggregate unemployment, of which roughly fifty percent can be attributed to longer unemployment spells for movers. The remaining fifty percent can be attributed to labor misallocation. When moving costs are eliminated, workers move to sectors where their productivity is highest. This movement provides firms with incentives to post more vacancies, which in turn leads to lower unemployment.

I then examine how the dispersion of sectoral shocks during the Great Recession has contributed to unemployment due to sectoral reallocation, or “move unemployment.” I study what would have happened in a world in which shocks across construction and non-construction in the Great Recession were more symmetric and the need for net labor real-
location eliminated. Contrary to a long-standing argument articulated in Lilien (1982), I find that the dispersion of shocks hardly increased move unemployment. While the outflow of labor from construction increased, the inflow decreased relative to the benchmark case with symmetric shocks. This result highlights the importance of accounting for gross labor reallocation. If net reallocation is partly accomplished through changes in the composition of gross reallocation, models in which gross and net flows of unemployed workers are equal would predict that higher amounts of net reallocation increase move unemployment.

My model economy consists of multiple sectors, each with many workers and firms. In each sector, firms and workers are matched according to a standard matching technology. Matches separate at an exogenous rate and unemployed workers choose to stay or move to other sectors. The choice is determined by sectoral job finding rates and wages, but also by an idiosyncratic taste component. The idiosyncratic taste component generates gross flows in excess of net flows. If a worker decides to move to another sector, she spends additional time in unemployment before she becomes available to the new sector’s labor market. Thus, the main adjustment cost in my model is extra time in unemployment. Because it takes time for labor to reallocate and moving costs prevent workers from fully exploiting the gains from arbitrage opportunities across sectors, the resulting labor allocation is one with higher unemployment relative to the benchmark without intersectoral mobility frictions.

Using Simulated Method of Moments, I calibrate the model to sectoral level data on movers and stayers as well as sectoral labor market objects from several micro data sources. To run counterfactuals, I estimate the history of sectoral shocks from 1977-2012 by using the model’s second order approximation around its deterministic steady state. Using data on employment in construction and the rest of the economy, I recover the sectoral shocks consistent with observed sectoral employment dynamics over this time period. I then use the calibrated model and the recovered shocks to study unemployment dynamics under several counterfactual scenarios.

In the first counterfactual, I ask how aggregate unemployment from 1977-2012 would
have evolved in the absence of moving costs. This allows me to gauge the importance of intersectoral mobility frictions in explaining aggregate unemployment. I find that, on average, unemployment would have been 0.66 percentage points lower in the absence of these frictions. This decline can be decomposed into two separate channels. The first channel is due to the time cost associated with gross reallocation of workers, or people transitioning to new sectors. This accounts for 0.34 percentage points of aggregate unemployment. The remaining 0.32 percentage points can be attributed to the improvement in labor market allocations when previously immobile workers reallocate to more productive sectors after moving costs are eliminated. This estimate is a lower bound since the model does not account for the movement of labor within the broad industry grouping of non-construction.

I then turn my attention to the 2008 recession, and ask whether the nature of the recession and its concentration in construction increased the need for sectoral reallocation and thus “move unemployment.” Lilien (1982) argues that the empirical correlation between dispersion in employment growth, a proxy for sectoral shocks, and unemployment is due slow intersectoral reallocation of labor. Under this hypothesis, the Great Recession might have seen more move unemployment as the recession was closely linked to the boom and bust in the the housing market which disproportionately affected construction workers. To evaluate this hypothesis, I run a counterfactual experiment in which the shocks to construction and non-construction beginning in 2008 were symmetric, eliminating any need for net reallocation of labor. I find that a more symmetric shock would not have decreased unemployment due to movers. The reasoning is as follows. While a more symmetric shock might have induced less outflows from construction, it would also have induced more inflows from the rest of the economy. Counterfactually, in the aggregate unemployment due to movers would be relatively unchanged.

The remainder of the paper is organized as follows. Section 2 discusses some related work. Section 3 develops the proposed model and its main features. Section 4 calibrates the model using sectoral level labor market data and evidence on movers and stayers taken from
several micro data sources. Section 5 performs the counterfactuals described above. Section 6 concludes and describes how the model can be applied to other topics and ongoing work.

2 Literature

The theory in this paper is a hybrid of three literatures on sectoral mobility and unemployment: the islands models of Lucas and Prescott (1974), the search models of Mortensen and Pissarides (1994), and models of labor mobility that draw from Discrete Choice Theory such as Artuc, et.al. (2010), Kline (2008), and Keenan and Walker (2011). Bridging these models together produces properties that are desirable in studying the quantitative importance of intersectoral labor mobility frictions in unemployment. I discuss how the model presented here relates to each of these literatures in turn. I then discuss papers that are closely related to the application in this paper.

I model labor mobility between distinct markets that is time consuming in a way that is similar to the island model of Lucas and Prescott (1974) and extensions of that model such as Alvarez and Shimer (2011). In these models, each market belongs to a continuum of markets. While this assumption provides tractability, it generates islands that have no real identity. Since my model consists of a discrete number of islands rather than a continuum, it becomes amenable to quantitative analysis in which events occur in specific and identifiable sectors. I can study environments in which certain sectors are in permanent decline, or cases in which some sectors are hit by larger shocks than others. Chang (2011) has a search-theoretic model of sectoral reallocation, but the analysis is limited to two sectors. Carrillos-Tudela and Visschers (2011), on the other hand, have a model of sectoral reallocation among many distinct markets, but these markets are again part of a continuum and thus subject to the same aforementioned criticism. Coen-Pirani (2010) similarly uses a continuum of markets.

I model unemployment within an island or sector using the search models of Mortensen and Pissarides (1994) in which workers and job-openings match via a constant returns to scale matching function. Since a model with a single sector precludes any discussion of
intersectoral mobility frictions, I introduce multiple sectors and costly intersectoral labor mobility. If an aggregate shock is experienced differentially across sectors - a hypothesis supported by empirical work in Abraham and Katz (1986) - the model generates much slower dynamics relative to the single-sector benchmark since an aggregate shock will lead to slow labor reallocation. Finally, the measured efficiency of the matching function in my model is endogenous and will depend on the total number of movers, which in turn will depend on the state of the economy and how much reallocation is taking place. In the standard one-sector search model, match efficiency is exogenous.

Lastly, I model the worker’s sectoral choice problem using methods developed in the discrete choice literature as in Kline (2008), Artuc, et. al. (2011), and Kennan and Walker (2011), all of whom take advantage of the Type I Extreme Value Distributions to formulate worker flows. Formulating idiosyncratic worker shocks as taste shocks (or shocks to moving costs) generates gross worker flows in excess of net worker flows, a feature of the data that is quantitatively large and usually ignored. In several counterfactual experiments, I show that the presence of gross flows in excess of net flows is an important feature of a model which tries to infer the effect of time-consuming labor mobility on unemployment. The composition of inflows and outflows across sectors can change in response to shocks rather than just the total number of movers.

The model provides a theoretical framework for thinking about the relationship between unemployment and vacancies in the face of sectoral shifts, a relationship studied by Lilien (1982) and Abraham and Katz (1986). These papers are largely empirical and test informal hypotheses. The model presented here formalizes these theories, but remains general enough to incorporate permanent sectoral declines as well as aggregate movements in demand that might impact sectors differentially over the cycle. While this paper focuses on the types of shocks described in Abraham and Katz (1986), the model would predict that permanent sectoral declines should coincide with permanent sectoral switches, while movements in aggregate demand which are experienced differentially by sectors over the cycle should induce
more temporary movements across sectors. The results also challenge the basic premise in Lilien (1982) that posits a direct link between net reallocation and unemployment. My results suggest that the relevant statistic is gross reallocation, since some changes in net reallocation are accomplished through the composition of gross reallocation.

The application of this paper is related to recent papers by Sahin, et. al. (2012), and Herz and van Rens (2011). These papers seek to measure the extent to which unemployment in the Great Recession is structural or caused by mismatch between available jobs and workers. Since I model labor mobility costs in a decentralized equilibrium, I can ask whether or not the observed unemployment patterns are constrained efficient, where the social planner is subject to the same frictions workers face when reallocating themselves across sectors. If the planner faces the same mobility costs and matching frictions as workers face, the observed level of unemployment is a constrained efficient response of the economy in the face of sectoral shocks, provided that the Hosios (1994) condition holds.

The counterfactuals I run for the recent recession in which I eliminate the relative boom and bust in construction are similar to exercises in Charles, Hurst, and Notowidigdo (2012). There are two main differences. First, I do not include Manufacturing as a separate sector, although this can certainly be added. Second, I do not have a labor force participation margin, though this could also be integrated easily. However, since my model provides a general equilibrium framework to analyze sectoral shocks, I can observe what happens counterfactually to aggregate productivity when shocks to certain sectors are shut off. Theoretically, the removal of the housing boom and bust in Charles, Hurst, and Notowidigdo (2012) does not hold aggregate demand fixed. Some of their results may be driven by the simple change in the aggregate frontier in the economy rather than the removal of the shock to the housing sector. In the structural model developed here, one could begin to think about disentangling these two effects.
3 Model

Each sector produces a homogeneous intermediate good using a linear production function whose only input is labor. The intermediate goods are aggregated via a CES aggregator to produce a final consumption good. In what follows, I suppress the time indices until they become necessary for clarity.

Every period, workers draw idiosyncratic sector-specific taste shocks that impact their mobility decisions. These shocks are assumed to be independently and identically distributed over time and across sectors.\(^1\) If a worker chooses to switch sectors, she must pay an additional time-cost of one extra period unemployed associated with switching sectors. This state, which I refer to as move unemployment, is distinct from stay unemployment in that workers in move unemployment have no prospect of getting hired.\(^2\) The state of move unemployment captures the time workers must spend retraining (if an island is viewed as a sector) before acquiring the necessary skills to be employable in a new sector, or the time spent moving to a new location (if an island is viewed as a geographical location).\(^3\) The empirical counterpart that will discipline these objects would be movers and stayers which we can observe retrospectively using several household surveys.\(^4\)

3.1 Final Goods Production

The N islands of the economy each produce an intermediate good \(y_n\) every period that is aggregated into a final good \(Y\) by final goods producers for consumption via the following

\(^1\)The taste shocks can be equally interpreted as utility costs to mobility since workers moving from \(n\) to \(j\) will lose \(\varepsilon_j - \varepsilon_n\) utils.

\(^2\)These notions are distinct from the notions of search and rest unemployment developed by Alvarez and Shimer (2011). Stayers in this model will still be actively searching for work on their island. Rest unemployed workers in Alvarez and Shimer (2011) remain on their island even though there is no immediate prospect for work.

\(^3\)In the model calibration, I think of this state as people who remain in the labor force but switch sectors. One could easily introduce a separate choice of non-participation in this setup and analyze non-employment rather than unemployment.

\(^4\)See Loungani and Rogerson (1989) and Murphy and Topel (1987) for studies of movers and stayers using the PSID and CPS respectively.
CES aggregator:

\[ Y = \left[ \sum_{n \in N} \left( \tau_n \frac{\frac{1}{\sigma}}{y_n} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \]

where \( \sigma \) represents the elasticity of substitution between goods and \( \tau_n \) represents sector \( n \)'s share of production in the final good so that \( \sum_{n \in N} \tau_n = 1 \). If we let \( P \) denote the price of the final consumption good and \( p_n \) denote the price of each intermediate good, it follows that the optimal demand for each intermediate good \( y_n \) by final goods producers is given by:

\[ y_n = \frac{Y \tau_n}{\left( \frac{P}{P^*} \right)^{\frac{1}{\sigma}}} \]

where \( P = \left[ \sum_{n \in N} \tau_n \left(p_n\right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \) is the “ideal” price index. Each island or sector thus faces a downward sloping demand curve. Finally, workers spend all their income on consumption of the final good.

### 3.2 Intermediate Goods Production and Sectoral Labor Markets

Each island produces an intermediate good with a linear production function whose only input is labor. One unit of labor in sector \( n \) produces \( \mu_n \) units of output. Detrended log labor productivity in each sector follows an AR(1) process so that:

\[ \log(\mu'_n) = \kappa_n \log(\mu_n) + \zeta_n \nu'_n \]

where \( \kappa_n \) represents the monthly autocorrelation of detrended log labor productivity in sector \( n \), \( \nu_n \sim N(0, 1) \) represents the innovations to log labor productivity in sector \( n \), and \( \zeta_n \) controls the variance of the series. I allow for correlations in the shocks (which I will call \( \phi \)) across sectors as discussed in more detail in Section 4.\(^5\)

While the final goods market is competitive, the labor markets within each island are

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\(^5\)Alternatively, I could have assumed there is one aggregate component of productivity and a sector-specific component that is independent across sectors. The two versions are equivalent since what ultimately matters is changes in relative sector sizes.
subject to standard search frictions; therefore, each sector will have an associated unemploy-
ment rate. For each island \( n \), let \( l_n \) denote the labor force size. The labor force will consist of employed workers \( e_n \) and unemployed workers of two types: movers and stayers. Stayers \( s_n \) will be unemployed workers who are searching for work in sector \( n \). Movers \( m_{nj} \) will be unemployed workers who were last in sector \( n \), but are moving to search for work in some sector \( j \).\(^6\) Thus, the total labor force size on sector \( n \) will be given by:

\[
l_n = s_n + \sum_{j \in N} m_{nj} + e_n
\]

The probability that stayers on island \( n \) meet jobs on island \( n \) is determined by the sector-specific matching function \( \Gamma_n(v_n, s_n) \), where \( v_n \) represents the total number of vacancies on the island. The fact that the matching function takes only stayers as inputs from the unemployment pool highlights the difference between the two states of unemployment. Stayers have the skills necessary to be hired instantaneously - they remain unemployed simply because it takes time for their resumes to reach potential employers. Movers, on the other hand, are still in the process of acquiring the skills necessary to become attractive hires. I make the standard assumption that \( \Gamma_n \) is constant returns to scale and has the particular form:

\[
(3.1) \quad \Gamma_n(v_n, s_n) = \Upsilon_n \cdot (v_n)^{1-g} (s_n)^g
\]

where \( g \in (0, 1) \) is the elasticity of the matching function and \( \Upsilon_n \) represents the sector-specific match efficiency. Letting \( \theta_n = \frac{v_n}{s_n} \) denote the island labor market tightness, the probability that vacancies in sector \( n \) turn into jobs is given by \( q_n(\theta_n) = \frac{\Upsilon_n(v_n, s_n)}{v_n} \). The probability that job seekers find jobs in sector \( n \) is given by \( f_n(\theta_n) = \frac{\Upsilon_n(v_n, s_n)}{s_n} \). Therefore, the transition probabilities satisfy the standard relationship \( f_n(\theta_n) = q_n(\theta_n) \theta_n \).

\(^6\)I choose to include these workers as part of sector \( m \) to more closely mimic what we observe in the data. In the CPS, we observe an unemployed workers sector of last employment, but cannot observe which sector they are moving to until they actually find a job. I discuss this in more detail in Section 4.
In the spirit of Kline (2008), Kennan and Walker (2011), and Artuc, et. al (2011), all workers draw a vector $\varepsilon$ of sector-specific idiosyncratic taste shocks $\varepsilon_n \sim Gumbel(-\rho \gamma, \rho)$ every period.\(^7\) I interpret these taste shocks as anything that might keep workers in a sector or geographic region that is unrelated to wages or the ability to find a job. For example, a worker might be unable to find a job as an artist, but they continue to search for work as an artist because this is what they enjoy doing most. In a spatial interpretation of the model, the tastes might include things like marriage or housing that would keep someone tied to a certain locale. The shocks are independently and identically distributed over time and across sectors.\(^8\) At the end of every period and after realizing their tastes, unemployed workers are able to move to the sector of their choice. This assumption ensures both that there are always some workers who will find it beneficial to change sectors (positive gross flows), and that labor mobility is multi-directional (gross flows in excess of net flows), even in the absence of sectoral productivity shocks.

The timing of the model is as follows. Time is discrete, and the economy begins with an allocation of workers in employment, stay unemployment, and move unemployment across all sectors $n \in N$, given sectoral productivities $\{\mu_n\}_{n=1}^N$. Employed workers work and earn wage $w_n$ and unemployed workers earn their value of leisure $b$. Afterward, separation of employed workers, absorption of movers into new sectors, and job-finding of stayers occur. Only after these labor market events occur do workers realize their taste shocks for the next period and make a move decision. The process starts over again after a new draw of sectoral productivity in each sector $\{\mu'_n\}_{n=1}^N$ has been realized. Figure 1 shows the timeline of the model graphically.

\(^7\)The Gumbel distribution is also known as the Type I Extreme Value Distribution. Without loss of generality, I set the mean of this distribution to 0, which requires setting the shape parameter to $-\rho \gamma$ where $\gamma \sim 0.5772$ is Euler’s constant.

\(^8\)A more plausible version of the model would be to have taste shocks that are correlated over time for an individual as well as correlated across specific sectors for different worker types (for example, high skill and low skill). If some workers are always more likely to move than others, we would see much higher mobility rates among these individuals. In addition, you would need large sectoral shocks to induce mobility by high skill types. These kind of implementations are certainly possible in a version of the model with simple moving costs when the state space is significantly lower.
t taste shocks realized productivity shocks

production, vacancies within sector reallocation intersectoral reallocation t + 1

Figure 1: Model Timeline

3.3 Workers

There are three distinct states of the labor force: employment, stay unemployment, and move unemployment. Let \( W_n, S_n, \) and \( M_{nj} \) represent their respective values. If \( \delta_n \) represents the exogenous separation probability in sector \( n \) and \( w_n \) represents the worker’s wage, the value of a job to an employed worker \( i \) in sector \( n \) (net of idiosyncratic taste shocks) is given by:

\[
W_n(\Omega) = w_n + [1 - \delta_n]E_{\Omega', \epsilon'} \left\{ (W_n(\Omega') + \epsilon'_{n,i}) \right\} \\
+ \delta_n E_{\Omega', \epsilon'} \left\{ \max \left( S_n(\Omega') + \epsilon'_{n,i}, \max_{k \neq n \in N} M_{n,k}(\Omega') + \epsilon'_{k,i} \right) \right\}
\]

(3.2) where \( \Omega = \{ s_n, e_n, m_{nj}, \mu_n \} \forall n, j \in N \} \) represents the state of the economy and \( \epsilon'_{n,i} \) represents the worker \( i \)'s taste draw for next period in sector \( n \).\(^9\) The present value of being an employed worker \( i \) in sector \( n \) is the earned wage \( w_n \) plus the continuation value. With probability \( 1 - \delta_n \), the worker remains employed in sector \( n \). With probability \( \delta_n \) the worker separates into stay unemployment, but is able to choose between remaining stay unemployed in \( n \) and becoming move unemployed from sector \( n \) to some other sector \( k \).\(^{10}\)

The value of being a stay unemployed worker in sector \( n \) for worker \( i \) (net of idiosyncratic

9In what follows, I show that the value functions do not depend on \( i \).

10Note that I do not allow workers to quit their jobs and search for other sectors, either through job-to-job transitions or through move unemployment. While these kinds of transitions are certainly present in the data (three percent of employed workers switch sectors every month according to Artuc, et. al. (2010)), the equivalent number for unemployed movers is ten percent. I choose to focus on reallocation through unemployment to keep the model as close to the Lilien (1982) hypothesis as possible. In addition, if I allowed workers to search on the job, each worker would have a different outside option which would in turn imply that the value functions are individual specific, significantly complicating the numerical solution of the model.
taste shocks) is given by:

$S_n(\Omega) = b + f_n(\theta_n) \beta \mathbb{E}_{\Omega', \epsilon'} \left\{ (W_n(\Omega') + \epsilon'_n) \right\}$

$+ [1 - f_n(\theta_n)] \beta \mathbb{E}_{\Omega', \epsilon'} \left\{ \max \left( S_n(\Omega') + \epsilon'_n, \max_{k \neq n \in N} M_{nk}(\Omega') + \epsilon'_k \right) \right\}$

where $b$ is the value of leisure for the unemployed. Unemployed workers earn a current period return of $b$, plus the expected future value of stay unemployment. With probability $f_n(\theta_n)$ these workers find a job in sector $n$. With probability $1 - f_n(\theta_n)$ these workers do not find a job and choose between remaining stay unemployed in sector $n$ or becoming move unemployed from sector $n$ to some other sector $k$. Finally, the value of being move unemployed from sector $n$ to sector $j$ (net of idiosyncratic taste shocks) is given by:

$M_{nj}(\Omega) = b + \beta \mathbb{E}_{\Omega', \epsilon'} \left\{ \max \left( S_j(\Omega') + \epsilon'_j, \max_{k \neq n \in N} M_{jk}(\Omega') + \epsilon'_k \right) \right\}$

In the current period, movers earn the value of leisure for unemployed. After one period, the worker is absorbed and becomes a stayer in $j$, but can choose whether or not to remain a stayer in $j$ or become a mover from sector $j$ to some other sector $k$. Thus, movers will spend one extra period in unemployment relative to stayers.\footnote{Since the data I use suggest that movers spend one month extra in unemployment relative to stayers, the one period assumption is what the calibration would ask for. However, in earlier versions of this paper there was an absorption parameter $\alpha_{nj}$ governing the length of time it takes workers to move from sector $n$ to sector $j$. One could calibrate this parameter to relative durations of movers and stayers by sector and assume that unemployed workers are stuck in move unemployment for a certain length of time with no option of turning around.}

To simplify the terms within the expectations in equations (3.2), (3.3), and (3.4), I use results from McFadden (1974) exploiting Type I Extreme Value Theory and integrate out future taste shocks:

$W_n(\Omega) = w_n + [1 - \delta_n] \beta \mathbb{E}_{\Omega'} \left\{ W_n(\Omega') \right\} + \rho \delta_n \beta \mathbb{E}_{\Omega'} \left\{ \log \left( \sum_{k \in N} \exp(\tilde{S}_{nk}(\Omega')/\rho) \right) \right\}$

$\tilde{S}_{nk}(\Omega')$
Now all value functions are independent of the worker’s future taste shocks, significantly simplifying the numerical computation of the model. The worker’s problem in unemployment is to choose whether to remain stay unemployed or to become move unemployed and transition to some other sector. To simplify notation, define $\tilde{S}_{nj}$ to as follows:

$$
\tilde{S}_{nj} = \begin{cases} 
  S_n & \text{for } j = n \\
  M_{nj} & \text{for } j \neq n
\end{cases}
$$

The probability, then, that a worker facing the reallocation choice in $n$ chooses to become move unemployed and move to some sector $j$ from $n$ at time $t$ is given by:

$$
\pi_{nj} = Pr\left(\tilde{S}_{nj}(\Omega') + \varepsilon'_{j,i} > \tilde{S}_{nk}(\Omega') + \varepsilon'_{k,i} \quad \forall k \in N\right)
$$

which reduces to:

$$
\pi_{nj} = \frac{1}{\sum_{k \in N} \exp\left(\frac{\tilde{S}_{nk}(\Omega') - \tilde{S}_{nj}(\Omega')}{\rho}\right)}
$$

The move probabilities specified in Equation 3.8 are the standard Logit probabilities from Discrete Choice Theory. The move probabilities imply that, on average, workers move in response to differences in sectoral payoffs. Furthermore, we can write the value of being a

\[\text{See Appendix section C for a derivation.}\]

\[\text{See Appendix section B for a derivation.}\]
stayer in sector \( n \) as a function of these move probabilities:

\[
S_n(\Omega) = b + f_n(\theta_n)\beta E \Omega' W_n(\Omega') + \rho[1 - f_n(\theta_n)]\beta E \Omega' S_n(\Omega') + \rho[1 - f_n(\theta_n)]\beta E \Omega' \{ \log [\pi_{nn}^{-1}] \}
\]

Therefore, the value of being a stayer in sector \( n \) can be decomposed into the value of finding a job in sector \( n \), the value of being a stayer next period in sector \( n \), and the option value of remaining in sector \( n \). The last-mentioned is given by the expected difference in sectoral payoffs next period, or the gain from being able to move from \( n \) to some \( j \) next period:

\[
\pi_{nn}^{-1} = \sum_{k \in N} \exp(\frac{\tilde{S}_{nk}(\Omega') - S_n(\Omega')}{\rho})
\]

### 3.4 Firms

Turning to the decisions of the firms, the value of a job to a firm is given by:

\[
J_n(\Omega) = p_n\mu_n - w_n + \beta E \Omega' \{ [1 - \delta_n]J_n(\Omega') + \delta_n V_n(\Omega') \}
\]

A firm earns \( p_n\mu_n \), but must pay the worker a wage \( w_n \). With probability \([1 - \delta_n]\) the match remains. With probability \( \delta_n \) the match exogenously blows up and the firm gets \( V_n \). The value of a vacancy to a firm is given by:

\[
V_n(\Omega) = -c_n + \beta E \Omega' \{ q_n(\theta_n)J_n(\Omega') + [1 - q_n(\theta_n)]V_n(\Omega') \}
\]

where \( c_n \) represents the flow cost of posting a vacancy in sector \( n \). Free entry every period in each sector drives the value of a vacancy in all sectors \( n \in N \) to zero.
3.5 Wages

To close the model, I assume that wages are rigid and fixed at their efficient deterministic steady state level.\textsuperscript{14} I choose a rigid wage model over a flexible wage model so that the model will be able to more closely match unemployment fluctuations.\textsuperscript{15}

In this setting, the efficient wage will be equal to the Nash bargained wage where firms and workers bargain over match surplus $W_n - S_n + J_n - V_n$ when the Hosios (1994) condition holds.\textsuperscript{16} If $\eta$ is the worker’s bargaining power and it is set equal to the elasticity of matching function $g$ and letting bars above variables denote the variable’s value in the deterministic steady state, efficient wages in the deterministic steady state will be given by:

\begin{equation}
\bar{w}_n = (1 - \eta)b + \eta\bar{p}_n\mu_n + \eta c_n\bar{\theta}_n + (1 - \eta)\rho\beta[1 - \delta_n - f_n(\bar{\theta}_n)]E_{t+1}\{\log(\pi_n^{-1})\}
\end{equation}

This wage resembles the standard wage equation in Pissarides (2001), for example, but has an extra positive term which accounts for the fact that workers are not allowed to move. As such, they must be compensated for the value of search in unemployment.

3.6 Inflows and Outflows

I am now able to characterize the stock of employed, stay unemployed, and move unemployed in each sector $n$ at the beginning of period $t + 1$. The stock of employed workers in sector $n$ at time $t + 1$ is given by:

\begin{equation}
\bar{e}_n = e_n[1 - \delta_n] + s_n f_n(\theta_n)
\end{equation}

\textsuperscript{14}I relegate the derivation of the efficient wage equation to the appendix, Section D.

\textsuperscript{15}We know from Shimer (2005) that wage rigidity significantly improves the ability of the model to match unemployment fluctuations. Another option would be to follow the calibration strategy outlined in Hagedorn and Manovski (2008). This would lead to a lower value of leisure and a lower bargaining power $\eta$. However, since this would require matching the elasticity of wages in two sectors, I would also need to have two different values of non-market activity which would in turn affect the level of intersectoral labor mobility. Therefore, I choose the simpler route of rigid wages.

\textsuperscript{16}See the appendix section E for the characterization and solution for the social planner’s problem in a two-sector economy.
That is, $1 - \delta_n$ employed workers on island $n$ from last period remain employed, while $f_n(\theta_n)$ stayers in $n$ find a job. The stock of stayers in sector $n$ at time $t + 1$ is given by:

$$s'_n = \pi_{nn} \left[ s_n \left[ 1 - f_n(\theta_n) \right] + \delta_n e_n + \sum_{k \in N} m_{kn} \right]$$

A fraction $\pi_{nn}[1 - f_n(\theta_n)]$ of stayers in $n$ from last period do not find a job and choose to remain stayers. The inflow consists of $\delta_n e_n$ employed workers from last period who lose their jobs and choose to remain stay unemployed, and movers searching in $n$ from all other sectors $k$ who get absorbed into sector $n$ and choose to remain stayers in $n$. Finally, the stock of movers in sector $n$ moving to sector $j$ at time $t + 1$ is given by:

$$m'_{nj} = \pi_{nj} \left[ e_n \delta_n + s_n[1 - f_n(\theta_n)] + \sum_{k \in N} m_{kn} \right]$$

That is, $\pi_{nj}\delta_n e_n$ employed workers in sector $n$ lose their job and choose to become move unemployed in $n$, moving toward sector $j$. A fraction $[1 - f_n(\theta_n)]\pi_{nj}$ stayers in $n$ do not find a job and choose to become move unemployed in $n$ to some sector $j$. Finally, some movers from sector $k$ who get absorbed in $n$ choose to turn their search efforts to some sector $j$ rather than remaining stayers in sector $n$.

### 3.7 Equilibrium

Letting the final consumption good be the numeraire of this economy, and normalizing the total labor force size to one, an equilibrium is an allocation $\{s_{n,t}, m_{nj,t}, e_{n,t}, \theta_{n,t} \forall n, j \in N\}_{t=1}^\infty$ and a set of prices $\{p_{n,t} \forall n \in N\}_{t=1}^\infty$, value functions $\{W_{n,t}, S_{n,t}, M_{nj,t} \forall n, j \in N\}_{t=1}^\infty$, and move probabilities $\{\pi_{nj,t} \forall n, j \in N\}_{t=1}^\infty$, such that given $\{s^0_n, m^0_{nj}, e^0_n \forall n, j \in N\}$, wages $\{\tilde{w}_n\}_{n=1}^N$, and the evolution of sectoral productivities $\{\mu_{n,t}\}_{n=1}^N$:

1. The free entry condition holds for all sectors $n \in N$, $V_n = 0$, which implies:

$$\frac{c_n}{\beta q_n(\theta_{n,t})} = \frac{p_{n,t} \mu_{n,t} - \tilde{w}_n}{1 - \beta(1 - \delta_n)} \quad \forall n \in N$$
2. Unemployed workers \( \{s_{n,t}, m_{n,j,t}\}_{n=1}^{N} \) choose where to search to maximize utility so that move probabilities satisfy Equation (3.8)

3. Given \( \{s_{n,t}, m_{n,j,t}, e_{n,t}\}_{n=1}^{N} \), firms in each sector \( n \in N \) optimally post vacancies \( v_{n} \) so as to maximize profits

4. The evolution of employment in each sector \( n \in N \) follows Equation (3.12)

5. The evolution of stay unemployment in each sector \( n \in N \) follows Equation (3.13)

6. The evolution of move unemployment in each sector \( n \in N \) follows Equation (3.14)

7. The intermediate goods market clears on every island:

\[
y_{n,t} = \mu_{n,t} e_{n,t} = \frac{Y_{t} \tau_{n}}{(p_{n,t})^{\sigma}} \quad \forall n \in N
\]

8. Value functions in each sector \( \{W_{n,t}, S_{n,t}, M_{n,j,t} \ \forall n, j \in N\} \) are given by Equations (3.5), (3.6), and (3.7).

### 3.8 Model Features

The model exhibits positive gross flows of unemployed workers across sectors, even absent sectoral productivity shocks. Since workers draw idiosyncratic shocks every period, there will always be a positive number of workers finding it optimal to switch sectors. This feature of the model accords well with the data, in which gross flows of workers across sectors are always positive and larger than net flows. For example, the CPS monthly data I use to categorize unemployed workers suggest that, on average, approximately ten percent of unemployed workers in any given month will find work in a sector other than their sector of last employment. This is about five times larger than the average net flows observed in the data.
Second, the model displays slow adjustment of sectoral labor allocations and thus aggregate unemployment in response to sectoral shocks.\textsuperscript{17} To see this, consider a two sector economy that is in a steady state where sectoral productivities are constant and net flows are zero. In response to a permanent shock which changes the desired allocation of labor across sectors, the economy does not adjust instantaneously since some workers still find it optimal to stay where they are given their current realizations of taste shocks. Net flows increase as a fraction of workers move to the relatively more productive sector, which then lowers the difference in values of unemployment across sectors. In the next period when taste shocks are drawn again, there is still positive net reallocation, but net reallocation declines as the difference in values across sectors declines. This process continues until the new desired allocation is achieved and net flows return to zero.

Third, consistent with evidence found in Loungani and Rogerson (1989), the model will feature an inflow of labor into cyclically sensitive sectors during booms and an outflow of labor from cyclically sensitive sectors during recessions. The intuition becomes clear when we examine the move probabilities: since workers care about differences in sectoral payoffs, sectors that are hit relatively worse during recessions will on average experience more workers deciding to leave. These same sectors will experience inflows during booms when they become relatively more productive.

4 Calibration

In this section I calibrate the model at a monthly frequency. I work with two sectors ($N = 2$), construction (C) and non-construction (NC). I choose this dichotomy so that I can analyze the movements in unemployment in construction in the counterfactual exercises that are specific to the 2008 recession. Given this two-sector calibration, there are 21 parameters governing the system, summarized in Table 1.

\textsuperscript{17}The impulse responses from the calibrated model suggest that when the economy is hit by a 1 standard deviation shock in one sector, it takes 3.3 years to reach the new steady state.
Table 1: Parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>elasticity of substitution</td>
</tr>
<tr>
<td>$\beta$</td>
<td>discount rate</td>
</tr>
<tr>
<td>$\tau_n$</td>
<td>sectoral CES demand shares ${C, NC}$</td>
</tr>
<tr>
<td>$\mu_n$</td>
<td>sectoral labor productivity ${C, NC}$</td>
</tr>
<tr>
<td>$g$</td>
<td>vacancy share in matching function</td>
</tr>
<tr>
<td>$\eta$</td>
<td>worker’s bargaining power</td>
</tr>
<tr>
<td>$\Gamma_n$</td>
<td>sectoral match efficiency ${C, NC}$</td>
</tr>
<tr>
<td>$b$</td>
<td>value of leisure for unemployed</td>
</tr>
<tr>
<td>$\delta_n$</td>
<td>sectoral separation probability ${C, NC}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>variance of taste shocks</td>
</tr>
<tr>
<td>$c_n$</td>
<td>sectoral vacancy creation cost</td>
</tr>
<tr>
<td>$\zeta_n$</td>
<td>sectoral variance parameter in AR(1) process ${C, NC}$</td>
</tr>
<tr>
<td>$\kappa_n$</td>
<td>sectoral autocorrelation parameter in AR(1) process ${C, NC}$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>correlation between sectoral innovations in AR(1) processes $\nu_n$</td>
</tr>
</tbody>
</table>

4.1 Calibrated Parameters

To reduce the number of parameters to be estimated, I fix the following parameters. I choose a monthly discount rate of $\beta = 0.997$, which corresponds to an annual interest rate of 4 percent, and an elasticity of substitution of $\sigma = 2.00$. Broda and Weinstein (2006) find that the median elasticity for three-digit sectors is about 2.2. I choose an elasticity lower than this median because the level of disaggregation here is lower. I calculate sectoral separation probabilities using sectoral monthly labor force data from the CPS. Following Shimer (2005), the empirical monthly separation probability is calculated via:

$$\delta_n(t) = \frac{u_n^{short}(t + 1)}{e_n(t)[1 - .5\hat{f}_n(t)]}$$

where $u_n^{short}(t + 1)$ corresponds to the level of short term unemployed workers from sector $n$ (workers who separated from their job within the last month), $e_n(t)$ corresponds to the level of employment in sector $n$ at time $t$, and $\hat{f}_n(t)$ corresponds to the monthly job finding...
probability of workers last employed in sector $n$. I find that the mean monthly separation probability from 1976-2002 for construction is $\delta_c = 0.05$, while the mean monthly separation probability for non-construction over the same time period is $\delta_{nc} = 0.03$.

The parameters $\tau_n$, the CES demand shares, will govern the employment shares across sectors. I choose these to match the average share of unemployment in construction and non-construction from 1976-2002. I set $\tau_C = 0.07$ and $\tau_{NC} = 0.93$, and assume that mean labor productivity in each sector is equal to one. I fit the AR(1) process for detrended log productivity in each sector, $\mu_n$, by matching data on sectoral employment dynamics.

Recall that the process for sectoral productivity is as follows:

$$\log(\mu'_n) = \kappa_n \log(\mu_n) + \zeta_n \nu'_n$$

The variance of the detrended log employment rate in construction is $1.78e - 02$, while in non-construction its value is $5.69e - 03$. The monthly autocorrelations, $\kappa_C, \kappa_{NC}$, for the detrended log employment rates are 0.93 and 0.89 respectively. Assuming $\nu_{\mu_n} \sim N(0,1) \ \forall n \in \{C, NC\}$, solving for the implied $\zeta_n$ results in setting $\zeta_C = 0.013$, $\kappa_C = 0.76$, $\zeta_{NC} = 0.004$, and $\kappa_{NC} = 0.90$. Finally, allowing the two processes to be correlated implies setting the correlation between $\nu_{\mu_C}$ and $\nu_{\mu_{NC}}$ to 0.80 to match the correlation in the data for detrended log sectoral productivities in these two sectors, 0.78. I fix $\eta = g$ so that the worker’s bargaining power is equal to the elasticity of the matching function and the Hosios (1994) condition holds if wages were flexible, as described in Section 3.5.

---

18 This job finding probability is not the same as $f$ in the model. $\hat{f}_n$ is the job finding probability of all workers who were last employed in sector $n$, regardless of whether they subsequently become movers or stayers.

19 Without data on intermediate goods prices, I cannot separately identify $\tau_n$ from $\mu_n$. Since shocks to these variables move all the endogenous variables in the same way (except for intermediate goods prices), I do not focus on separately identifying the two. Thus, the shocks to labor productivity I identify are really combinations of supply and demand shocks. Furthermore, the model allows one normalization anyway: since doubling the parameters $\{b, c_n, \mu_n, \rho\}$ doubles all the value functions and output, but does nothing to the allocation of workers across sectors, I am permitted to normalize one of the $\mu_n$ regardless.

20 One could also include these parameters as part of the SMM procedure described below, but given the computational intensity of the algorithm, I begin by calibrating them independently.

21 See the Appendix Section F for a detailed derivation of how to recover the implied parameters governing the AR(1) processes given the parameters governing the AR(1) processes for employment in the data.
Table 2: Average Monthly Gross Flows of Unemployed Workers: 1976-2008

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>NC</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.69</td>
<td>0.30</td>
</tr>
<tr>
<td>NC</td>
<td>0.06</td>
<td>0.94</td>
</tr>
</tbody>
</table>

The remaining parameters are estimated via Simulated Method of Moments (SMM). This reduces the system to six parameters which need to be estimated, \( H = [\gamma_C \gamma_{NC} \rho \sigma_C \sigma_{NC} g] \).

4.2 Estimated Parameters

Before launching into the SMM estimation of the remaining parameters, I describe here how I construct model moments that will be equivalent to moments we observe in the data. Respondents in the CPS are interviewed for four consecutive months, then interviewed for another 4 consecutive months 8 months after the first rotation. In addition, when we look at the CPS respondents, we can observe a worker’s sector of last employment, but we cannot observe where they are searching. Therefore, limiting the sample to workers who were in the labor force throughout the first four month period, I identify movers in the CPS in any given month to be any unemployed workers who were last employed in some sector \( n \) who subsequently find a job in sector \( j \) within the months I can follow them.\(^{22}\) Table 2 reports summary statistics from the CPS on movers out of total unemployment.

In my model, there are some workers labeled as “movers” from \( n \) to \( j \) who will ultimately find a job in a sector other than \( j \). For example, suppose a stayer in sector \( n \) receives a vector of taste shocks in period \( t \) that compels her to become a mover from \( n \) to \( j \). She reaches sector \( j \) in period \( t+2 \), but randomly gets a vector of taste shocks that compel her to move back to sector \( n \). To be consistent with the definitions in the CPS, if this unemployed worker never finds employment in a new sector, we should not count her as a mover. Thus, I must

\(^{22}\) Some unemployed workers do not find a job within this four month period and cannot be categorized as movers or stayers. I assume all these unclassified workers are stayers (“thirty percent of unemployed). In future work, I intend to exploit the full-panel structure of the CPS and to allow right-censored unemployment spells of workers to be categorized as censored.
correctly determine the fraction of model movers who are the data equivalent of movers described above. Similarly, I must categorize a worker as belonging to sector $n$ only when her sector of last employment was sector $n$.

Fortunately, the model setup allows me to do this easily. To do so, I take the simulated model and create a dataset equivalent to the CPS as follows. Using the simulated time path for the value functions, I take random Gumbel draws of the taste shocks for two sectors for one million people (enough to eliminate small simulation errors due to the draws from the T1EV) over 250 months (approximately 20 years).\footnote{In future work, I plan to only track workers for four months in exactly the same way as the CPS rather than assuming that the inferences I draw from the 4 month panel in the CPS are representative.} The realized taste shocks for each individual combined with the value functions are all that is necessary to compute decision rules for each unemployed worker according to Equation 3.8. Once I follow the decision rules, I can track their employment histories and categorize data movers and stayers (as opposed to model movers and stayers) and the sectors they belong to in the same way as was done in the CPS. To deal with the fact that in the beginning of this simulated CPS sample I do not know where workers were last employed, I drop the first half of the sample. This is long enough to be able to completely categorize a worker’s sector of last employment. In what follows, all moments that are reported are the moments taken from this simulated dataset so that the model moments and moments calculated from the data are equivalent.

I estimate the remaining six parameters (one match efficiency per sector, one vacancy flow cost per sector, the variance of the taste-shocks, and the elasticity of the matching function) using the Simulated Method of Moments. The moment condition is of the form:

$$E[G(H_0)] = 0$$

where $H_0$ is the true value of $H = [\Upsilon_C \ Upsilon_{NC} \ \rho \ \epsilon_C \ \epsilon_{NC} \ \theta \ ]$. The SMM estimator is then given
by:

\[
\hat{H} = \arg \min_{H} [G(H)'W G(H)]
\]

where \( W \) is a 6 by 6 weighting matrix and \( G(H) \) is the 6 by 1 vector of moments that are a function of the parameters to be estimated, \( H = [\Upsilon_C \ Upsilon_{NC} \ \rho \ c_C \ c_{NC} \ g] \). The six moment conditions used to estimate the parameters are as follows. The match efficiencies as well as the vacancy creation costs will in part determine the unemployment rates and mover rates across sectors, so I include these as moments in the SMM procedure. All else equal, a higher match efficiency will lead to a lower unemployment rate as well as a lower mover rate as job-finding probabilities rise. Next, the parameter \( \rho \) governs how likely it is for unemployed stayers to decide to move, so I choose to match the fraction of movers out of unemployed workers in each sector.\(^{24}\) Finally, the elasticity of the matching function will determine the relationship between labor market tightness and job-finding probabilities in time series data. Thus, the moment I choose to match is the coefficient on labor market tightness from the

\(^{24}\)In future work, I plan to use the responsiveness of labor mobility to differences in sectoral payoffs as the moment to calibrate \( \rho \). Higher values of \( \rho \) correspond to less responsive labor mobility to differences in sectoral job-finding probabilities. Letting \( X = M_{nj}(\Omega') - S_n(\Omega') \)

\[
\frac{\partial \pi_{nn}}{\partial [X]} = \frac{1}{\rho} \frac{\exp(\frac{X}{\rho})}{\left[ \sum_{k \in N} \exp(\frac{X}{\rho}) \right]^2}
\]

The limit of this derivative as \( \rho \) goes to infinity is given by

\[
- \left[ \lim_{\rho \to \infty} \frac{1}{\rho} \right] \left[ \lim_{\rho \to \infty} \frac{\exp(\frac{X}{\rho})}{\left[ \sum_{k \in N} \exp(\frac{X}{\rho}) \right]^2} \right] = 0 \Rightarrow 1 = 0
\]

Thus, if the variance of the shocks is large, there will be several workers realizing taste shocks that will induce mobility regardless of sector differences in job-finding probabilities and wages. Any changes in these differences will not change the move decisions of workers. On the other hand, if the variance in taste shocks is small, there will be a large number of workers close to the cutoff point for movement. In this case, even slight changes in sectoral job-finding probabilities will induce labor mobility so that labor mobility will be more responsive to movements in sectoral productivity over the cycle.
Table 3: Results from SMM Estimation

<table>
<thead>
<tr>
<th>Model Data</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>stayer share of unemployed, construction</td>
<td>0.81</td>
<td>0.69</td>
</tr>
<tr>
<td>stayer share of unemployed, non-construction</td>
<td>0.97</td>
<td>0.94</td>
</tr>
<tr>
<td>unemployment rate, construction</td>
<td>0.11</td>
<td>0.09</td>
</tr>
<tr>
<td>unemployment rate, other</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>job-finding probability, construction</td>
<td>0.44</td>
<td>0.41</td>
</tr>
<tr>
<td>job-finding probability, non-construction</td>
<td>0.53</td>
<td>0.40</td>
</tr>
<tr>
<td>aggregate matching function regression</td>
<td>0.64</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Table 4: Other Moments

<table>
<thead>
<tr>
<th>Model Data</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>relative durations (movers/stayers)</td>
<td>1.71</td>
<td>1.57</td>
</tr>
<tr>
<td>employment share, construction</td>
<td>0.07</td>
<td>0.07</td>
</tr>
</tbody>
</table>

following regression in the CPS:

\[
\ln \left( \frac{h_{nt}}{u_{nt}} \right) = \ln(\Phi_t) + \ln(\tilde{\Upsilon}_n) + (1 - \tilde{g}) \ln \left( \frac{v_{nt}}{u_{nt}} \right)
\]

where \( \tilde{\Upsilon}_n \) is the sector-specific match-efficiency parameter estimated in the data, \( \Phi_t \) is the time-varying match-efficiency, \( h_{nt} \) are the number of hires in sector \( n \) at time \( t \), \( v_{nt} \) is the number of vacancies in sector \( n \) at time \( t \), and \( u_{nt} \) is the number of unemployed workers in sector \( n \) at time \( t \). I can run a similar regression with my simulated CPS dataset.

I use the method of Simulated Annealing to search for the parameters that minimize the moment function given by Equation 4.1. The algorithm is explained in detail in the appendix of Dell (2011). Table 4 reports the results from the estimation.

5 Counterfactual Experiments

In this section, I assess the importance of slow intersectoral labor reallocation in explaining aggregate unemployment, both historically and in the recent recession. In Section 5.2, I compare unemployment in the model with moving costs (the “true” model) to unemployment
when intersectoral labor reallocation is frictionless.\footnote{I am in the process of solving the model when moving costs are infinite.}

I then turn my focus to the recent recession in which the dispersion of shocks across sectors may have been large given the linkages of the recession to the boom and bust in the housing market. I analyze the Lilien (1982) hypothesis and examine how move unemployment would have changed if the shocks had been more symmetric and looked more like an aggregate shock rather than a shock which required net reallocation of labor across sectors. This is described in Section 5.3. Finally, Section 5.4 examines how the boom in construction prior to the recession may have created a misallocation of labor, and led to a larger need for net reallocation of construction workers once the 2008 recession reversed the run-up in the housing market.

5.1 Recovering the Shocks

To run these counterfactuals, I first recover the shocks which hit construction and non-construction from 1977-2012 that are consistent with observed sectoral employment dynamics in the data conditional on the estimated parameters from Section 4. I take a second order approximation of the model around its deterministic steady state which gives an approximate rule for how the model’s endogenous variables respond following exogenous shocks to sectoral productivities.\footnote{In particular, I first solve for the shocks using a first order approximation of the model, and use these recovered shocks as the starting values for the search with the second order approximation. I use the second order approximation because it is more accurate, especially in episodes when there were large swings in employment from the steady state.}

Recall that the evolution of sectoral labor productivities follows:

\[
\log(\mu'_n) = \kappa_n \log(\mu_n) + \zeta_n \nu'_n
\]

Thus, combining the second order approximation with a time series on two of the model’s endogenous variables implies that solving for the underlying shocks reduces to solving a system of two non-linear equations for every period \(t\) in the shocks, \(\nu_{\mu C,t}\) and \(\nu_{\mu NC,t}\). Given the time-series on employment in construction and the rest of the economy, I assume the

\[
\log(\mu'_n) = \kappa_n \log(\mu_n) + \zeta_n \nu'_n
\]
economy was in a steady state in January 1977 and back out the time path of the shocks consistent with observed sectoral employment dynamics. The time series for employment can be found in Figure 2 while the recovered paths of sectoral productivity can be found in Figure 3.27 A more complete description of how I recover the shocks is described in the Appendix, Section G.

**Figure 2: Time Series of Sectoral Employment**

![Time Series of Sectoral Employment](image)

*Notes:* Both series were constructed using data on employment from the Current Employment Statistics and labor force data from the CPS. To be consistent with the model, the series created is employment in each sector relative to the total labor force. The red line connected by hollow circles represents the hp-filtered trend of monthly employment in construction relative to the total labor force, with smoothing parameter 14400. The blue solid line represents the hp-filtered trend of monthly employment in non-construction relative to the total labor force, with smoothing parameter 14400. The dotted lines represent their unfiltered values.

27The employment series I use is total employment in each sector relative to the aggregate labor force so that the model generated $e_n$, which is employment in each sector relative to the total labor force when the labor force is normalize to 1, will be equivalent to the time series in the data and I can use the second order approximation directly.
Figure 3: Estimated Paths of Sectoral Productivity

Notes: The red line connected by hollow circles represents the hp-filtered trend of monthly labor productivity in construction with smoothing parameter 14400. The blue solid line represents the hp-filtered trend of monthly labor productivity in non-construction with smoothing parameter 14400. The dotted lines represent their unfiltered values.

Since I fit the labor productivity shocks to movements in employment, construction labor productivity displays larger variance over the cycle. In addition, the shocks are able to pick up the housing market boom, which drew many workers into construction. This phenomenon corresponds to the larger deviation in sectoral shocks beginning in the early 2000s. The deviation is so large because the relative shift of employment into construction during the boom was unprecedentedly large.

5.2 Counterfactual 1: Frictionless Benchmark

In this exercise, I ask how much unemployment over the cycle is caused by the presence of intersectoral mobility frictions in the form of extra time spent unemployed. Given the two-sector calibration, the measured fraction of unemployment attributed to these frictions will be a lower bound, as the estimate does not account for the labor mobility between sectors I have lumped into non-construction. Nonetheless, the exercise provides an idea of the magnitudes of overall unemployment from this channel.

I solve the model in which labor mobility across sectors takes no extra time, thus re-
moving the state of model move unemployment.\textsuperscript{28} Workers are now choosing between stay unemployment between sectors. In this two sector case, the relevant probability of moving becomes:

\[
\pi_{12} = \frac{1}{1 + \exp(S_1 - S_2)} = 1 - \pi_{21}
\]

I then construct the second order approximation of the model without moving time around its deterministic steady state, and use the approximation combined with the recovered shocks from Section 5.1 to trace out the path of unemployment when labor can move freely across sectors. Figure 4 depicts both the realized path of unemployment as well as the path of unemployment in the hypothetical world with no moving costs.

**Figure 4:** Hypothetical Aggregate Unemployment Rate without Moving Costs

![Figure 4](image)

*Notes:* The black line represents the unemployment rate constructed using the estimated shocks found in 5.1 combined with the second order approximation of the model with moving time, which mimics the dynamics of unemployment in the data. The pink line connected by squares represents the hypothetical unemployment rate in the model without moving time using the same estimated shocks.

The overall level of unemployment in the hypothetical economy in which moving takes no extra time is always lower than the unemployment in the “true” economy. On average, the unemployment rate falls by 0.67 percentage points. This reduction comes from two different channels. First, holding the number of movers fixed, but lowering their unemployment

\textsuperscript{28}That is, there will still be data movers, but I remove the notion of extra time spent in unemployment in the model.
duration lowers unemployment as the flow into employment increases. Second, the absence of moving costs allows for a more efficient allocation of labor, as workers who previously did not move choose to reallocate to the more productive sector. The better allocation of labor incentivizes vacancy creation and thus lowers unemployment.

To tease out the importance of these two effects, I compute a simple calculation. I take all movers in the true model, then assume these workers would find a job according to the sectoral job finding probabilities of the true model. I find that 0.34 percentage points of unemployment can be attributed to the decline in unemployment duration associated with these movers. The remaining 0.32 percentage points can be attributed to the more efficient allocation of labor following the elimination of moving costs.

I also examine how these types of moving costs impact the Beveridge curve relationship (the relationship between vacancies and unemployment) as well as the matching function one would estimate from the data. Figure 5 plots the relationship between vacancies and unemployment in the hypothetical world without moving costs as well as in the world with moving costs. The world with moving costs is associated with an outward shift of the Beveridge curve relative to the frictionless world so that the vacancy rate is 0.01 percentage points higher at every level of unemployment; the job-finding probability is 15.21 percent lower at every level of labor market tightness. Thus, the model has the power to generate endogenous shifts in the Beveridge curve.29

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29While I only have a two-sector calibration with one extra period of unemployment for switchers, one can imagine that an N-sector calibration where switching time is origin-and-destination specific would lead to constant inward and outward shifts of the Beveridge curve. The amount of movers as a fraction of unemployment will change depending on how costly it is to switch sectors. If relative demand shifts occur between sectors that are closely related in terms of human capital so that moving takes no time, this might induce an inward shift. Conversely, if relative demand shifts occur between sectors that are more different so that moving takes longer, this would induce an outward shift.
Figure 5: Hypothetical Aggregate Beveridge Curve without Moving Costs

Notes: The black dots represent the unemployment rate and vacancy rate constructed using the estimated shocks found in 5.1 combined with the second order approximation of the model with moving time. The pink squares represent the hypothetical unemployment rate and vacancy rate in the model without moving time using the same estimated shocks.

5.3 Counterfactual 2: Symmetric Shocks in the 2008 Recession

Lilien (1982) argues that the empirical correlation between dispersion in employment growth, a proxy for the dispersion in sectoral shocks, and aggregate unemployment is evidence that some unemployment is due to the slow movement of labor across sectors in response to sectoral shifts. The underlying assumption is that dispersion of shocks leads to an increase in net reallocation of labor which in turn increases aggregate unemployment through higher unemployment duration for movers. According to this logic, we might expect that the recent recession, which affected the construction sector relatively worse than other sectors of the economy, increased the need for net reallocation and thus unemployment due to movers.

To evaluate the validity of the Lilien (1982) hypothesis in the most recent downturn, I ask what would have happened to move unemployment if shocks to labor productivity in construction and non-construction were more symmetric. Specifically, I construct a hypothetical path for productivity in the two sectors so that (i) aggregate employment (and thus unemployment) are equal to the same values as in the true economy over the length of the
recession and (ii) the employment shares in construction and non-construction are constant beginning in 2008.\textsuperscript{30} In this way, the hypothetical shocks will lead to an economy in which the unemployment rate is the same as in the true economy, but there is no need for net reallocation.

The hypothetical shocks can be found in Figure 6. The shocks which would have been necessary to keep net reallocation at zero are more symmetric. Hypothetical labor productivity in construction is higher relative to its estimated productivity, while hypothetical labor productivity in non-construction is relatively lower, bringing the two productivity levels closer together. Figure 7 plots the realized and hypothetical path of unemployment due to gross movers (move unemployment) and the fraction of unemployment due to net movers in both the hypothetical and true scenario.\textsuperscript{31}

**Figure 6:** Estimated and Hypothetical Paths of Sectoral Productivity in construction

![Graph showing estimated and hypothetical paths of sectoral productivity in construction](image)

*Notes:* The black solid line represents the estimated path of sectoral productivity in construction, while the red starred line represents the hypothetical path of sectoral productivity in construction. The hypothetical shocks are constructed so that aggregate unemployment remains the same in the two counterfactuals, but so the employment shares in the hypothetical world remain constant beginning in 2008, as described in Section 5.3.

\textsuperscript{30}The first restriction is so that I hold the depth of the recession the same. The second restriction is to remove any need for net reallocation of labor.

\textsuperscript{31}If gross reallocation is given by $m_{12} + m_{21}$, then net reallocation is given by $|m_{12} - m_{21}|$.
Figure 7: Fraction of Unemployment Due to Gross Reallocation (Move Unemployment) and Net Reallocation

Notes: The black solid line in the left panel represents the move unemployment rate constructed using the estimated shocks found in 5.1, while the black dashed line in the right panel represents the fraction of unemployment due to net movers constructed using the same estimated shocks. The pink line connected by circles in the left panel represents the hypothetical move unemployment rate in the hypothetical scenario, while the pink line connected by circles in the right panel represents the hypothetical fraction of unemployment due to net movers.

By construction, the fraction of unemployment due to net movers depicted in the right panel of Figure 7 tends to zero since the shocks require no net reallocation. Net reallocation does not immediately jump to zero because some leftover reallocation must take place depending on where the economy begins. I return to the position of the economy beginning in 2008 in the next section. Gross reallocation, on the other hand, does not tend to zero. In fact, in this specific scenario gross reallocation in the hypothetical world where shocks are more symmetric increases relative to the true scenario in which shocks are more dispersed.\textsuperscript{32}

\textsuperscript{32}The reason for the increase is because, in such a model, whenever the sectors become more similar to one another, gross reallocation increases.
Figure 8: Estimated and Hypothetical Number of Movers in construction and non-construction

Notes: The dotted lines represent the estimated path of movers in construction (red) and the solid line represents the estimated path of movers in non-construction (blue). The red line connected by circles represents the hypothetical path of movers in construction and the blue line connected by stars represents the hypothetical path of movers in non-construction. The hypothetical shocks are constructed so that aggregate unemployment remains the same in the two counterfactuals and the employment shares in the hypothetical world remain constant beginning in 2008, as described in Section 5.3. All values are expressed relative to their values in 2008m1.

What is the reason a decline in net reallocation is not associated with a similar decline in gross reallocation? The intuition is as follows. Consider a world in which gross reallocation is large; therefore, at any point in time, workers are moving back and forth simultaneously between these two sectors. When a shock comes that requires net labor reallocation from construction to non-construction, this reallocation does not necessarily occur through a simple increase in the number of movers from construction to non-construction. Instead, some who may have previously moved from non-construction to construction no longer move and more people move from construction to non-construction. In the aggregate, the level of total movers may remain unchanged. That is, when gross reallocation is not equal to net reallocation, some of the response to sector-specific shocks works through changes in the composition of gross reallocation.

To see this, I plot the number of movers in both construction and non-construction in
the hypothetical and true scenario relative to their initial values before the new shock (when they are equal) in Figure 8. As the intuition describes, the world with more symmetric shocks features both fewer movers from construction to non-construction as well as more movers from non-construction to construction.

5.4 Counterfactual 3: No Housing Boom

Taking the Lilien (1982) logic one step further, the housing boom might have been responsible for some unemployment if it induced a large fraction of labor to move to construction in the boom period that only needed to reallocate again during the bust. In this section, I perform a similar exercise as the previous counterfactual, except I solve for the path of shocks that would keep sectoral employment shares constant beginning in 2002, when the housing boom started to take off. The hypothetical paths for sectoral productivity for this exercise can be found in Figure 9.

**Figure 9:** Estimated and Hypothetical Paths of Productivity in construction and non-construction

![Graph showing estimated and hypothetical paths of productivity in construction and non-construction](image)

**Notes:** The black solid line represents the estimated path of sectoral productivity in construction, while the red starred line represents the hypothetical path of sectoral productivity in construction. The hypothetical shocks are constructed so that aggregate unemployment remains the same in the two counterfactuals, but so the employment shares in the hypothetical world remain constant beginning in 2008, as described in Section 5.3.

33 The rise in house prices started earlier, but I want to analyze the interaction between the housing boom that occurred at the same time as the boom period prior to the recession.
Figure 10: Estimated and Hypothetical Paths of Gross Reallocation (Move Unemployment)

Notes: The black solid line in the left panel represents the move unemployment rate constructed using the estimated shocks found in 5.1. The pink line connected by circles represents the hypothetical move unemployment rate.

Consistent with the results from earlier experiments, while the housing boom increased the need for net reallocation during the recession, it hardly increased the amount of unemployment due to movers. Aggregate gross reallocation over the entire period is basically unchanged. The first result might be surprising since the counterfactual effectively shuts down large intersectoral net labor movements. Again, the underlying reason is that these net flows were effectively happening through the composition of gross reallocation and not through its level. Second, gross reallocation would have been lower during the housing boom and slightly higher during the bust. The reason for the asymmetry is that the construction sector was large between 2002 and 2007. In the counterfactual construction is significantly smaller thus reducing the number of workers flowing into and out of that sector.\textsuperscript{34} The reason for the slight increase in gross reallocation during the recession is similar. Note, however, that the effect for the recession is much smaller compared to Figure 7 owing to the fact that Figure 7 starts at the housing buildup, whereas this counterfactual does not allow for the buildup.

These last two counterfactuals combined suggest that changes in net labor reallocation

\textsuperscript{34}This is the same force driving the results behind the increase in gross flows in Counterfactual 2.
do not necessarily increase unemployment due to movers. While asymmetric shocks induce more movers out of the sector hit relatively worse, they also induce less movement from relatively better off sectors.

5.4.1 An Illustrative Example

I have argued that a shock that requires net reallocation across sectors hardly increases gross reallocation and therefore move unemployment. To illustrate this, consider a simplified version of the model. Suppose the economy consists of two sectors \( n = \{1, 2\} \) and movers do not need to spend additional time in unemployment. Instead, movers are immediately absorbed within one period. Then, the assignment of unemployed workers is a repeated static choice. An unemployed worker in sector 1 chooses to move or stay according to

\[
\max \{S_1 + \varepsilon_1, S_2 + \varepsilon_2\}
\]

In the steady state, gross (G) and net (N) flows are given by:

\[
G = \pi_{12}U_1 + \pi_{21}U_2
\]

\[
N = |\pi_{12}U_1 - \pi_{21}U_2| = 0
\]

where

\[
\pi_{12} = \frac{1}{1 + \exp[S_1 - S_2]} = 1 - \pi_{21}
\]

and \( U_n \) for \( n \in \{1, 2\} \) represents the level of unemployment in sector \( n \). From the above, we can write that in steady state:

\[
\frac{\pi_{12}}{\pi_{21}} = \frac{U_2}{U_1} = \exp[S_2 - S_1] = \tilde{d}
\]

Now consider a shock that increases \( S_2 \) relative to \( S_1 \) so that \( X = S_2 - S_1 \) increases, but
corresponds to the same level of aggregate unemployment. Write $U_2 = \bar{d}U_1$

$$\frac{\partial G}{\partial X} = \frac{\exp(S_1 - S_2)}{[1 + \exp(S_1 - S_2)]^2}U_1 - \frac{\exp(S_1 - S_2)}{[1 + \exp(S_1 - S_2)]^2} \bar{d}U_1$$

$$\frac{\partial N}{\partial X} = \frac{\exp(S_1 - S_2)}{[1 + \exp(S_1 - S_2)]^2}U_1 + \frac{\exp(S_1 - S_2)}{[1 + \exp(S_1 - S_2)]^2} \bar{d}U_1$$

Note that $\frac{\partial N}{\partial X} > 0$, but $\frac{\partial G}{\partial X}$ will depend on the sign of $1 - \bar{d}$. Putting the two together:

$$|\frac{\partial G}{\partial X}| = \frac{\partial N}{\partial X} |\frac{1 - \bar{d}}{1 + \bar{d}}|$$

For there to be any meaningful movements in gross flows when net flows increase, one of the following must be true. Either:

- $\bar{d} = 0$, which is the case when we have one sector, and on impact gross flows equal net flows

- The change in net flows must be large. Let $\tilde{d} = \frac{1 - \bar{d}}{1 + \bar{d}}$ and consider the percentage change in gross flows. Since gross flows are about ten times net flows in the data, we can write:

$$|\frac{\Delta G}{G}| = \frac{\Delta N}{G} |\tilde{d}| = \frac{\Delta N}{10N} |\tilde{d}|$$

Then for gross flows to increase by ten percent, it must be that

$$\frac{\Delta N}{N} \tilde{d} = 1$$

Now suppose sector 1 represents construction and sector 2 represents non-construction. In this case, $\tilde{d} \sim .8$. Then $\frac{\Delta N}{N}$ would need to be 1.25, corresponding to an increase in net flows of over a one hundred percent. Of course, this example is highly stylized. The shocks I analyze in the counterfactuals are temporary shocks, so movements in net flows would be much lower than the ones I calculate above under more permanent shocks. Nonetheless, this
simple example shows that gross flows move one-for-one with net flows only in extreme cases of the world. It also suggests that more permanent shocks which do increase net flows may have significant impacts on gross flows and thus unemployment, for example, the long run shift from Manufacturing to Services.

6 Conclusion

This paper develops a tractable multisector equilibrium search model of labor reallocation to study the importance of intersectoral mobility frictions in explaining aggregate unemployment. In a version of the model calibrated to construction and non-construction, I find that mobility frictions in the form of higher unemployment durations significantly contribute to unemployment. First, these frictions impede the efficient movement of labor across sectors in response to sector-specific shocks. Second, they increase average unemployment duration in the aggregate by increasing the unemployment duration for those who choose to move in response to those shocks. Given the two sector calibration, this is likely a lower bound for the importance of these types of mobility frictions.

I then ask whether the importance of labor reallocation in explaining aggregate unemployment changed in the recent recession, when differences in sectoral shocks were pronounced. According to Lilien (1982), one might expect that the concentration of the recession in sectors closely tied to the housing market might increase the need for reallocation of workers in these sectors to other sectors of the economy, thereby increasing aggregate unemployment. While the nature of the sectoral shocks in the 2008 recession did require net labor reallocation, I find no significant increase in unemployment due to movers over this time period. I similarly study the importance of the housing boom in generating a large degree of misallocation given the nature of the shocks during the recession. The recession essentially overturned the pre-recession run-up in the share of unemployment in construction, so the housing boom is responsible for a large need for net reallocation thus unemployment. Consistent with the results from earlier experiments, while the housing boom increased the need
for net reallocation during the recession, it hardly increased the amount of unemployment due to movers.

These results highlight the importance of accounting for gross labor reallocation when quantifying the impact of reallocation on unemployment. In my model simulations, much of the response to sector-specific shocks works through changes in the composition of gross reallocation so that increased need for net reallocation hardly affects the total level of movers. Even though net reallocation moves in response to sector-specific shocks, which is consistent with the logic of Lilien (1982), the total number of movers remains relatively unchanged. The outflow from less productive sectors increases as the outflow from more productive sectors concurrently declines.

While I use the model to focus on the most recent recession and its uniqueness in terms of its relationship to the housing boom and construction workers, it has broader applications. In future work, I intend to use the model to study the effect of trade liberalization on unemployment dynamics in Manufacturing and Services. Artuc, et. al. (2010) have a similar setup, but they study the impact of trade liberalization on employment and wage dynamics. Given their emphasis on the welfare effects of trade liberalization, a natural extension would be to include unemployment, since these costs can be large and significant contributions to welfare losses, at least in the short run.\(^{35}\) While there are several papers that have begun to study the effects of trade liberalization on the labor market, to my knowledge none have incorporated its impact on unemployment dynamics.\(^{36}\)

One could also amend the model to think about more simple forms of mobility costs by removing the state of move unemployment and introducing a utility cost that might be sector or location specific, as in Kline (2008).\(^{37}\) This formulation of the model could be used in several studies. For example, one could measure mobility costs across different dimensions (space, sectors, or both) and think about how these costs might have changed over time.

\(^{35}\)See Davis and Wachter (2011), for example.

\(^{36}\)See, for example, Kamborou (2009) and Dix-Carneiro (2010).

\(^{37}\)The model becomes computationally simple when the extra state variable of move unemployment is removed.
both in absolute terms and relative to one another. This line of inquiry might be useful in understanding why interstate migration in the U.S. has declined drastically since the 1980s. For example, this decline may be driven by a decline in intersectoral moving costs that allows workers to more easily adjust to local shocks by changing sectors rather than locations.\textsuperscript{38} As another example, one could estimate spatial moving costs over time, and ask whether they have increased in the recent recession to test the “house lock” hypothesis.

Finally, the model could be related to recent research on Jobless Recoveries such as Berger (2012) and Jaimovich and Siu (2012). It is possible that in the past three jobless recoveries, output growth was not mirrored by employment growth because the growth in labor productivity was not experienced equally across sectors. If some workers need to leave declining sectors to enter new ones, we would observe longer spells of non-employment. Measured output per worker would grow coming out the recession as a larger fraction of production is undertaken by workers in more productive sectors. Since these kinds of structural changes are more permanent, net reallocation might lead to increases in gross reallocation and thus aggregate unemployment.

Perhaps the biggest opening for future work within this theoretical setting is in the modeling of the taste shocks. Thus far, I have assumed that shocks are iid over time and across sectors. One can imagine that certain workers are more likely to move than others. Workers who have accumulated human capital in a specific sector should be less willing to move in response to sectoral shocks. Introducing idiosyncratic shocks that are correlated over time and across sectors would give rise to rich unemployment dynamics within skill or age groups.

\textsuperscript{38}In work in progress with Erik Hurst and Kerwin Charles, we explore such a hypothesis.
A Some Useful Type 1 Extreme Value Properties

1. If $\varepsilon_n \sim Gumbel(\bar{\varepsilon}_n, \rho)$ and $\varepsilon_j \sim Gumbel(\bar{\varepsilon}_j, \rho)$, then
   
   $\varepsilon_n - \varepsilon_j \sim LOGISTIC(0, \rho)$

2. For $\varepsilon \sim LOGISTIC(0, \rho)$ random variable,
   
   $E(\varepsilon) = 0, \quad E(\varepsilon^2) = \frac{\pi^2}{3} \rho^2$

3. If $\varepsilon_n \sim Gumbel(\bar{\varepsilon}_n, \rho)$ for iid $n = 1, 2, 3, \ldots N$, then
   
   $\max(\varepsilon_n) \sim Gumbel(\log \left\{ \sum \exp(\bar{\varepsilon}_n) \right\}, \rho)$

4. For $\varepsilon_n \sim Gumbel(\bar{\varepsilon}_n, \rho)$,
   
   $E(\varepsilon_n) = \bar{\varepsilon}_n + \gamma \rho, \quad \text{Var}(\varepsilon_n) = \frac{\pi^2}{6} \rho^2$

B Derivation of equation (3.8)

Using the definition of $\tilde{S}_{nj}$, the probability that a stayer in $n$ chooses to become a mover from $n$ to $j$ is given by:

$\pi_{nj} = Pr \left( \tilde{S}_{nj}(\Omega') + \varepsilon_{ji} \geq \max_{k \neq n} \tilde{S}_{nk}(\Omega') + \varepsilon_{ki} \right)$

Using results from A, this becomes:

$\pi_{nj} = \frac{\exp(\tilde{S}_{nj}(\Omega')/\rho)}{\sum_{k \in N} \exp(\tilde{S}_{nk}(\Omega')/\rho)}$
C Derivation of Equations (3.5), (3.6), and (3.7)

We begin with the value of employment for a worker in an arbitrary sector $n$:

$$W_n(\Omega) = w_n$$

$$+ \beta \mathbb{E}\left\{ [1 - \delta_n] (W_n(\Omega') + \epsilon_{n,i}') + \delta_n \max\left( S_n(\Omega') + \epsilon_{n,i}', \max_{k \neq n \in N} M_{nk}(\Omega') + \epsilon_{k,i}' \right) \right\}$$

Integrating out the idiosyncratic taste shocks gives:

$$\beta \mathbb{E}\left\{ \delta_n \max\left( S_n(\Omega') + \epsilon_{n,i}', \max_{k \neq n \in N} M_{nk}(\Omega') + \epsilon_{k,i}' \right) \right\}$$

$$= \beta \mathbb{E}_{\Omega} \mathbb{E}_{\epsilon}\left\{ \delta_n \max\left( S_n(\Omega') + \epsilon_{n,i}', \max_{k \neq n \in N} M_{nk}(\Omega') + \epsilon_{k,i}' \right) \right\}$$

where the first expectation is taken over the aggregate state, and the second refers to the expectation over the idiosyncratic taste shocks. Simplifying further we get:

$$\beta \mathbb{E}_{\Omega} \mathbb{E}_{\epsilon}\left\{ \delta_n \max\left( S_n(\Omega') + \epsilon_{n,i}', \max_{k \neq n \in N} M_{nk}(\Omega') + \epsilon_{k,i}' \right) \right\}$$

$$= \beta \delta_n \mathbb{E}\left\{ \rho \log\left[ \sum_{k \in N} \exp(\tilde{S}_{nk}(\Omega')/\rho) \right] \right\}$$

where I use the fact that the expectation of a T1EV($-\rho\gamma, \rho$) variable is zero.

D Derivation of the Wage Equation Under Nash Bargaining

First derive the surplus for the worker, $W_n - S_n$:

$$(D.1) \quad W_n(\Omega) - S_n(\Omega) = w_n - b + \beta [1 - \delta_n - f_n(\theta_n)] \mathbb{E}\left\{ W_n(\Omega') - \rho \log\left[ \sum_{k \in N} \exp(\tilde{S}_{nk}(\Omega')) \right] \right\}$$
Applying the wage sharing rule \((1 - \eta)[W_n - S_n] = \eta J_n\) and substituting in for \(J_n(\Omega)\) gives:

\[
(1 - \eta)(w_n - b) + (1 - \eta)\beta[1 - \delta_n - f_n(\theta_n)] E\left\{W_n(\Omega') - \rho \log \left[ \sum_{k \in N} \exp(\tilde{S}_{nk}(\Omega')/\rho) \right] \right\}
\]

\[
= \eta(p_n - w_n) + \eta\beta(1 - \delta_n) E\{J_n(\Omega')\}
\]

Solving for \(w_n\) gives:

\[
w_n = (1 - \eta)b + \eta p_n
\]

\[
- (1 - \eta)\beta[1 - \delta_n - f_n(\theta_n)] E\left\{W_n(\Omega') - \rho \log \left[ \sum_{k \in N} \exp(\tilde{S}_{nk}(\Omega')/\rho) \right] \right\}
\]

\[
+ \eta\beta(1 - \delta_n)J_n(\Omega')
\]

Using the wage sharing rule for next period, \((1 - \eta)[W_n(\Omega') - S_n(\Omega')] = \eta J_n(\Omega')\) gives:

\[
w_n = (1 - \eta)b + \eta p_n
\]

\[
- (1 - \eta)\beta[1 - \delta_n - f_n(\theta_n)] E\left\{W_n(\Omega') - \log \left[ \sum_{k \in N} \exp(\tilde{S}_{nk}(\Omega')/\rho) \right] \right\}
\]

\[
+ (1 - \eta)\beta(1 - \delta_n) E\{W_n(\Omega') - S_n(\Omega')\}
\]

Adding and subtracting \((1 - \eta)\beta[1 - \delta_n - f_n(\theta_n)]S_n(\Omega')\) gives:

\[
w_n = (1 - \eta)b + \eta p_n
\]

\[
- (1 - \eta)\beta[1 - \delta_n - f_n(\theta_n)] E\left\{W_n(\Omega') - S_n(\Omega') + S_n(\Omega') - \rho \log \left[ \sum_{k \in N} \exp(\tilde{S}_{nk}(\Omega')/\rho) \right] \right\}
\]

\[
+ (1 - \eta)\beta(1 - \delta_n) E\{W_n(\Omega') - S_n(\Omega')\}
\]
Using the free entry condition, we know that $\beta \mathbb{E}\{J_n(\Omega')\} = \frac{c_n}{d_n(\theta_n)}$ so that:

$$w_n = (1 - \eta) b + \eta p_n + \eta c_n \theta_n$$

$$- (1 - \eta) \beta [1 - \delta_n - f_n(\theta_n)] \mathbb{E} \left\{ S_n(\Omega') - \rho \log \left[ \sum_{k \in N} \exp(\tilde{S}_{nk}(\Omega')/\rho) \right] \right\}$$

which is equivalent to:

$$(D.2) \quad w_n = (1 - \eta) b + \eta p_n + \eta c_n \theta_n - \rho (1 - \eta) \beta [1 - \delta_n - f_n(\theta_n)] \mathbb{E} \left\{ \log \left[ \frac{\exp(S_n(\Omega')/\rho)}{\sum_{k \in N} \exp(\tilde{S}_{nk}(\Omega')/\rho)} \right] \right\}$$

Using the equation for the probability of remaining a stayer on island $n$, we can also write this as:

$$w_n = (1 - \eta) b + \eta p_n + \eta c_n \theta_n - \rho (1 - \eta) \beta [1 - \delta_n - f_n(\theta_n)] \mathbb{E} \log [\pi nn]$$

**E  Efficiency**

A natural question is whether or not observed responses in unemployment over the cycle are efficient. In this section, I set out the social planner’s problem and compare its properties with the decentralized equilibrium. In this multisector model, the key equilibrium objects are the number of vacancies posted by firms, as well as the move decisions of workers. Thus, the question here is whether or not the wage rates, intermediate goods prices, and cutoff probabilities (described in more detail below) found in the decentralized equilibrium lead to the same outcomes chosen by a social planner, in which case the decentralized equilibrium is constrained efficient. I assume that the planner, in deciding how many vacancies to post, does not take into account the effect of his cutoff choice on the pool of people who will be able to make move decisions, but solve that problem in the next section. If I instead let the planner internalize the effect of market tightness on the pool of workers making a move
decision, the resulting wage that would decentralize the economy would be

\[ w^*_n = w^nash_n + (1 - \eta)\rho [1 - \beta(1 - \delta_n - f_n(\theta_n)) \log[1 - H(x_{nj})]] \]

where the Hosios (1994) condition still applies. In this scenario, the wages that decentralize the planner’s solution are lower than the nash-bargained wages. Since firms do not account for how their vacancy choices will impact the pool of workers making move decisions today, they over-post vacancies. The planner would like to keep some workers unemployed so that they can capitalize on taste-shock differences. Since this is an artifact of the way I model taste shocks, I assume that the planner, like the firms, does not take into account the effect of that choice on the pool of people who will be able to make move decisions. Finally, the planner is subject to the same moving frictions and search frictions outlined in the decentralized economy. In what follows, I draw heavily from results previously derived in Cameron, et. al. (2007). However, since the models are different in significant ways I re-derive their results when necessary.

**E.1 Two-Sector Case**

Let \( E_\varepsilon \left\{ \int x_{nj}[\varepsilon_j - \varepsilon_n]h(\varepsilon_j - \varepsilon_n)d(\varepsilon_j - \varepsilon_n) \right\} = H(x_{nj})x_{nj} - \rho \log(1 + \exp(x_{nj}/\rho)) \) be the cost of moving workers from sector \( n \) to sector \( j \) given some cutoff for movement \( x_{nj} \), where \( H(\varepsilon) \) is the CDF of the difference between two Gumbel random variables (Logistic) and \( h(\varepsilon) \) is the corresponding PDF. Then the planner’s problem can be formulated as follows:

\[
T(e_n, s_n, m_{nj}) = \max_{\{x_{nj}\}} -\beta \sum_{n \in N} \left[ \frac{s_n'}{1 - H(x_{nj})} \right] [H(x_{nj})x_{nj} - \rho \log(1 + \exp(x_{nj}/\rho))] + \max_{\{\theta_n\}} \left[ \sum_{n \in N} (\tau_n)^\frac{1}{2} (\mu_n e_n)^{\frac{\sigma - 1}{\sigma}} \right]^\frac{\sigma}{\sigma - 1} + \sum_{n \in N} \left( b s_n - \theta_n s_n c_n + b \sum_{j \in N} m_{nj} \right) + \beta T(e'_n, s'_n, m'_{nj})
\]
subject to:

\[ e'_n = e_n[1 - \delta_n] + s_n f_n(\theta_n) \]

\[ s'_n = (1 - H(x_{nj})) \left[ s_n [1 - f_n(\theta_n)] + \delta_n e_n + \sum_{k \in N} m_{kn} \right] \]

\[ m'_{nj} = H(x_{nj}) \left[ e_n \delta_n + s_n[1 - f_n(\theta_n)] + \sum_{k \in N} m_{kn} \right] \]

The first order conditions are given by:

\[ \theta_n : \frac{c_n}{\beta f'(\theta_n)} = T_{e_n} - T_{s_n} (1 - H(x_{nj})) - T_{m_{nj}} H(x_{nj}) \]

(E.1)

\[ x_{nj} : x_{nj} = -(T_{s_n} - T_{m_{nj}}) \]

simplifying gives:

(E.2)

\[ \frac{c_n}{\beta f'(\theta_n)} = T_{e_n} - T_{s_n} \]

The envelope conditions are given by:

(E.3)

\[ T_{e_n} = p_n + \beta \delta_n [\rho \log(1 + \exp(x_{nj}/\rho))] + \beta T_{e_n}(1 - \delta_n) + \beta T_{s_n} \delta_n \]

(E.4)

\[ T_{s_n} = b - \theta_n c_n + \beta [1 - f_n(\theta_n)][\rho \log(1 + \exp(x_{nj}/\rho))] + \beta T_{e_n} f_n(\theta_n) + \beta T_{s_n}[1 - f_n(\theta_n)] \]

Note that if the planner were to take into account the effect of \( \theta_n \) on the pool of workers who can choose whether or not to move, the first order condition would be give by:

\[ \frac{c_n}{\beta f'(\theta_n)} = \rho \log(1 - H(x_{nj})) + T_{e_n} - T_{s_n} \]
Now solving for $T_e - T_s$ gives:

$$T_e - T_s = \frac{p_n - b + \theta_n c_n - \beta(1 - \delta_n - f_n(\theta_n)) [\rho \log(1 + \exp(x_{nj}/\rho))]}{1 - \beta(1 - \delta_n - f_n(\theta_n))}$$

Plugging this into the FOC for theta gives:

$$\frac{c_n}{\beta f'(\theta_n)} = \frac{p_n - b + \theta_n c_n - \beta(1 - \delta_n - f_n(\theta_n)) [\rho \log(1 + \exp(x_{nj}/\rho))]}{1 - \beta(1 - \delta_n - f_n(\theta_n))}$$

Multiplying through and using the fact that $f'(\theta_n) = (1 - g)q_n(\theta_n)$ gives:

$$c_n [1 - \beta(1 - \delta_n - f_n(\theta_n))] = \beta f'(\theta_n) (p_n - b - \beta(1 - \delta_n - f_n(\theta_n)) [\rho \log(1 + \exp(x_{nj}/\rho))]) + (1 - g) \beta q_n(\theta_n) \theta_n c_n$$

Canceling out $\beta f_n(\theta_n) c_n$ on both sides gives:

$$c_n [1 - \beta(1 - \delta_n)] = \beta (1 - g) q_n(\theta_n) (p_n - b - \beta(1 - \delta_n - f_n(\theta_n)) [\rho \log(1 + \exp(x_{nj}/\rho))]) - g \beta q_n(\theta_n) \theta_n c_n$$

Dividing back through by $\beta q_n(\theta_n)$ gives:

$$\frac{c_n}{\beta q_n(\theta_n)} = \frac{(1 - g) (p_n - b - \beta(1 - \delta_n - f_n(\theta_n)) [\rho \log(1 + \exp(x_{nj}/\rho))]) - g \theta_n c_n}{1 - \beta(1 - \delta_n)}$$

which is equivalent to:

$$\frac{c_n}{\beta q_n(\theta_n)} = \frac{p_n - gp_n - (1 - g)b - g\theta_n c_n + (1 - g)\beta(1 - \delta_n - f_n(\theta_n)) [\rho \log(1 - H(x_{nj})]}{1 - \beta(1 - \delta_n)}$$

Then if the Hosios (1994) condition holds ($g = \eta$), the decentralized economy will be con-
strained efficient since the above will boil down to:

\[
\frac{c_n}{\beta q_n(\theta_n)} = \frac{p_n - u_{n,nash}^n}{[1 - \beta(1 - \delta_n)]}
\]

### E.2 N-Sector Case

*Note: This proof is still incomplete.* Following the proof given in Cameron, et. al. (2002), define \(D^{ij}(\varepsilon; \Omega)\) to be a function which gives the fraction of workers who are unemployed with idiosyncratic shocks \(\varepsilon = \{\varepsilon_1, ..., \varepsilon_N\}\) in sector \(i\) making a move decision who will move to sector \(j\) given the state of the economy \(\Omega\). Of course, the necessary constraint is that:

\[
\sum_{j=1}^{N} D^{ij}(\varepsilon; \Omega) = 1 \quad \forall i \in N
\]

The social planner wishes to maximize:

(E.5)

\[
E_{\{\Omega_t\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[ \max_{D_{nk}} \left( e_{n,t} \delta_n + s_{n,t} [1 - f_n(\theta_{n,t})] + \sum_{j \neq n \in N} m_{jn,t} \right) \right] \int \cdots \int \sum_{k \in N} (D^{nk} \varepsilon_k) \prod_{k=1}^{N} (h(\varepsilon_k) d\varepsilon_k) + \sum_{n \in N} \left( \max_{\theta_n} \left\{ (\tau_n)^{\frac{1}{\sigma}} \left( \mu_{n,t} e_{n,t} \right)^{\frac{\sigma - 1}{\sigma}} \right\} - s_{n,t} \theta_{n,t} c_n + b[s_{n,t} + \sum_{j \neq n \in N} m_{nj,t}] \right)
\]

over \(\theta_n\) \(\forall n \in N\) and the functions \(D^{nk}\) \(\forall k \in N\) subject to:

\[
e_{n,t+1} = e_{n,t} [1 - \delta_n] + s_{n,t} f_n(\theta_{n,t})
\]

\[
s_{n,t+1} = \left( 1 - \sum_{k \neq n \in N} \int \cdots \int D^{nk} \prod_{k=1}^{N} h(\varepsilon_k) d\varepsilon_k \right) \left[ s_{n,t} [1 - f_n(\theta_{n,t})] + \delta_n e_{n,t} + \sum_{j \in N} m_{jn,t} \right]
\]

\[
m_{nk,t+1} = \int \cdots \int D^{nk} \prod_{k=1}^{N} h(\varepsilon_k) d\varepsilon_k \left[ e_{n,t} \delta_n + s_{n,t} [1 - f_n(\theta_{n,t})] + \sum_{j \in N} m_{jn,t} \right]
\]
and the adding up constraint on the $D^{nk}$. In what follows, I assume that the planner does not take into account the effect that his choice of vacancies has on the pool of workers that will be making a move decision. In this scenario, the planner would think that firms are over posting vacancies since some of those vacancies turn into matches which then remove the ability of these workers to respond to their taste-shock realizations. Therefore, I assume that the planner faces the same knowledge that firms face when posting vacancies.\textsuperscript{40}

The first three terms represent the current period return. The last term represents the value of taste shocks, conditional on move decisions of workers. Note that the economy I have set out does not have any externalities beyond those governing the frictional search process - labor reallocation has no congestion, so we might guess that a standard Hosios (1994) condition is going to hold.

Note that the first order condition for the planner w.r.t. $\theta_n$ is given by:

\begin{equation}
\frac{c_n}{\beta f'(\theta_n,t)} = \frac{p_{n,t+1} \mu_{n,t+1} + G_{nn} \theta_{n,t+1} c_n - b}{(1 - g) q_n(\theta_n,t)}
\end{equation}

Now, use the fact that $f'(\theta_n,t) = (1 - g) \theta_n^{-g} = (1 - g) q_n(\theta_n,t) = (1 - g) \frac{f_n(\theta_n,t)}{\theta_n}$ and rewrite the above as:

\begin{equation}
\frac{c_n}{\beta q_n(\theta_n,t)} = (1 - g) \times [p_{n,t+1} \mu_{n,t+1} - b] + (1 - g) c_n \theta_n
\end{equation}

\textsuperscript{40}For a discussion of this in the two-sector case, see the next section.
Remember that the decentralized equilibrium has the following:

\[
\frac{c_n}{\beta q_n(\theta_{n,t})} = \frac{[1 - \eta](\bar{p}_{n,t}\mu_{n,t} - b) - \eta c_n \theta_n + \rho(1 - \eta)\hat{f}[1 - \delta_n - f(\theta_n)]E\log(\pi_{nn})}{1 - \beta(1 - \delta_n)}
\]

Solve for the wage that makes these two statements equal:

\[\text{(E.9) } \frac{p_n\mu_n - w_n}{1 - \beta(1 - \delta_n)} = (1 - g) \times [p_{n,t+1}\mu_{n,t+1} + \theta_{n,t+1}c_n - b]\]

**Proposition 1.** Any equilibrium maximizes the planner’s problem, provided that the Hosios (1994) condition holds, \(g = \eta\)

*Proof.* Fix the initial allocation of labor \(\{s^0_n, m^0_{nj}, e^0_n \forall n, j \in N\}\). Following the proof in Cameron, et. al. (2002), for any date \(t > 0\), define the public history variable \(H_t = \{\Omega_0, ..., \Omega_t\}\) and for any worker define the history of private shocks \(H_t' = \{\varepsilon_0, ..., \varepsilon_t\}\). The decision for moving can be written as a function \(d^{ij}_t(H_t, \varepsilon_t)\) where \(d^{ij}_t(H_t, \varepsilon_t) = 1\) if a workers is in sector \(i\) making a move decision after aggregate history \(H_{t-1}\) and faces idiosyncratic shocks \(\varepsilon_t\) moves to \(j\) and \(d^{ij}_t(H_t, \varepsilon_t) = 0\) otherwise. From this rule along with indicator functions for separation and job-finding, we can figure out the allocation \(\{s_{n,t}, m_{nj,t}, e_{n,t} \forall n, j \in N\}\) at date \(t\) from \(H_{t-1}\) and the location of every workers from \(H_{t-1}\) and \(H_{t-1}'\). We can now summarize this information by writing three vector-valued functions. Call the first \(\pi^*_t(H_{t-1}; H_{t-1}', i)\) where \(\pi^*_t = 1\) if a person is in sector \(j\) at time \(t\) and \(\pi^*_t = 0\) otherwise. Call the second \(\xi_t(H_{t-1}; H_{t-1}', i)\) where \(\xi_t = 1\) if a person is employed at time \(t\) and \(\xi_t = 0\) otherwise. Finally, call the third \(\xi^*_t(H_{t-1}; H_{t-1}', i)\) where \(\xi^*_t = 1\) if a person is a stayer at time \(t\) and \(\xi^*_t = 0\) otherwise.

Define the following indicators function. Let \(I_\delta = 1\) if an employed worker separates, \(I_t = 1\) if a stayer finds a job. Now suppose that the functions \(\tilde{D}^{ij}\) with associated moving functions, allocations, and location functions \(\tilde{d}^{ij}, \tilde{\Omega}, \text{and } \tilde{\pi} \text{ respectively. Consider an alternative feasible
function $\hat{D}^{ij}$. From final goods producer optimization, it must be that:

$$
E_{\{\Omega\}_{t=0}^{\infty}} \beta^t \sum_{n \in N} \left( (\tau_n)^{\frac{1}{\sigma}} (\hat{y}_{n,t})^{\frac{\sigma - 1}{\sigma}} \right) - \sum_{n \in N} \tilde{p}_{n,t}(\hat{y}_{n,t})\hat{y}_{n,t}
$$

(E.10)

$$
\geq E_{\{\Omega\}_{t=0}^{\infty}} \beta^t \sum_{n \in N} \left( (\tau_n)^{\frac{1}{\sigma}} (\hat{y}_{n,t})^{\frac{\sigma - 1}{\sigma}} \right) - \sum_{n \in N} \tilde{p}_{n,t}(\hat{y}_{n,t})\hat{y}_{n,t}
$$

From worker optimality, it must be that:

$$
E_{\{\Omega\}_{t=0}^{\infty}} \beta^t \sum_{n \in N} \tilde{\pi}^n_t(H_{t-1}; H'_{t-1}; i) (\nonumber

\tilde{\xi}(H_{t-1}; H'_{t-1}; i) \cdot \left\{ \tilde{w}_t^n + I_\delta \varepsilon_t^n + [1 - I_\delta] \sum_{k \in N} \tilde{d}^{nk}_t(H_t, \varepsilon_t) \right\} 

+ \tilde{\xi}^s_t(H_{t-1}; H'_{t-1}; i) \cdot \left\{ b + I_f \varepsilon_t^n + [1 - I_f] \sum_{k \in N} \tilde{d}^{nk}_t(H_t, \varepsilon_t) \right\} 

\left[ 1 - \tilde{\xi}^s_t(H_{t-1}; H'_{t-1}; i) - \tilde{\xi}_t(H_{t-1}; H'_{t-1}; i) \right] \cdot \left\{ b + \sum_{k \in N} \tilde{d}^{nk}_t(H_t, \varepsilon_t) \right\})

(E.11)

\geq E_{\{\Omega\}_{t=0}^{\infty}} \beta^t \sum_{n \in N} \tilde{\pi}^n_t(H_{t-1}; H'_{t-1}; i) (\nonumber

\tilde{\xi}(H_{t-1}; H'_{t-1}; i) \cdot \left\{ \tilde{w}_t^n + I_\delta \varepsilon_t^n + [1 - I_\delta] \sum_{k \in N} \tilde{d}^{nk}_t(H_t, \varepsilon_t) \right\} 

+ \tilde{\xi}^s_t(H_{t-1}; H'_{t-1}; i) \cdot \left\{ b + I_f \varepsilon_t^n + [1 - I_f] \sum_{k \in N} \tilde{d}^{nk}_t(H_t, \varepsilon_t) \right\} 

\left[ 1 - \tilde{\xi}^s_t(H_{t-1}; H'_{t-1}; i) - \tilde{\xi}_t(H_{t-1}; H'_{t-1}; i) \right] \cdot \left\{ b + \sum_{k \in N} \tilde{d}^{nk}_t(H_t, \varepsilon_t) \right\})
$$
Summing this over all workers gives:

$$
\sum_{n \in N} \tilde{e}_{n,t} \tilde{\omega}_{n,t} + b[\tilde{s}_{n,t} + \sum_{j \in n} \tilde{m}_{j,n,t}] + \left( \varepsilon_{n,t} \delta_n + \tilde{s}_{n,t} [1 - f_n(\tilde{\theta}_{n,t})] + \sum_{j \neq n \in N} \tilde{m}_{j,n,t} \right) \times
$$

\[
\int \cdots \int \sum_{k \in N} (\bar{D}^{nk}\varepsilon_k) \prod_{k=1}^N (h(\varepsilon_k) d\varepsilon_k)
\]

(E.12)

$$
\geq \sum_{n \in N} \tilde{e}_{n,t} \tilde{\omega}_{n,t} + b[\tilde{s}_{n,t} + \sum_{j \in n} \tilde{m}_{j,n,t}] + \left( \varepsilon_{n,t} \delta_n + \tilde{s}_{n,t} [1 - f_n(\tilde{\theta}_{n,t})] + \sum_{j \neq n \in N} \tilde{m}_{j,n,t} \right) \times
$$

\[
\int \cdots \int \sum_{k \in N} (\bar{D}^{nk}\varepsilon_k) \prod_{k=1}^N (h(\varepsilon_k) d\varepsilon_k)
\]

Finally, from the intermediate firm’s optimality condition, we have that:

(E.13)

$$
\mathbf{E}(\Omega)_{t=0}^{\infty} \beta^t \mathbf{I}_v (-c_n) + \mathbf{I}_j [\tilde{p}_n,t(\tilde{y}_{n,t}) \mu_{n,t} - \tilde{\omega}_{n,t}]
$$

$$
\geq \mathbf{E}(\Omega)_{t=0}^{\infty} \beta^t \mathbf{I}_v (-c_n) + \mathbf{I}_j [\tilde{p}_n,t(\tilde{y}_{n,t}) \mu_{n,t} - \tilde{\omega}_{n,t}]
$$

Adding Equation E.10, E.13, and Equation E.12, and canceling terms gives:

(E.14)

$$
\mathbf{E}(\Omega)_{t=0}^{\infty} \beta^t \sum_{n \in N} \left\{ (\tau_n) \frac{1}{\sigma} (\tilde{\omega}_{n,t})^{\frac{\sigma}{\sigma-1}} \right\} + \sum_{n \in N} b[\tilde{s}_{n,t} + \sum_{j \in n} \tilde{m}_{j,n,t}] - c_n \tilde{\omega}_{n,t}
$$

$$
+ \left( \varepsilon_{n,t} \delta_n + \tilde{s}_{n,t} [1 - f_n(\tilde{\theta}_{n,t})] + \sum_{j \neq n \in N} \tilde{m}_{j,n,t} \right) \times \int \cdots \int \sum_{k \in N} (\bar{D}^{nk}\varepsilon_k) \prod_{k=1}^N (h(\varepsilon_k) d\varepsilon_k)
$$

$$
\geq \mathbf{E}(\Omega)_{t=0}^{\infty} \beta^t \sum_{n \in N} \left\{ (\tau_n) \frac{1}{\sigma} (\tilde{\omega}_{n,t})^{\frac{\sigma}{\sigma-1}} \right\} + \sum_{n \in N} b[\tilde{s}_{n,t} + \sum_{j \in n} \tilde{m}_{j,n,t}] - c_n \tilde{\omega}_{n,t}
$$

$$
+ \left( \varepsilon_{n,t} \delta_n + \tilde{s}_{n,t} [1 - f_n(\tilde{\theta}_{n,t})] + \sum_{j \neq n \in N} \tilde{m}_{j,n,t} \right) \times \int \cdots \int \sum_{k \in N} (\bar{D}^{nk}\varepsilon_k) \prod_{k=1}^N (h(\varepsilon_k) d\varepsilon_k)
$$

We know that in the decentralized equilibrium,

$$
\frac{c_n}{\beta q_n(\theta_{n,t})} = \frac{[1 - \eta](\tilde{p}_n,t \mu_{n,t} - b) - \eta c_n \theta_n + \rho (1 - \eta) \beta [1 - \delta_n - f(\theta_n)] \mathbf{E} \log(\pi_m)}{1 - \beta (1 - \delta_n)}
$$

\[\square\]
Table 5: Detrended Log Sectoral Employment in the Data

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>NC</th>
</tr>
</thead>
<tbody>
<tr>
<td>monthly autocorrelation</td>
<td>0.928</td>
<td>0.890</td>
</tr>
<tr>
<td>variance(*1000)</td>
<td>0.317</td>
<td>0.032</td>
</tr>
</tbody>
</table>

F  Calibrating Sectoral AR(1) Parameters

I have assumed that detrended log labor productivity in each sector follows:

\[ \log(\mu'_n) = \kappa_n \log(\mu_n) + \zeta_n \nu'_n \]

Therefore, the variance for detrended log labor productivity will be given by:

\[ \text{Var} [\log(\mu_n)] = \frac{\zeta_n^2}{1 - \kappa_n^2} \]

where I use the assumption that \( \nu_n \sim N(0, 1) \). The autocorrelation is simply \( \kappa_n \). I thus set each \( \kappa_n \) to the autocorrelations of the detrended log sectoral employment series in the data, reported in Table 5. Given these values for \( \kappa_n \), I can back out the implied value for \( \zeta_n \) that would make the variance of my AR(1) match the variance of the detrended log employment series. Letting hats denote my own estimates of these numbers,

\[ \hat{\zeta}_n = \text{Var}[\log(e_n)] \cdot [1 - \hat{\kappa}_n^2] \]

Now we have pinned down \( \{\kappa_n, \zeta_n\} \) for \( n \in \{C, NC\} \). The only thing left to pin down is the parameter \( \phi \) which governs the correlation between \( \log(\mu_C) \) and \( \log(\mu_{NC}) \). Given the above AR(1) assumption, the following holds:

\[ \text{Cov} [\log(\mu'_C) - \kappa_C \log(\mu_C), \log(\mu'_NC) - \kappa_{NC} \log(\mu_{NC})] = \zeta_C \cdot \zeta_{NC} \text{Cov} [\nu'_C, \nu'_{NC}] \]

Thus, given the \( \zeta_n \) described above, I set \( \phi = 0.80 \).
G Recovering the Shocks

G.1 First Order Approximation

To fix ideas, start with the first order approximation of the model. In the two sector version, I have 26 endogenous variables, of which 8 are state variables. Let $T_{26 \times 1}$ denote these endogenous variables and let $R_{8 \times 8}$ denote the state variables. Then the first order approximation solves for a linear relationship between $T$, $R$, and the innovations in the model $\nu_n$:

$$
\begin{bmatrix}
T_{t+1} - \bar{T}
\end{bmatrix} = \bar{T} + A_1 \begin{bmatrix} R_t - \bar{R} \end{bmatrix} + A_2 \nu_{t+1}
$$

where $\nu$ is a $2 \times 1$ vector of the two innovations to sectoral productivity.\(^{41}\) Thus, to recover the innovations, take the following steps:

1. Start in period $t = 0$ in the steady state so that $R_t - \bar{R} = 0$. In my calculations, I use January 1977 as the steady state.

2. Using data on employment in the two sectors in February 1977 (date $t + 1$) and the first order approximation implies that there is exactly one solution for $\nu_{t+1}$ that would produce the observed employment in $t + 1$. Solve for the implied $\nu_{t+1}$

3. Once you have uncovered $\nu_{t+1}$, use the first order approximation again to solve for the remaining endogenous variables in $T$. Now we have the vector $T_{t+1}$ and thus $R_{t+1}$.

4. Repeat steps 2 through 3 until the end of the series

G.2 Second Order Approximation

Because the time series on employment tends to deviate from its mean values over some periods, I choose to use a second order approximation to more accurately match the data. The algorithm is the same as described above, except that the approximation is no longer linear.

\(^{41}\)I use Dynare to numerically solve my model and thus recover the different $A$ coefficients.
there might be multiple shocks that can generate the same employment data. Therefore, I carry out the first order approximation as above, and use the recovered shocks there as starting values in the search for the true shocks under the second order approximation. The second order approximation has the form:

\[
[T_{t+1} - \bar{T}] = ([\bar{T} + 0.5A_0^2 + A_1 [R_t - \bar{R}] + A_2\nu_{t+1}

+ 0.5A_3 [(R_t - \bar{R}) \otimes (R_t - \bar{R})] + 0.5A_4 [(\nu_{t+1}) \otimes (\nu_{t+1})] + A_5 [(R_t - \bar{R}) \otimes (\nu_{t+1})])
\]

1. Start in period \( t = 0 \) in the steady state so that \( R_t - \bar{R} = 0 \). Again, I use January 1977 as the steady state.

2. Using data on employment in the two sectors, the second order approximation, and the recovered shocks from the first order approximation, solve for the implied \( \nu_{t+1} \).

3. Once you have uncovered \( \nu_{t+1} \), use the approximation again to solve for the remaining endogenous variables in \( T \). Now we have the vector \( T_{t+1} \) and thus \( R_{t+1} \).

4. Repeat steps 2 through 3 until the end of the series.

H Simulated Method of Moments

The updating algorithm I use in Simulated Method of Moments is Simulated Annealing as described in Kirkpatrick, Gellat, and Vecchi (1983). The algorithm is useful for non-convex problems in which gradient methods might produce local minima. This procedure allows the objective function to increase in value at some points over the search. I begin with the identity weighting matrix. I then carry out the following procedure:

1. Guess an initial value for the parameters.

2. Simulate the model and retrieve the time series for value functions.

3. Using the value functions, simulate Gumbel draws for individuals to create a “Simulated CPS” dataset.
4. Calculate the moments of the “Simulated CPS” dataset that are comparable to the actual CPS as described in Section 4.

5. Form the moment function.

6. Update the parameters space.

7. After the algorithm has converged once, compute the optimal weighting matrix as described in Gourieroux and Monfort (1997) and rerun these steps (but substituting the parameters achieved on the first iteration for the initial guess) until convergence is achieved again.
References


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Jaimovich, N. and H. E. Siu (2012, August). The Trend is the Cycle: Job Polarization and Jobless Recoveries.


