Means of Payment and Timing of Mergers and Acquisitions in a Dynamic Economy*

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Abstract

We develop a theory of acquisition timing and means of payment for financially constrained firms. Bidders with private valuations choose when to approach the target and whether to bid in cash or stock. By linkage principle, bidders prefer to bid in cash, but are limited by cash constraints. The model delivers many implications, both novel and consistent with existing evidence. First, high-synergy targets are approached when they are young and small, and are acquired for cash. Low-synergy targets are acquired after they have grown, and for stock. This selection is consistent with higher takeover premiums observed in cash versus stock deals. Second, cash constraints need not have a monotonic effect on acquisition timing. Third, acquisitions can be caused by shocks not only to fundamentals but also to financial constraints. Finally, some targets are never acquired despite positive synergies.

Keywords: Auctions, financial constraints, cash constraints, mergers and acquisitions, real options, security design.

The decision to acquire a target is one of the most important choices that the firm's management and board of directors face, with the potential to gain or lose millions and billions in profit.[1] It is therefore important to understand how these multifaceted decisions are made and what factors affect them. Among the most important choices in acquisitions are timing and the means of payment: a bidder must decide when to approach the target with an offer and which form of payment to use. These choices appear to be correlated with various firm-specific and economy-wide characteristics. In particular, it is known that mergers often occur in waves that are correlated with periods of economic expansion and easy access to external financial markets, or financing constraints.[2] In this paper, we provide a theoretical analysis of acquisitions based on two simple ideas: (i) a bidder can choose when to approach the target with an offer; (ii) its ability to pay cash is limited by a financing constraint. We show that a simple real options model of acquisitions has the potential to match much of the empirical evidence on the relation between M&A activity and economic shocks, financing constraints, means of payment, and the distribution of gains among the contest participants. In addition, we provide several novel predictions.

More specifically, we consider a dynamic model in which there are three agents: a target and two potential bidders. The target is a growth firm: its assets and cash flows grow over time with some uncertainty. Both bidders are mature companies: the bidder's assets and cash flows do not grow unless it acquires the target.[3] The bidders have privately-known synergies with the target: an acquisition improves productivity of the target in a combined company by a bidder-specific multiple. At any time each bidder can approach the target with an offer. Once a bidder makes a bid, the auction between the first bidder and the competitor is initiated, and the bidder who submits the highest bid wins the auction. A

[1] In 2007 alone, the value of world-wide deal volume exceeded $4.8 trillion.

[2] For evidence on procyclicality of mergers and acquisitions (M&A) see Mitchell and Mulherin (1996) and Maksimovic and Phillips (2001). For evidence on relation of M&A to the costs of borrowing see Harford (2005).

[3] An alternative interpretation of the framework is that the target's assets and cash flows change relative to those of the bidders.

bidder's decision when to approach the target reflects the following trade-off. On one hand, approaching the target early leads to an earlier increase in its productivity. On the other hand, a deal involves a cost: If the bidder loses the auction, its post-merger value will diminish, because it will face a stronger competitor. If the bidder's valuation of the target is low, it is optimal to wait until the target grows in size so that the increase in its productivity outweighs the cost of the acquisition.

The second building block of the model is information asymmetry between the target and the bidders. Similarly to the literature on auctions, but unlike the prior literature that considers takeovers in the real options framework, we assume that potential synergies from acquiring the target are the private information of the bidder. As shown in the literature on securities auctions, this feature makes bids in stock and in cash not equivalent, in contrast to the case when bidders do not have any private information. Specifically, because the value of a bid in stock (but not in cash) depends on the bidder's private information, it is costlier for a bidder to separate itself from a marginally lower type in a stock auction than in a cash auction. Even if both stock bidders offer the same proportion of the combined company to the target's shareholders, the bidder with the higher valuation will end up paying more in cash equivalent. Because of this effect, which is a version of the linkage effect (Milgrom and Weber, 1982), each bidder wants to bid in cash whenever possible. The ability to do this is, however, limited by the financing constraint of the bidder. We model it by assuming that the bidder cannot pay in cash above a certain limit.

We initially solve for the equilibrium initiation strategies and terms of takeovers when means of payment are exogenous: both bidders submit bids in cash; in stock; or one bidder submits bids in cash and the other in stock. These cases correspond to special cases of the general model, in which financing constraints of each bidder are either infinite or zero. Then, we provide solution of the general model with arbitrary cash constraints. The model delivers a number of implications relating the timing of mergers, synergies, and means of payment.

We show that high-synergy targets are typically acquired young and for cash, while low-synergy targets are typically acquired old (if at all, despite positive synergies) and for stock. Intuitively, if the bidder expects high synergies, it does not pay off to wait, so the target is acquired when small. As a result, for an acquirer, the required payment is likely to be below the financing constraint, leading to deals done in cash. Because of high synergies, such deals are also likely to result in high takeover premiums (relative to the current value of the target under its current management). Thus, the model predicts that in a sample of deals, cash deals can be associated with higher takeover premiums, despite that stock deals are perceived as more expensive by bidders. This finding is broadly consistent with empirical evidence (e.g., Betton, Eckbo, and Thorburn, 2008). While this evidence can seem inconsistent with predictions of security-bid auctions literature, it becomes consistent once dynamic selection of targets by bidders into cash and stock deals is taken into account.

The model delivers interesting comparative statics as to which deals are likely to be done in cash versus in stock and when. For example, all else equal, the option to delay approaching the target is more valuable if the value of the target's assets is more volatile. Thus, such targets are acquired later, when the financial constraint of the acquirer is less likely to be satisfied, and hence are more likely to be done in stock. All else equal, stock deals for these targets are also, on average, better than stock deals for lower-risk targets: they have higher average synergies and higher average takeover premiums.

Surprisingly, the financing constraint of a bidder can have a non-monotonic effect on its initiation strategy. There are two effects in play. On one hand, a less-constrained bidder is more likely to have enough cash to finance the winning bid and avoid paying in stock. Thus, it obtains a higher expected surplus from the auction compared to a more-constrained bidder. All else equal, this higher payoff from option exercise leads to an earlier exercise, i.e., an earlier initiation of the deal. On the other hand, a less-constrained bidder has a weaker incentive to further accelerate initiation to win the contest in cash while the target is small. For such bidder, it is cheaper to postpone the deal without having to pay in stock later.

If the two bidders have asymmetric financing constraints, their initiation strategies are asymmetric as well. Therefore, as time goes by, the posterior distributions of valuations of the two bidders become different, even if the initial distributions are the same. In equilibrium, the initiating bidder does not always win. Bidders who approach the target and whose highest bid is in cash are not only cash-rich firms but also firms with high valuations of the target. As a result, for some parameters of the model, they are more likely to win than initiating bidders whose highest bid is in stock. This implication is consistent with empirical evidence (e.g., Betton, Eckbo, and Thorburn, 2008) that cash bids are less often rejected than stock bids in favor of a rival bid.

Taken together, our results suggest that the decision to initiate a takeover contest is driven by three fundamental factors: the technological shocks that affect total gains from the acquisition, the cash constraints of the bidder, and the bidder's perception of the cash constraints of other potential bidders. While the importance of technology shocks has often been acknowledged in the prior literature (e.g., Mitchell and Mulherin, 1996, Jovanovic and Rousseau, 2002, Lambrecht, 2004), the effect of financing constraints has received less attention. We show that financing constraints have subtle but important impact on mergers and provide a number of implications relating them to merger activity. In particular, consistent with the empirical findings of Harford (2005), cash constraints can significantly delay acquisitions in our model.

Our paper is related to two strands of research. First, it is related to literature that studies mergers and acquisitions as real options. Lambrecht (2004) studies a setting in which mergers are driven by economies of scale and shows that the merger takes place once the price of the industry output rises to a sufficiently high threshold, thereby providing a rationale for the procyclicality of mergers. Hackbarth and Morellec (2008) apply a similar framework to a setting with incomplete information between the market and the merging firms to study the dynamics of stock returns and risk in M&A.[4] Versions of a real options framework have been applied to study takeovers in declining industries (Lambrecht and Myers, 2007), the effects of bidders' capital structure (Morellec and Zhdanov, 2008) and

[4]See also Morellec and Zhdanov (2005).

industry structure (Hackbarth and Miao, 2012). All these papers assume that the target and the acquirer have the same information about the value of the combined company. This assumption has a crucial effect on the role of means of payment in takeovers. If all bidders and the target have the same information about the gains from the deal, takeover battles in stock and in cash are equivalent. In contrast, we follow the traditional literature on auctions in assuming that bidders have private information about their valuations of the target. Thus, bids in cash and stock are no longer equivalent, which leads to implications relating the timing of mergers and means of payment to synergies and other characteristics.

Second, our paper is related to information theories of means of payment in mergers and acquisitions and, more generally, in auctions in which bidders can make bids in securities.[5] These models are static, and do not explore strategic timing in the presence of financing constraints. Perhaps, the most relevant paper in this literature is Fishman (1989), as it delivers many of our empirical implications using a different mechanism, in a static model with a two-sided information asymmetry between bidders and the target.[6] The advantage of a stock bid is that it reduces the adverse selection problem, inducing a more efficient accept/reject decision of the target. A cash bid is, however, used when a bidder has a high enough valuation to preempt competition by signaling a high valuation. In contrast to Fishman (1989), our paper shows that a one-sided information asymmetry in which only bidders have private information is sufficient to capture empirical evidence on means of payment, once dynamic aspects are taken into account. It also explains why stock bids are often perceived as more expensive by bidders, yet look smaller in the data. The way to test the relative importance of the two explanations for the observed means of payment would be to account for the timing of acquisitions, such as size of the target and its age,

[5]For example, bids in auctions of oil leases are often combinations of an upfront cash payment and a claim on future cash flows from the lease. Auctions with contingent payments are studied by Hansen (1985), Rhodes-Kropf and Viswanathan (2000), DeMarzo, Kremer, and Skrzypacz (2005), and Gorbenko and Malenko (2011). Skrzypacz (2013) provides a review of the literature. The choice of timing is also important in these deals and is explored by Cong (2012).

[6]Other models of means of payment based on two-sided information asymmetry are provided by Hansen (1987), Eckbo, Giammarino, and Heinkel (1990), and Berkovitch and Narayanan (1990). Shleifer and Vishny (2003) and Rhodes-Kropf and Viswanathan (2004) develop theories relating means of payment in mergers to merger waves.

and financing constraints of bidders.

The remainder of the paper is organized in the following way. Section I outlines the setup of the model. Section II solves the model with exogenous means of payment, or equivalently, special cases of the general model in which cash constraints are either perfect or non-existent. Section III solves the general model. Section IV provides the comparative statics analysis. Section V studies the properties of the equilibrium and the predictions of the model, and discusses testable hypotheses. Section VI concludes. All proofs appear in Appendix A. Appendix B contains the details of numerical solutions.

I Model Setup

We consider a setting in which the risk-neutral target attracts two potential risk-neutral acquirers, or bidders. The roles of the target and the bidders are exogenous. The value of the target as a separate entity at time t is given by X_t, where X_t evolves as a geometric Brownian motion:

$$dX_t = \mu X_t dt + \sigma X_t dB_t, \; X_0 = x. \tag{1}$$

Here, μ and $\sigma > 0$ are constant growth rate and volatility, and dB_t is the increment of a standard Brownian motion. The discount rate is constant at r. To guarantee finite values, we assume that $r > \mu$. Process $(X_t)_{t>0}$ is a reduced-form specification of the present value of the target's assets. For example, this value can be obtained by assuming that the target produces cash flow $(r - \mu) X_t$ per unit of time. We interpret X_t as the current size of the target. It accounts for all exogenous shocks to their value, such as changes in the price of the final product and inputs, as well as for the endogenous response of the target firm to them.[7] The initial value of each bidder as a separate entity is constant at Π_b.[8] If bidder i

[7]In this paper, we focus on fundamental rather than market prices of the target (that is, prices clear of market expectations about the potential acquisition). This is consistent with related empirical studies, in which target prices are typically cleared of pre-acquisition runups.

[8]Bidders' values are equal for simplicity of exposition; this assumption does not affect the main trade-offs of the model. This setup captures a situation in which a relatively mature company aims to acquire a growing company. An additional assumption could be that the growth rate of the target decreases as

acquires the target at time t, the value of the combined firm is

$$\Pi_b + v_i X_t, \tag{2}$$

where $v_i \in [\underline{v}, \bar{v}]$, $\bar{v} > \underline{v} > 1$ is the multiple that characterizes an improvement in operations of the target due to a change in ownership.[9] We refer to v_i as bidder i's valuation of the target. Importantly, each bidder's valuation is its private information that is known to it before the start of the acquisition process.[10] Each valuation is an i.i.d. draw from distribution with p.d.f. $f(v) > 0$ on $[\underline{v}, \bar{v}]$. Each bidder knows its valuation, but not the valuation of its competitor, except for the distribution. We assume that the distribution of valuations satisfies the restriction that the payoff of the winning bidder monotonically increases in its valuation v in all specifications.[11] This assumption intuitively means that the direct effect on the winner's payoff of having a higher valuation is stronger than the indirect effect of a higher expected payment.

To have a non-trivial timing of the acquisition, the deal has to entail a cost. We capture this cost by assuming that the losing bidder is also affected by the acquisition: its value changes from Π_b to $\Pi_o < \Pi_b$. Intuitively, the acquisition makes the winning bidder a stronger competitor for the losing bidder, resulting in the lower post-acquisition value of the latter.[12] For example, the recent acquisition of Instagram by Facebook made Facebook a stronger competitor for other social network firms. This loss in the losing bidder's value

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it grows, so that it becomes a more mature company. Although more realistic, this assumption results in less tractability and does not alter the economics behind our results. Similarly, it is possible to extend our setup by allowing bidders to grow over time. Our results hold in this setup as long as the cash balances of each bidder, defined below, do not grow at a faster rate than the target.

[9] Allowing $\underline{v}$ below 1 does not add to the model intuition in any way.

[10] Introducing the additional private information that the bidder can learn at the beginning of the contest does not affect the results of the model qualitatively. It is only the ex-ante private information that defines bidders' strategies to initiate the takeover contest.

[11] For example, in the model of Section II.B this restriction is equivalent to a restriction that $v - \mathbb{E}[w|w \leq v]$ is a strictly increasing function of v. An example of distribution that satisfies these restrictions is uniform distribution.

[12] Spiegel and Tookes (2013) quantify this effect at 1.86% of the rival firm value on average. Horizontal mergers also feature an opposite effect, because the losing bidder faces fewer competitors. This effect is not present in our setup, because the target is not a direct competitor of the bidder.

is a source of delay of the acquisition in the model. Of course, other potential sources of delay such as direct costs of initiating the takeover contest are possible too. We denote the value loss of the losing bidder as $\Delta \equiv \Pi_b - \Pi_o$.

In practice, acquisitions by strategic buyers are usually initiated by a potential bidder, rather than the target (Fidrmuc et al., 2012). To reflect this practice, we assume that each bidder has a real option to approach the target at any time. If a bidder approaches the target at time t, the takeover contest is initiated and both bidders compete for the target in an open ascending-bid (English) auction.

The term English auction refers to a wide variety of open ascending-bid auction formats, which can differ in their precise rules. The most convenient formalization of English auctions is the "button" auction introduced by Milgrom and Weber (1982). Initially, all bidders are active. An auctioneer sets the price at zero and gradually raises it. A bidder confirms its participation continuously until the raising price forces it to withdraw from the auction. As soon as only one bidder remains, it is declared the winner and pays the current price.

Each bidder can submit bids in the form of cash or stock of the combined company, or any mixture of the two. In Sections II.B and II.C we generalize the "button" auction of Milgrom and Weber (1982) to the cases of stock and mixed bids. In contrast to the static auction literature and natural to the dynamic setting of our problem, each bidder needs to decide not only how much to bid, but also when to approach the target, if it has not been approached by the other bidder yet. The assumption that bidders but not the target determine the timing of a takeover can be justified by the absence of commitment to sell on the part of the target. If the target announces that it wants to sell itself before any bidder is ready to make a move, the bidders can always delay the acquisition by not participating in the target-initiated process. On the other hand, when the bidder with positive value is ready to make an offer then, consistent with the "Revlon Duty," the target's board would be responsible to consider all offers and accept the highest bid offered provided that it exceeds the value of the target under the current management.[13]

Because bidders have private information about their valuations, they have incentives to pay in cash whenever possible. Specifically, bidding in cash leaves more surplus to bidders because each bidder finds it cheaper to separate from lower types compared to bidding in stock.[14] However, the ability to submit bids in cash is limited, because the bidder's internal cash inventory is finite and borrowing from outside investors can be expensive. For simplicity, we assume that bidder i has only C_i units of cash, and the cash constraints are infinitely rigid after that: in other words, bidder i can bid up to C_i units of cash, but cannot bid above that at any cost. Parameter C_i can be interpreted as a solution to the cash inventory problem of a firm, trading off the lower need to access external finance (the benefit) against the carry cost of cash inventory, e.g., due to the agency problems.[15] First, we consider a model with exogenous means of payment. Then, we make them endogenous by introducing cash constraints into the model.

II Initiation with Exogenous Means of Payment

First, we solve the model with exogenous means of payment. In particular we consider three cases. In the first case, both bidders always compete in cash bids. In the second case, both bidders always compete in stock bids. Finally, in the third case, one bidder competes in cash bids and the other bidder competes in stock bids. While the means of payment are exogenous in this part of the paper, this model will provide some intuition behind the model with endogenous means of payment.

[13]In our setup, it can be shown that given the equilibrium payoff of the target, it does not have an incentive to delay the acquisition, after it is approached by a bidder.

[14]This result was first established by Hansen (1985). See Rhodes-Kropf and Viswanathan (2000) and DeMarzo, Kremer, and Skrzypacz (2005) for generalizations of it.

[15]For cash inventory problems of this kind see Miller and Orr (1966), Bolton, Chen, and Wang (2011), and Hugonnier, Malamud, and Morellec (2012).

II.A Two Cash Bidders

Consider the case in which both bidders make offers in cash. Suppose that a takeover contest is initiated at time t and both bidders compete for the target in an English auction. A weakly dominant strategy for bidder i is to bid up to b_i, where

$$\Pi_b + v_i X_t - b_i = \Pi_o. \tag{3}$$

Intuitively, each bidder is willing to bid up to a point at which its payoff from acquiring the target ($\Pi_b + v_i X_t - b_i$) is equal to its payoff from losing the contest to the rival (Π_o). Therefore, the bidder with the highest valuation acquires the target and pays $\min(v_1, v_2) X_t + \Delta$.

Conjecture that the bidder with valuation v finds it optimal to approach the target at threshold $\bar{X}_c(v)$ provided that the other bidder has not approached the target yet, where $\bar{X}_c(v)$ is decreasing in v. If the bidder with valuation v wins the auction for target X_t against the bidder with valuation w, the change in its value relative to the stand-alone level is

$$(v - w) X_t - \Delta. \tag{4}$$

If, on the other hand, the bidder loses, the corresponding difference is $-\Delta$. If the bidder with valuation v follows the strategy of approaching the target at threshold $\bar{X}$, its expected payoff at the initial date is

$$\begin{aligned} &\left(\frac{X_0}{\bar{X}}\right)^{\beta} \int_{\underline{v}}^{\bar{X}_c^{-1}(\bar{X})} \left(\bar{X} \max\{v - w, 0\} - \Delta\right) dF(w) \\ &+ \int_{\bar{X}_c^{-1}(\bar{X})}^{\bar{v}} \left(\frac{X_0}{\bar{X}_c(w)}\right)^{\beta} \left(\bar{X}_c(w) \max\{v - w, 0\} - \Delta\right) dF(w), \end{aligned} \tag{5}$$

where $\beta > 1$ is the positive root of the fundamental quadratic equation $\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - r = 0$. The second multiplicand in each term of (5) is the change in the value of the bidder from the auction relative to the stand-alone level, and the first multiplicand is the price of the contingent claim that pays \$1 at the time when the auction occurs. The first (second) term of (5) reflects the case in which the bidder with valuation v (its competitor) approaches the target. Maximizing (5) with respect to $\bar{X}$ and applying the equilibrium condition that the maximum is reached at $\bar{X}_c(v)$, we obtain

$$\bar{X}_c(v) = \frac{\beta}{\beta - 1} \frac{\Delta}{v - \mathbb{E}[w|w \leq v]}. \tag{6}$$

This equation is intuitive. Because of the option to delay approaching the target, a bidder approaches the target only at a point when its expected surplus from initiating the contest exceed the costs by a high enough margin. The increase in the target's efficiency that is captured by the acquirer in expectation is $(v - \mathbb{E}[w|w \leq v]) X_t$, and the cost of approaching the target is Δ. The term $\beta / (\beta - 1) > 1$ captures the degree to which the option to delay approaching the target is important.

By assumption, the bidder's surplus from winning the auction less what it pays in expectation, $v - \mathbb{E}[w|w \leq v]$, is increasing in v. Hence, $\bar{X}_c(v)$ is indeed decreasing in valuation v. This monotonicity has two implications. First, targets with higher potential synergies are acquired earlier. Second, the bidder that approaches the target is the bidder with the higher valuation. In this model, it follows that in equilibrium, the bidder that approaches the target always wins the auction.[16] In a more general setting, in which bidders can update their valuations after the contest initiation (e.g., during due diligence), this result would not hold, but the bidder that initiates the contest would always win with a higher probability than its competitor, provided that the degree of initial information is the same for both bidders.

Another interesting property of (6) is that all bidders with valuations $v > \underline{v}$ find it optimal to approach the target at some finite $\bar{X}_c(v)$. This is because, as (4) shows, there always exists high enough X_t such that the winning bidder receives a positive surplus for any $w < v$. As long as $v > \underline{v}$, the competitors have lower valuations than the initiating bidder with probability one which results in a positive expected surplus for high enough

[16]This result does not hold in the general model of Section III, in which the bidders are asymmetric with respect to their means of payment or cash constraints.

X_t and, a result, in a finite initiation threshold.

The equilibrium is summarized in the following proposition:

Proposition 1. *The symmetric equilibrium in the joint entry-bidding game when both bidders always make bids in cash is as follows. A bidder with the valuation v approaches the target at threshold $\bar{X}_c(v)$, given by (6), provided that no bidder has approached the target before. In the auction, bidder i progressively increases its bid until it reaches $b_i = v_i X_t + \Delta$ or until the other bidder drops out. The bidder with the higher valuation initiates the auction and acquires the target.*

In the special case of the uniform distribution of v over $[\underline{v}, \bar{v}]$, $\mathbb{E}[w|w < v] = (\underline{v} + v)/2$. Therefore,

$$\bar{X}_c(v) = \frac{\beta}{\beta - 1} \frac{2\Delta}{v - \underline{v}}. \tag{7}$$

It is easy to see that $\bar{X}_c(v)$ is indeed a decreasing function of v.

II.B Two Stock Bidders

Now, consider the case in which both bidders make offers in stock. The English auction in the case of stock bids is formalized in the following way. The auctioneer sets the initial fraction of the combined company to zero and gradually raises it. Each bidder confirms its participation continuously until it chooses to withdraw from the auction irreversibly. As soon as only one bidder remains, it is declared the winner and pays the current fraction of the combined company. It immediately follows that the weakly dominant strategy for bidder i is to bid up to α_i, where

$$(1 - \alpha_i)(\Pi_b + v_i X_t) = \Pi_o \tag{8}$$

Intuitively, as in the case of cash offers, each bidder is willing to bid up to a level at which its remaining payoff ($(1 - \alpha_i)(\Pi_b + v_i X_t)$) is equal to its payoff from losing the auction to the rival (Π_o). It is easy to show that the value of this offer to the target is equal to (3). Suppose that a bidder with valuation v wins against the bidder with valuation $w \leq v$. Then, it pays fraction

$$1 - \frac{\Pi_o}{\Pi_b + wX_t} \tag{9}$$

of the combined company to the target. Note that the winner's payment depends on its valuation, while the share it pays is determined by the valuation of its competitor. Because the value of the stock payment increases in the bidder's valuation, the value of the winner's payment in this case is higher than in the case of an all-cash auction against the same competitor. More precisely, the change in the winner's value relative to the stand-alone level is

$$\frac{\Pi_o (v - w) X_t}{\Pi_b + wX_t} - \Delta. \tag{10}$$

Note that $\Pi_o / (\Pi_b + wX_t) < 1$. It immediately follows that a bidder always obtains less surplus from competing in stock than in cash.

Conjecture that bidder with valuation v finds it optimal to approach the target at threshold $\bar{X}_s(v)$, provided that the other bidder has not approached the target yet, where $\bar{X}_s(v)$ is decreasing in v. If the bidder with valuation v follows the strategy of approaching the target at threshold $\bar{X}$, its expected payoff at the initial date is

$$\begin{aligned} &\left(\frac{X_0}{\bar{X}}\right)^{\beta} \int_{\underline{v}}^{\bar{X}_s^{-1}(\bar{X})} \left(\frac{\Pi_o}{\Pi_b + w\bar{X}} \bar{X} \max\{v - w, 0\} - \Delta\right) dF(w) \\ &+ \int_{\bar{X}_s^{-1}(\bar{X})}^{\overline{v}} \left(\frac{X_0}{\bar{X}_s(w)}\right)^{\beta} \left(\frac{\Pi_o}{\Pi_b + w\bar{X}_s(w)} \bar{X}_s(w) \max\{v - w, 0\} - \Delta\right) dF(w), \end{aligned} \tag{11}$$

Similarly to the case of two cash bidders, the first (second) term of (11) reflects the case in which the bidder with valuation v (its competitor) initiates the auction. Maximizing (11) with respect to $\bar{X}$ and applying the equilibrium condition that the maximum is reached at

$\bar{X}_s(v)$, we obtain

$$\mathbb{E}\left[\frac{\Pi_o\left(\Pi_b+\frac{\beta}{\beta-1}w\bar{X}_s(v)\right)}{\left(\Pi_b+w\bar{X}_s(v)\right)^2}(v-w)\,|w\le v\right]\bar{X}_s(v)=\frac{\beta}{\beta-1}\Delta. \tag{12}$$

As with (6), the left-hand side is a strictly increasing function of $\bar{X}$, which verifies the conjecture that the optimal approaching policy of each bidder is given by the upper trigger $\bar{X}_s(v)$. In particular, monotonicity implies that if the trigger exists, it is unique. However, (12) does not have a solution for some $v\in[\underline{v},\bar{v}]$. By monotonicity, the highest value of the left-hand side of (12) is

$$\lim_{\bar{X}\to\infty}\mathbb{E}\left[\frac{\Pi_o\left(\Pi_b+\frac{\beta}{\beta-1}w\bar{X}\right)\bar{X}}{\left(\Pi_b+w\bar{X}\right)^2}(v-w)\,|w\le v\right]=\frac{\beta}{\beta-1}\Pi_o\mathbb{E}\left[\frac{v-w}{w}|w\le v\right]. \tag{13}$$

This value decreases in v and reaches zero when $v=\underline{v}$.[17] Thus, once v decreases to a sufficiently low level v^*, given by

$$\mathbb{E}\left[\frac{v^*-w}{w}|w\le v^*\right]=\frac{\Delta}{\Pi_o}, \tag{14}$$

no bidder finds it optimal to approach the target, even though it is socially optimal to do so when X_t is high enough. The intuition can be seen from (10). As $X_t\to\infty$, the bidder with valuation v only obtains a limited revenue, $\Pi_o\frac{v-w}{w}-\Delta$, in a contest against the bidder with valuation w. The reason for the limited bidder revenue in the case of stock bids is that in the model, the relative size of the bidders and the target changes in time. As X_t increases, for the same v, the bidder has to give away a larger portion of the combined company to the target. As a result, the expected revenue of the bidder with valuation v

[17] To see that the value decreases in v, differentiate it with respect to v. The derivative is

$$-\frac{\beta}{\beta-1}\Pi_o\int_{\underline{v}}^{v}\frac{v-w}{w}\frac{f(w)\,f(v)}{F(v)^2}dw<0.$$

is also limited from above as $X_t \to \infty$. For sufficiently low v, the bidder prefers to remain stand-alone; the threshold v^* is given above.

The equilibrium is summarized in the following proposition:

Proposition 2. *The symmetric equilibrium in the joint entry-bidding game when both bidders always make bids in stock is as follows. If the valuation of a bidder is $v > v^*$, where v^* is defined by (14), then it approaches the target at threshold $\bar{X}_s(v)$, given by (12), provided that no bidder has approached the target before. If $v \leq v^*$, then a bidder never approaches the target first. In the auction, bidder i progressively increases its bid until it reaches $\alpha_i(v_i)$, given by (8) or until the other bidder drops out. If $\max(v_1, v_2) > v^*$, the bidder with the higher valuation initiates the auction and acquires the target. If $\max(v_1, v_2) \leq v^*$, the takeover never occurs.*

While there is no analytical solution for $\bar{X}_s(v)$, it is easy to study its properties. In particular, it is interesting to see how (12) relates to (6). For this purpose, it is convenient to decompose (12) into two parts:

$$\mathbb{E}\left[\frac{\Pi_o(v-w)\bar{X}}{\Pi_b + w\bar{X}}|w \leq v\right] + \frac{1}{\beta - 1}\mathbb{E}\left[\frac{\Pi_o(v-w)w\bar{X}^2}{\left(\Pi_b + w\bar{X}\right)^2}|w \leq v\right] = \frac{\beta}{\beta - 1}\Delta. \tag{15}$$

The left-hand side of (15) consists of two components. The first component is the surplus that the bidder obtains in expectation. It is always below the left-hand side of (6), because separation is costlier is stock than in cash. If this were the only term on the left-hand side of (15), then each bidder would always find it optimal to approach the target later if it bids in stock. However, (15) contains an additional positive second term. It corresponds to the effect that the delay causes the surplus of the bidder to increase at a slower pace when the bidder makes bids in stock. Alternatively, one can think of this term as a part of the delay cost on the right-hand side of (15): when X_t is higher, further delay is less costly to the bidder as further increase in X_t has a negative effect of a smaller magnitude on the bidder revenue. The magnitude of this effect depends on the value of delay parameter $\beta/(\beta-1)$. The following proposition shows that if $\beta/(\beta-1)$ is not too high, then this additional effect is dominated by the first effect, so the bidder always approaches the target earlier if it bids in cash:

Proposition 3. *Suppose that the measure of the option value of delay, $\beta/(\beta-1)$, is not too high:*

$$\frac{\beta}{\beta-1}<2\frac{\Pi_b}{\Pi_o}. \tag{16}$$

Then, $\bar{X}_s(v)>\bar{X}_c(v)$ for any *v.*

For standard parameters, the multiplier of the delay option, $\beta/(\beta-1)$, does not exceed 2. As a consequence, condition (16) holds, so other things equal the bidder approaches the target later if it bids in stock. However, if the multiplier of the delay option is very high, then the stock bidder can approach the target earlier than the cash bidder, despite obtaining a lower fraction of the total surplus in expectation. As an example, consider an extreme case in which the multiplier of the delay option is infinite, $\beta/(\beta-1)\to\infty$ (or equivalently, $\beta\to 1$). Then, the optimal threshold of the cash bidder is $\bar{X}_c(v)\to\infty$ for all v. By contrast, (15) implies that the optimal threshold of the stock bidder $\bar{X}_s(v)$ solves

$$\mathbb{E}\left[\Pi_o\frac{(v-w)\,w\bar{X}_s(v)^2}{\left(\Pi_b+w\bar{X}_s(v)\right)^2}|w\leq v\right]=\Delta. \tag{17}$$

In particular, it is finite for all $v>v^*$. Thus, if $\beta/(\beta-1)$ is very high, then stock bidders with high enough v approach the target earlier than cash bidders. However, it is never the case that stock bidders approach the target earlier than cash bidders uniformly for all v: if v is low enough, $\bar{X}_c(v)$ is always below $\bar{X}_s(v)$ even if condition (16) does not hold.

In the rest of the paper we assume that realistic condition (16) holds. We refer to this case as the "normal" case.

II.C Cash vs. Stock Bidder

Finally, consider the case in which one bidder makes bids in cash and the other bidder makes bids in stock. In this case, as well as in the model with endogenous means of payment of Section III, bidders make bids in different securities. Before proceeding with the analysis, we extend the formalization of the English auction for bids from different security sets. The following definition puts a formal structure on the English auction:

Definition (English auction for bids in combinations of stock and cash). The auctioneer sets the starting price to zero and gradually rises it. A price p corresponds to either a payment of p dollars in cash or a payment of any $b \in [0, p]$ dollars in cash and a fraction $\alpha(b, p)$ in the stock of the combined company defined below. As p gradually rises, a bidder confirms its participation until it decides to withdraw from the auction. As soon as only one bidder remains, it is declared the winner and pays any element of its choice from set $\{(b, \alpha(b, p), b \in [0, p])\}$, corresponding to price p at which its competitor dropped. $\alpha(b, p)$ is such that a bidder who withdraws at price p is indifferent between all elements of set $\{(b, \alpha(b, p), b \in [0, p])\}$:

$$\alpha(b, p) = \frac{p - b}{\Pi_o + p}. \tag{18}$$

This formalization extends the standard "button" model of an English auction for all-cash bids (Milgrom and Weber, 1982), as well as the analogous model for all-stock bids (Hansen, 1985). If bidders always bid in cash, the definition is equivalent to an auction in which the seller gradually rises the cash price, which the winner pays once its rival withdraws. Similarly, if bidders always bid in stock, the definition is equivalent to an auction in which the seller gradually rises the proportion of the combined company, which the winner pays once its rival withdraws. The indifference condition for $\alpha(b, p)$ means that the decision of a bidder to drop from the auction is only driven by its valuation and not the security it is bidding with. To obtain (18), note that the bidder with valuation v withdraws at price p if and only if

$$(1-\alpha)\left(\Pi_b + vX_t\right) = b + \Pi_o \;\Rightarrow\; vX_t + \Pi_b = \frac{b+\Pi_o}{1-\alpha}. \tag{19}$$

The indifference condition requires $(b+\Pi_o)/(1-\alpha)$ to be the same for all $b \in [0,p]$ and yields (18). In the case considered here, the stock bidder does not have cash, so for it, $b=0$, $\alpha(b,p) = \frac{p}{\Pi_o+p}$.

Without loss of generality, we call the cash bidder "bidder 1" and the stock bidder "bidder 2." Suppose that an auction takes place at time t. If the valuation of bidder 1 is equal to v_1, bidder 1 is willing to offer up to $b(v_1) = p(v_1) = v_1 X_t + \Delta$. If the valuation of bidder 2 is equal to v_2, bidder 2 is willing to offer up to $\alpha(v_2)$, given by (8). The cash value of this bid is equal to $p(v_2) = v_2 X_t + \Delta$. If $v_1 > v_2$, then bidder 1 is the winning bidder, and it pays according to the maximum value that bidder 2 is willing to offer, $v_2 X_t + \Delta$. If $v_2 > v_1$, then bidder 2 is the winning bidder, and it has to pay a fraction of the combined company corresponding to the maximum cash bid that bidder 1 is willing to offer, $\alpha(v_1) \equiv \alpha(b(v_1), p(v_1))$. In other words, the winning bidder of each type (cash or stock) makes a payment that only depends on its but not its competitor's valuation.

We assume that the equilibrium in strictly decreasing initiation strategies of both bidders, $\bar{X}_1(v)$ and $\bar{X}_2(v)$, exists. This is the case in all of our numerical examples. We do not make any assumptions about ordering of the two strategies but later provide conditions under which such ordering can be established. First, if bidder 1 with valuation v approaches the target at threshold $\bar{X}$, its expected payoff at the initial date equals[18]

$$\begin{aligned}
&\left(\frac{X_0}{\bar{X}}\right)^{\beta} \int_{\underline{v}}^{\bar{X}_2^{-1}(\bar{X})} \left(\bar{X} \max\{v-w,0\} - \Delta\right) dF(w) \\
&+ \int_{\bar{X}_2^{-1}(\bar{X})}^{\bar{v}} \left(\frac{X_0}{\bar{X}_2(w)}\right)^{\beta} \left(\bar{X}_2(w) \max\{v-w,0\} - \Delta\right) dF(w).
\end{aligned} \tag{20}$$

[18]Here and hereafter, we use $\bar{X}_i^{-1}(\bar{X})$, $i=\{1,2\}$ instead of the more precise $\min\{\bar{X}_i^{-1}(\bar{X}), \bar{v}\}$ to save on notation.

Intuitively, if valuation of bidder 2 is below $\bar{X}_2^{-1}(\bar{X})$, bidder 1 initiates the auction at threshold $\bar{X}$. Otherwise, the auction is initiated by bidder 2. If the auction is initiated at some X_t and valuation of bidder 1, v, is above valuation of bidder 2, w, then bidder 1 wins the auction, makes a payment in cash and is left with the revenue equal to $X_t\,(v-w)-\Delta$. If $v<w$, it loses the auction and suffers the loss of Δ. Maximizing (20) with respect to $\bar{X}$ and applying the equilibrium condition that the maximum is reached at $\bar{X}_1\,(v)$, we obtain

$$\bar{X}_1\,(v)=\frac{\beta}{\beta-1}\frac{\Delta}{v-\mathbb{E}\left[w|w\leq\Omega(v)\right]}\Psi(v), \tag{21}$$

where for bidder i and its competitor $-i$, $\Omega(v)=\min\left\{v,\bar{X}_{-i}^{-1}\left(\bar{X}_i(v)\right)\right\}$ and $\Psi\left(v\right)\equiv\max\left\{1,\frac{F\left(\bar{X}_{-i}^{-1}\left(\bar{X}_i(v)\right)\right)}{F(v)}\right\}$. Note that (21) is very similar to (6). To see the intuition for the difference, consider, without loss of generality, $\bar{X}_1\,(v)<\bar{X}_2\,(v)$. Then for bidder 1, $\Omega(v)=v$, $\Phi(v)\geq 1$. Consequently, bidder 1 delays approaching the target compared to the case in which it faces another cash bidder: $\bar{X}_1(v)\leq\bar{X}_c(v)$. Intuitively, because other things equal bidder 2 with the same valuation approaches the target later than bidder 1, upon approaching bidder 1 faces a stronger competitor than if it faced a cash bidder. Because of this, bidder 1 faces a lower probability of winning the auction, which decreases its expected surplus. Consequently, it further delays approaching the target.

Second, if bidder 2 with valuation v approaches the target at threshold $\bar{X}$, its expected payoff at time 0 is equal to

$$\begin{aligned}&\left(\frac{X_0}{\bar{X}}\right)^{\beta}\int_{\underline{v}}^{\bar{X}_1^{-1}(\bar{X})}\left(\frac{\Pi_0}{\Pi_b+w\bar{X}}\bar{X}\max\left\{v-w,0\right\}-\Delta\right)dF(w)\\&+\int_{\bar{X}_1^{-1}(\bar{X})}^{\bar{v}}\left(\frac{X_0}{\bar{X}_1\,(w)}\right)^{\beta}\left(\frac{\Pi_0}{\Pi_b+w\bar{X}_1(w)}\bar{X}_1(w)\max\left\{v-w,0\right\}-\Delta\right)dF(w).\end{aligned} \tag{22}$$

This expression is similar to (20), with the only difference that bidder 2 pays stock if it wins the contest and is left with the revenue equal to $\left(\frac{\Pi_0}{\Pi_b+wX_t}X_t\max\left\{v-w,0\right\}-\Delta\right)$. Maximizing (22) with respect to $\bar{X}$ and applying the equilibrium condition that the maximum is reached at $\bar{X}_2(v)$, we obtain

$$\mathbb{E}\left[\frac{\Pi_o\left(\Pi_b+\frac{\beta}{\beta-1}w\bar{X}_2(v)\right)}{\left(\Pi_b+w\bar{X}_2(v)\right)^2}(v-w)\,|w\leq\Omega(v)\right]\bar{X}_2(v)=\frac{\beta}{\beta-1}\Delta\Psi(v). \tag{23}$$

Note that (23) is very similar to (12). To see the intuition for the difference, again, consider $\bar{X}_1(v)<\bar{X}_2(v)$, so that for bidder 2, $\Omega(v)<v$ and $\Psi(v)=1$. Because w takes lower values compared to the case in which bidder 2 faces another stock bidder, bidder 2 accelerates approaching the target: $\bar{X}_2(v)\geq\bar{X}_s(v)$. Intuitively, because other things equal bidder 1 with the same valuation approaches the target earlier than bidder 2, upon approaching bidder 2 faces a weaker competitor than if it faced another stock bidder. Because of this, bidder 2 obtains a higher expected surplus from the auction, which accelerates its decision to approach the target.

The equilibrium is summarized in the following proposition:

Proposition 4. *The separating equilibrium in the joint entry-bidding game between the stock and the cash bidder takes the following form. The initiation strategy of bidder 1 (the cash bidder) with valuation* v_1 *is to approach the target at threshold* $\bar{X}_1(v_1)$*, given by (21), provided that no bidder has approached the target before. The initiation strategy of bidder 2 (the stock bidder) with valuation* $v_2>v_2^*$ *is to approach the target at threshold* $\bar{X}_2(v_2)$*, given by (23), provided that no bidder has approached the target before. If* $v_2\leq v_2^*$*, then bidder 2 never approaches the target first. The boundary type* v^* *is given by*

$$v_2^*=\frac{\Pi_b}{\Pi_o}\underline{v}>\underline{v}. \tag{24}$$

In the auction, bidder 1 progressively increases its cash bid until it reaches $b(v_1)=v_1X_t+\Delta$ *or until bidder 2 drops out; bidder 2 progressively increases its stock bid until it reaches fraction* $\alpha(v_2)$*, given by (8) or until the other bidder drops out.*

As in the case of two stock bidders, expecting low revenue from acquiring the target in stock, the stock bidder does not initiate the takeover contest for valuations equal to or below v_2^*. There is no analytical solution for the jointly determined $\bar{X}_1(v)$ and $\bar{X}_2(v)$ but two closed form equations can be obtained for $\bar{X}_1^{-1}(X)$ and $\bar{X}_2^{-1}(X)$ which make the numerical analysis of the strategies easy. Appendix B provides more detail.

Proposition 5 establishes ordering of strategies in the three cases of exogenous means of payment for standard parameters, such that $\beta/(\beta-1)$ does not exceed 2:

Proposition 5. *If equilibria in strictly decreasing initiation strategies $\bar{X}_c(v)$; $\bar{X}_s(v)$; and $\bar{X}_1(v)$, $\bar{X}_2(v)$ exist and $\frac{\beta}{\beta-1} < 2\frac{\Pi_b}{\Pi_o}$ then the strategies are ordered: $\bar{X}_s(v) > \bar{X}_2(v) > \bar{X}_1(v) > \bar{X}_c(v)$ for any v.*

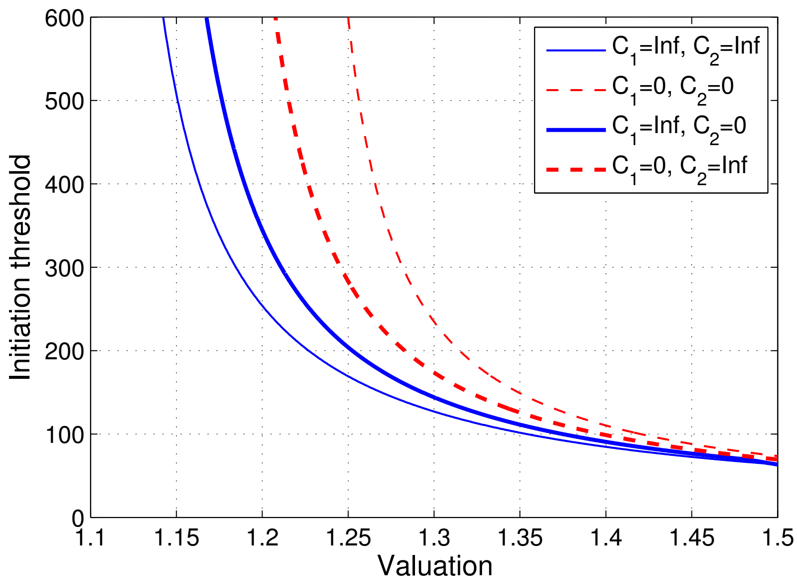

Figure 1: Initiation strategies of cash and stock bidders facing different types of competitors. The figure shows the optimal initiation strategies of bidders as a function of their valuations, v. The thin solid (thin dashed) line is the strategy of a cash (stock) bidder facing another cash (stock) bidder; the thick solid (thick dashed) line is the strategy of a cash (stock) bidder facing a stock (cash) bidder.

For the numerical example, we choose the benchmark model parametrization: $r = 0.05$,

$\mu = 0.01$, $\sigma = 0.25$, $\underline{v} = 1.1$, $\bar{v} = 1.5$, $v \sim \text{Uniform}[\underline{v}, \bar{v}]$, $\Pi_b = 100$, $\Pi_o = 95$. These values are also reported in Table I. Specifically, the benchmark case considers acquisition of a target whose assets grow at the risk-adjusted rate μ, typically used in dynamic models of the firm, and who has the average COMPUSTAT asset volatility σ. The losing bidder's profits are 5% below the pre-acquisition levels. The average synergies are equal to 30% of the target's core business. The interest rate is set at constant 5%. The benchmark parametrization satisfies $\beta/(\beta-1) < 2$.

Figure 1 shows the four thresholds for our benchmark parametrization as a function of bidders' valuations, v. A higher probability of losing the takeover contest makes a cash bidder that competes against a stock bidder more cautious compared to the case when it competes against another cash bidder. As a result, its initiation threshold increases. The opposite is also true: a lower probability of losing the takeover contest makes a stock bidder more aggressive when it competes against a cash bidder. As a result, its initiation threshold decreases. Another interesting result is that competing against a cash bidder not only directly accelerates the initiation by a stock bidder but also makes stock bidders with lower valuations willing to initiate in the first place: $v_2^* < v^*$. As a result, both effects combine so that initiation of takeover contests is sped up and happens more frequently when at least one of the bidders is able to bid cash.

III Initiation with Endogenous Means of Payment

In the previous section we solved for the optimal initiation thresholds assuming that means of payment of each bidder is exogenous. In this section, we make them endogenous by introducing cash constraints of bidders to the economy: specifically, bidder i can only bid up to $C_i \geq 0$ in cash. Throughout the section, we assume that the equilibrium with strictly decreasing initiation strategies of both bidders, $X_1(v)$ and $X_2(v)$, exists. This is the case in all of our numerical examples.

Consider the auction that occurs at time t. Suppose that valuations of bidder i and j are

v and $w < v$, respectively. At any price p during the course of the auction at which bidder j has not withdrawn yet, bidder i prefers not to withdraw. Therefore, bidder j withdraws first. Because at the withdrawal point, bidder j is indifferent between all elements of the set (b, α), its withdrawal price point p is such that it is indifferent between acquiring the target for p in cash and taking the outside option: $p = wX_t + \Delta$. Once bidder j withdraws, bidder i selects the element of the set (b, α) yielding the lowest expected payment. Because bidder j is indifferent between all elements of the set and the valuation of bidder i is higher, the payment of bidder i is minimized if the cash amount in the payment is maximized. If $C_i \geq wX_t + \Delta$, bidder i acquires the target by paying $wX_t + \Delta$ in cash. Otherwise, bidder i acquires the target by paying C_i in cash and fraction

$$\alpha\left(C_i, wX_t + \Delta\right) = 1 - \frac{\Pi_o + C_i}{\Pi_b + wX_t} \tag{25}$$

of the combined company. In the former case, the change in the value of bidder i relative to the stand-alone value is

$$\left(v - w\right) X_t - \Delta. \tag{26}$$

In the latter case, it is

$$\begin{aligned} & \frac{\Pi_o + C_i}{\Pi_b + wX_t}\left(v - w\right) X_t - C_i - \Pi_b \\ = \quad & \left(\Pi_o + C_i\right) \frac{\left(v - w\right) X_t}{\Pi_b + wX_t} - \Delta. \end{aligned} \tag{27}$$

Consider the decision of bidder i with valuation v to approach the target. If bidder i approaches the target at threshold $\bar{X}$, its expected payoff at the initial date equals

$$\begin{aligned} & \left(\frac{X_0}{\bar{X}}\right)^\beta \int_{\underline{v}}^{\bar{X}_{-i}^{-1}(\bar{X})} \left(\min\left\{\frac{\Pi_o + C_i}{\Pi_b + w\bar{X}}, 1\right\} \bar{X} \max\left\{v - w, 0\right\} - \Delta\right) dF\left(w\right) \\ + & \int_{\bar{X}_{-i}^{-1}(\bar{X})}^{\bar{v}} \left(\frac{X_0}{\bar{X}_{-i}\left(w\right)}\right)^\beta \left(\min\left\{\frac{\Pi_o + C_i}{\Pi_b + w\bar{X}_{-i}\left(w\right)}, 1\right\} \bar{X}_{-i}(w) \max\left\{v - w, 0\right\} - \Delta\right) dF\left(w\right). \end{aligned} \tag{28}$$

Intuitively, if the valuation of the competitor is below $\bar{X}_{-i}^{-1}\left(\bar{X}\right)$, bidder i approaches the target at threshold $\bar{X}$. Otherwise, the competitor approaches the target at threshold $\bar{X}_{-i}(w)$. In both cases, if $v > w$, bidder i wins the auction and makes a payment either in cash or in a combination of cash and stock. If $v < w$, it loses the auction and suffers the loss of Δ. Maximizing (28) with respect to $\bar{X}$ and using the equilibrium condition that the maximum is reached at $\bar{X}_i(v)$, we obtain

$$\begin{aligned} & \mathbb{E}\left[\min\left\{\frac{\Pi_o + C_i}{\Pi_b + w\bar{X}_i(v)}, 1\right\}(v-w)\,|w \leq \Omega(v)\right]\bar{X}_i(v) \\ & + \frac{1}{\beta-1}\int_{\min\left(\frac{C_i-\Delta}{\bar{X}_i(v)},\Omega(v)\right)}^{\Omega(v)} \left(\Pi_o + C_i\right)\frac{(v-w)\,w\bar{X}_i(v)^2}{\left(\Pi_b + w\bar{X}_i(v)\right)^2}\frac{f(w)}{F(\Omega(v))}dv \\ = & \frac{\beta}{\beta-1}\Delta\Psi(v), \end{aligned} \tag{29}$$

where $\Omega(v) \equiv \min\left\{v, \bar{X}_{-i}^{-1}\left(\bar{X}_i(v)\right)\right\}$ and $\Psi(v) \equiv \max\left\{1, \frac{F\left(\bar{X}_{-i}^{-1}\left(\bar{X}_i(v)\right)\right)}{F(v)}\right\}$. The system of equations (29) for bidders 1 and 2 jointly determines equilibrium thresholds $\bar{X}_1(v)$ and $\bar{X}_2(v)$. Note that this solution embeds solutions for three special cases, studied in Section II. The following proposition summarizes the equilibrium:

Proposition 6. *The equilibrium in the general model takes the following form. Bidder i with valuation $v_i > v_i^*$ approaches the target at threshold $\bar{X}_i(v_i)$, provided that it has not been approached before, where $\bar{X}_i(v)$ satisfies (29) and v_i^* is defined in Appendix A. If $v_i \leq v_i^*$, bidder i never approaches the target first. Once the auction is initiated at time t, bidder i progressively increases its bid until the price reaches $vX_t + \Delta$ or the other bidder drops out. If the other bidder drops out, bidder i acquires the target for cash, if its cash balance exceeds the price at which the other bidder drops out, and for cash and stock paying C_i in cash, otherwise.*

As long as $C_1 < \infty$ and $C_2 < \infty$, each bidder never approaches the target for valuations equal to or below, correspondingly, v_1^* and v_2^*. Appendix B provides more detail on the numerical solution for $\bar{X}_1(v)$ and $\bar{X}_2(v)$.

Figure 2, Panel A shows the four thresholds (cash vs. cash bidders, stock vs. stock bidders, and bidders with internal cash $C_1 = 125$ and $C_2 = 0$ competing against each other) for our benchmark parametrization as a function of bidders' valuations, v. An interesting new effect compared to the case of exogenous means of payment is that for intermediate valuations, constrained bidders can choose to accelerate initiation even relative to the case of two cash bidders. This happens because they attempt to "fit into" their cash constraints. Consider Figure 2, Panels B and C that show expected bidder revenue from non-cash and cash-only deals. As the valuation of bidder 1 decreases, it initiates contests for a larger target and eventually finds itself unable to complete all deals in cash (the dashed vertical line on the right-hand side of all panels). At this stage, bidder 1 trades off costs of inefficiently early initiation against its benefits (a smaller probability that the deal is non-cash, resulting in a higher expected revenue from the auction). If the latter dominates, bidder 1 can approach a smaller target compared to the case when it is unconstrained ($C_1 = 0$) or even to the case when both bidders are unconstrained. As the valuation of bidder 1 decreases even further (beyond the dashed vertical line on the left-hand side of all panels), any successful contest requires the payment of at least C_1 that makes fitting into cash not possible. Then, bidder 1's initiation threshold increases faster, similarly to an all-stock bidder.

Consider bidder 2 who competes against bidder 1 with $C_1 < \infty$ instead of $C_1 \to \infty$. Bidder 1 attempts to fit into cash and, for intermediate valuations, accelerates its initiation compared to $C_1 \to \infty$, so bidder 2 becomes a stronger bidder with higher expected revenues. As a result, it is optimal for bidder 2 to also accelerate initiation for intermediate valuations.

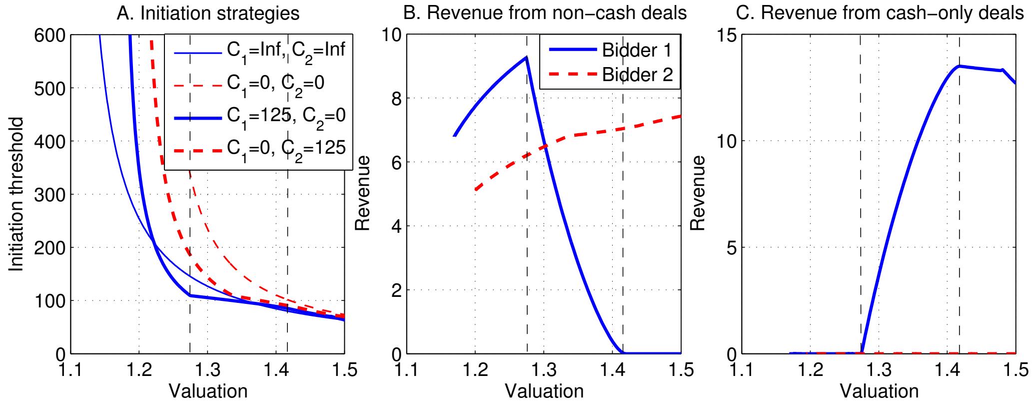

Figure 2: Initiation strategies of cash and stock bidders facing different types of competitors. Panel A shows the optimal initiation strategies of bidders as a function of their valuations, v. The thin solid (thin dashed) line is the strategy of a cash (stock) bidder facing another cash (stock) bidder; the thick solid (thick dashed) line is the strategy of a bidder with internal cash $C_1 = 125$ ($C_1 = 0$) facing a bidder with internal cash $C_2 = 0$ ($C_2 = 125$). Panels B and C show the part of expected bidder revenue from non-cash and cash-only deals (first line of (29)) for bidders with internal cash $C_1 = 125$ and $C_2 = 0$.

IV Comparative Statics

In this section, we investigate the effects of target and bidder characteristics on initiation strategies. Proposition 7 establishes comparative statics results for the general model of Section III:

Proposition 7. *Assume that each bidder is, in any combination, either severely constrained ($C_i < \Delta$) or unconstrained ($C_i \to \infty$), and that (16) holds. Consider an equilibrium in decreasing initiation strategies $\bar{X}_i(v)$. For any v, $\bar{X}_i(v)$, $i \in \{1, 2\}$:*

1. increase in μ;
2. increase in σ;
3. decrease in r;
4. increase in Δ (keeping Π_b fixed);
5. weakly decrease in Π_b (keeping Δ fixed).

Proposition 7 provides sufficient conditions for monotone comparative statics; in addition, the numerical analysis shows that the same results hold for a wide range of cash constraints of both bidders. These results are intuitive. *(1)* When μ is higher, bidders wait longer before approaching the target: the present value of costs associated with losing the deal increases due to X_t reaching the initiation threshold of a competitor faster, and this increase dominates an increase in the present value of synergies in case of success. *(2)* For the same reason, when the discount rate r is lower, the costs of losing the deal loom larger, so the takeover contest is initiated later. *(3)* Similarly, higher σ implies a higher likelihood of the competitor reaching the initiation threshold fast, which in turn increases costs of losing the deal and leads to delay in initiation. *(4)* When costs of losing the contest, Δ, are high, the winning bidder has to pay more to separate itself from the losing bidder: the value of the winning bidder's outside option (losing) is a negative function of Δ. As a result, the bidders' expected payoffs from the contest decrease, so they initiate later. *(5)* The additional restrictions here make the motive to fit into cash weak, resulting in monotone comparative statics. The initiation strategies of two unconstrained bidders competing with each other are constant in Π_b keeping Δ fixed. For a severely constrained bidder, however, a larger Π_b results in its bidding a smaller portion of the combined company, which leads to earlier initiation, no matter the constraints of the competitor.

In case *(5)*, it is easy to notice that when an unconstrained bidder competes against a severely constrained bidder, its initiation threshold also decreases in Π_b. The reason is that a higher Π_b speeds up initiation by the constrained bidder. Thus, conditional on the constrained bidder not initiating yet, the unconstrained bidder faces, on average, a weaker competitor. As a result, at any hypothetical initiation threshold, the expected payoff of the unconstrained bidder from initiating the contest is higher, leading to a lower initiation threshold.

Figure 3 shows the comparative statics of the four equilibrium initiation strategies corresponding to the model in Sections II.A–II.C. The strategies are built for the benchmark model parametrization, for a bidder with the average valuation, $v = 1.3$. The comparative statics are with respect to the five model parameters highlighted in Proposition 7 as well as the dispersion of the bidders' valuations. As the dispersion of the valuations increases, a bidder with valuation v becomes better separated from bidders with lower valuations, and therefore on average pays less in a successful contest. As a result, the bidder initiates the contest earlier. The initiation strategies seem to be particularly sensitive to the costs of losing the deal and the dispersion of the bidders' valuations. In fact, when costs of losing the deal (the dispersion of valuations) are sufficiently high (low), the stock bidder with the average valuation never initiates the contest: its valuation is below the threshold v^* (v_2^*) obtained in Proposition 2 (3).

Figure 4 depicts the comparative statics of the four equilibrium initiation strategies (two unconstrained bidders, two extremely constrained bidders ($C_1 = C_2 = 0$), and bidders with internal cash $C_1 = 125$ and $C_2 = 0$ competing against each other) for the benchmark model parametrization as a function of the same six model parameters. The strategies are plotted for the bidder with the average valuation, $v = 1.3$. Incentives to fit into cash constraints are strong when μ, σ or P_b are higher, and when r is smaller. In all these cases, the combined company has a higher expected value. When means of payment are endogenous, the bidders are unwilling to share this highly-valued company with the target and choose to predominantly pay cash at the cost of earlier initialization.

Figure 5 shows the comparative statics of the optimal initiation strategies for the benchmark model parametrization and bidders with cash constraints C_1 and $C_2 = 0$, with respect to C_1. The strategies are calculated for the bidder with the average valuation, $v = 1.3$. For intermediate ranges of C_1, bidder 1 has incentives to fit into cash and bidder 2, recognizing that now it faces a weaker competitor, follows by decreasing its own initiation threshold. For low and high values of C_1, all deals either require all available cash to be done or are always done in cash only, weakening the motives to fit into cash. As a result, strategies of both cash-constrained bidders lie between the strategies of two unconstrained and two extremely constrained bidders competing against each other.

In unreported results, we extend the model to the case of more than two bidders under

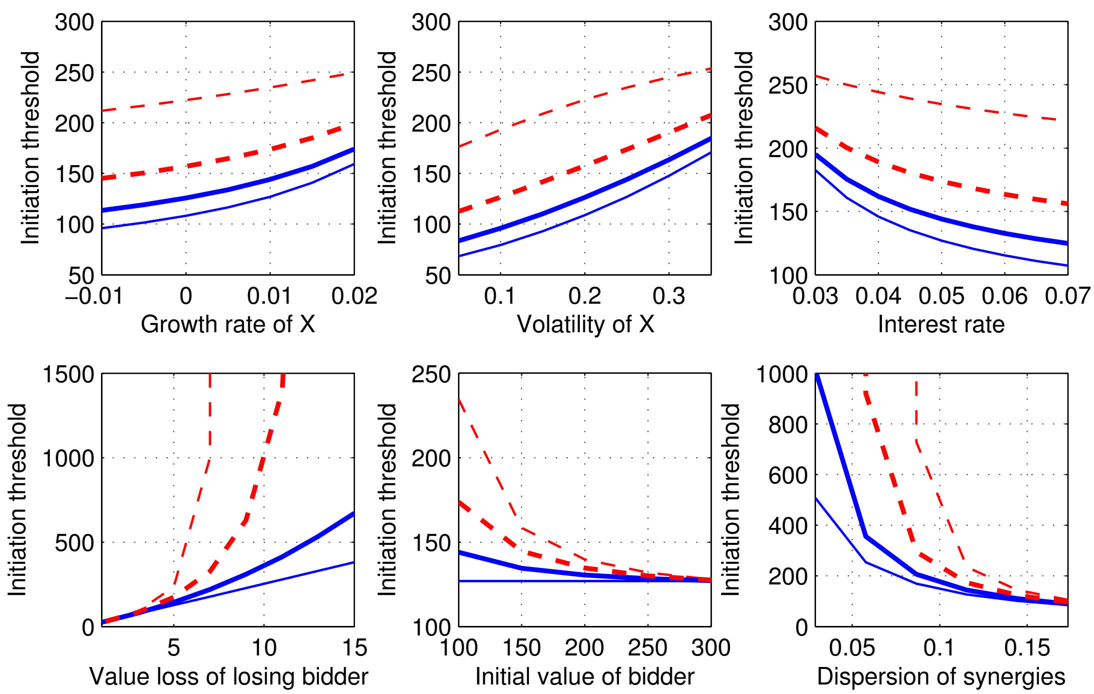

Figure 3: Initiation strategies of cash and stock bidders as a function of model parameters. The figure shows the comparative statics of the four initiation strategies for the benchmark model parametrization (Table I) and a bidder with the average valuation, $v = 1.3$. The thin solid (thin dashed) line is the strategy of a cash (stock) bidder facing another cash (stock) bidder; the thick solid (thick dashed) line is the strategy of a cash (stock) bidder facing a stock (cash) bidder. The comparative statics are with respect to *(i)* the growth rate of a target's assets, μ, *(ii)* the volatility of a target's assets, σ, *(iii)* the interest rate, r, *(iv)* costs of losing the contest, Δ, *(v)* the initial value of bidders, P_b (keeping Δ fixed), and *(vi)* the dispersion of the bidders' valuations, $D(v)$ (keeping the average valuation fixed).

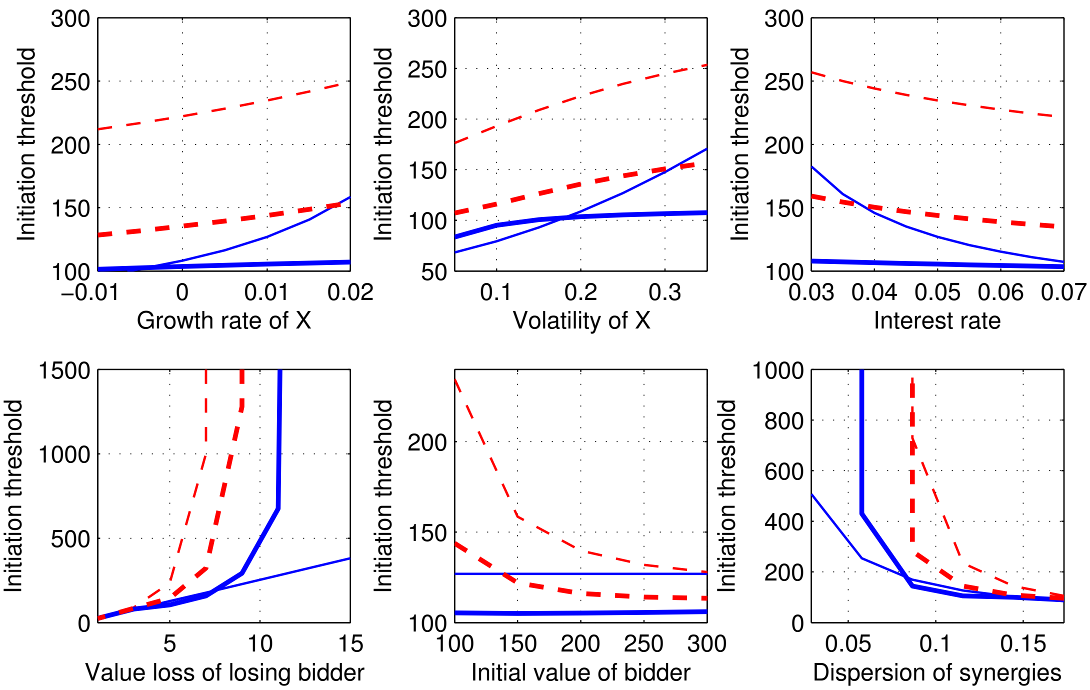

Figure 4: Initiation strategies of cash-constrained bidders as a function of model parameters. The figure shows the comparative statics of the four initiation strategies for the benchmark model parametrization (Table I) and a bidder with the average valuation, $v = 1.3$. The thin solid (thin dashed) line is the strategy of an unconstrained (extremely constrained) bidder facing another unconstrained (extremely constrained) bidder; the thick solid (thick dashed) line is the strategy of a bidder with cash $C_1 = 125$ ($C_1 = 0$) facing a bidder with cash $C_2 = 0$ ($C_2 = 125$). The comparative statics are with respect to *(i)* the growth rate of a target's assets, μ, *(ii)* the volatility of a target's assets, σ, *(iii)* the interest rate, r, *(iv)* costs of losing the contest, Δ, *(v)* the initial value of bidders, Π_b (keeping Δ fixed), and *(vi)* the dispersion of the bidders' valuations, $D(v)$ (keeping the average valuation fixed).

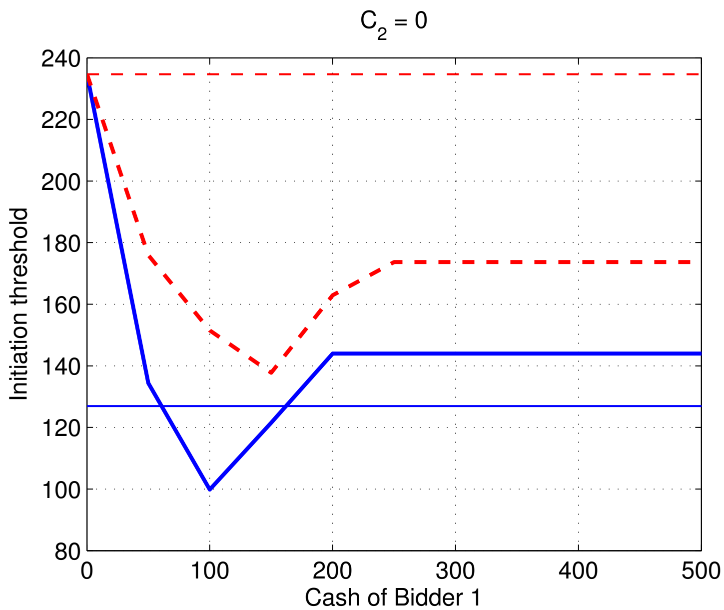

Figure 5: Initiation strategies of cash-constrained bidders as a function of asymmetries in cash constraints. The figure shows the comparative statics of the four initiation strategies for the benchmark model parametrization (Table I) as a function of cash constraints of bidder 1, C_1. The comparative statics are calculated for the bidder with the average valuation, $v = 1.3$. The thick solid (thick dashed) line is the strategy of a bidder with cash C_1 ($C_2 = 0$) facing a bidder with cash $C_2 = 0$ (C_1). The thin solid (thin dashed) line is the strategy of an unconstrained (extremely constrained) bidder facing another unconstrained (extremely constrained) bidder.

the assumption that all bidders are either unconstrained or extremely constrained. If a bidder faces more potential competitors, its expected payoff from initiation decreases, which results in later initiation.

V Analysis

The results obtained in previous sections yield a rich set of predictions. Below, we list and discuss each of them.

V.A Initiation of Takeover Contests, Means of Payment, and Premiums

A1. *Companies acquired in stock are larger and older than companies acquired in cash.* Bidders with low valuations have higher benefits to wait until the target grows large and end up paying in stock, as compared to bidders with intermediate to high valuations who wait less, possibly trying to "fit into" their cash constraints, and pay in cash. This result can be important for empirical research as it establishes a reverse link between means of payment in takeover contests and size/age of targets. Not only are large companies acquired in non-cash deals because the bidders do not have sufficient cash; such companies were allowed to grow large because potential synergies were not high enough for bidders to acquire them small.

Figure 6 shows, for the benchmark parametrization and $C_1 = 125$, $C_2 = 0$, probabilities that cash and non-cash deals are completed in years 1, 2–5, 6–10, 11–25, and 26–100[19] as well as average acquisition size in deals completed by the end of year 1, 5, 10, 25, and 100. The starting value of the target is such that it is on the verge of being acquired by the highest bidder with the lowest cash constraints: $X_0 = \bar{X}_1(\bar{v})$. Cash deals mostly happen within the first five years of the target's life while non-cash

[19] Formally, for each given realization of the two bidders' valuations, v_1 and v_2, the conditional probability

deals reach their peak in years 2–5 and continue to be dominant types of acquisition in years 6–10. Cash deals are on average smaller and the gap in average size of cash and non-cash deals increases with the sample horizon as more and more non-cash deals are made for large targets by bidders with the lowest valuations.

While we do not directly model shocks to cash constraints, the above results make it evident that takeover activity can be spurred by two types of shocks: technology shocks that affect the gains from a takeover and shocks to bidders' cash constraints. The effect of technology shocks is clear: a contest is triggered once technology shocks shoot the cash flow variable X_t up to the upper acquisition threshold. The effect of shocks to cash constraints is more subtle. According to a naive argument, cash constraints should have no effect, because even if a bidder is cash constrained, it can always pay the target the proportion of the stock of the combined firm. In the setting with bidders' private information about valuations, this naive argument is not valid, because a severely cash constrained bidder initiates a contest at a higher threshold than an unconstrained bidder. As a result, the change in economic environment that relaxes the bidders' cash constraints decreases the threshold on the level of cash flows at which each bidder initiates an acquisition and thereby sparks merger activity.[20]

A2. *Companies acquired in cash generate higher total premiums (target+bidder) than com-*

that a contest is initiated over a finite time horizon T is

$$\mathbb{P}[\text{acquisition}|v_1, v_2, X_t, T] = \min\left\{1, N\left(\frac{-\log\frac{\min\{\bar{X}_1(v_1),\bar{X}_2(v_2)\}}{X_t} + (\mu - \sigma^2/2)T}{\sigma\sqrt{T}}\right)\right.$$
$$\left. + \exp\left\{\frac{2(\mu-\sigma^2/2)\log\frac{\min\{\bar{X}_1(v_1),\bar{X}_2(v_2)\}}{X_t}}{\sigma^2}\right\} N\left(\frac{-\log\frac{\min\{\bar{X}_1(v_1),\bar{X}_2(v_2)\}}{X_t} - (\mu-\sigma^2/2)T}{\sigma\sqrt{T}}\right)\right\}.$$

Then, the conditional probability that a contest is initiated over a finite time horizon T for any v_1 and v_2 is

$$\mathbb{P}[\text{acquisition}|X_t, T] = \mathbb{E}_{v_1,v_2}\left[\mathbb{I}[v_1 > v_1^*, v_2 > v_2^*]\mathbb{P}[\text{acquisition}|v_1, v_2, X_t, T]\right],$$

where $\mathbb{I}[\cdot]$ is the indicator function equal to one if the condition in brackets is satisfied and zero otherwise.

[20] Allowing cash constraints to evolve in time, e.g., switch from high to low level only strengthens this result. Intuitively, in states with low cash, bidders have extra incentives to delay the acquisition until their cash constraints are relaxed and acquisition in mostly cash is possible.

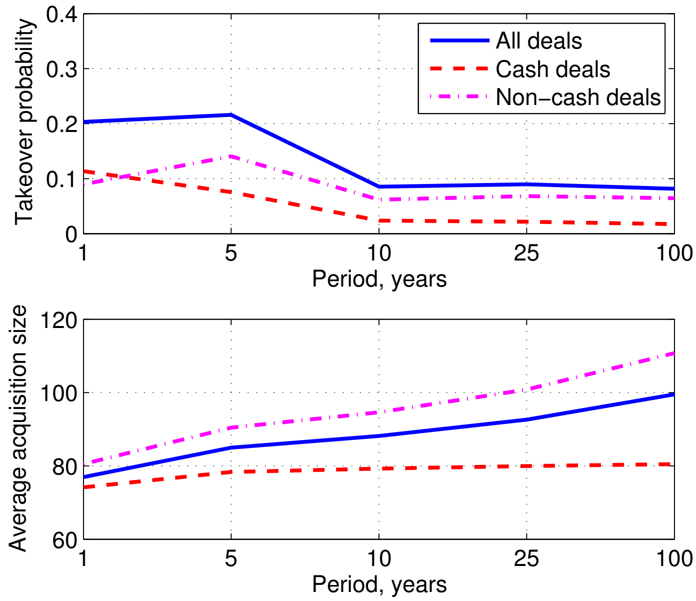

Figure 6: Takeover probability and average acquisition size in cash and non-cash deals. The figure corresponds to prediction A1. For the benchmark parametrization (Table I) and $C_1 = 125$, $C_2 = 0$, the top panel shows the frequency of takeovers initiated and completed in years 1, 2–5, 6–10, 11–25, and 26–100. The bottom panel shows the average acquisition size in deals completed by the end of years 1, 5, 10, 25, and 100. The starting value of the target is $X_0 = \bar{X}_1(\bar{v})$. The solid line corresponds to all types of deals. The dashed (dash-dotted) line corresponds to cash (non-cash) deals.

panies acquired in stock.

Because bidders with intermediate to high valuations acquire targets in cash, the total premium in cash deals (as a percentage of the target's value) is higher.

A3. *For some parameterizations of the model, bidders pay higher takeover premiums to acquire companies in cash.*

Despite the fact that bidders give away a smaller portion of valuations in cash acquisitions, they are the bidders with higher valuations. They give away a smaller portion of a larger pie. As a result, there exist parameterizations for which the effect of a pie increase dominates the effect of a smaller pie share and cash bidders on average pay higher takeover premiums (as a percentage of the target's value).

Figure 7 shows the average takeover premiums in cash and non-cash deals, both conditional on observing the highest bidder valuation and sample-wide unconditional, where the sample consists of takeover contests that differ only in valuations of participating bidders. As expected, the conditional takeover premiums are higher in non-cash deals for any value of highest valuation. However, in the case when both bidders have non-zero internal cash (Panels B and D), best deals are done exclusively in stock while worst deals are done exclusively in combinations on cash and stock which leads to an inverse relationship between the sample-wide unconditional average takeover premiums. This result is obtained without assuming either adverse selection about the bidders' assets or private information of the acquirer about its own firm as in the previous literature. It is the takeover timing-determined positive correlation between cash deals and high-synergy deals that is responsible for the result.

An empirical implication of A3 is that, if a good proxy of synergies can be found, then conditional on this proxy, takeover premiums in cash deals should be lower than those in non-cash deals. A good proxy of synergies can be difficult to find; an alternative indirect approach is to estimate unobserved synergies from observed bids

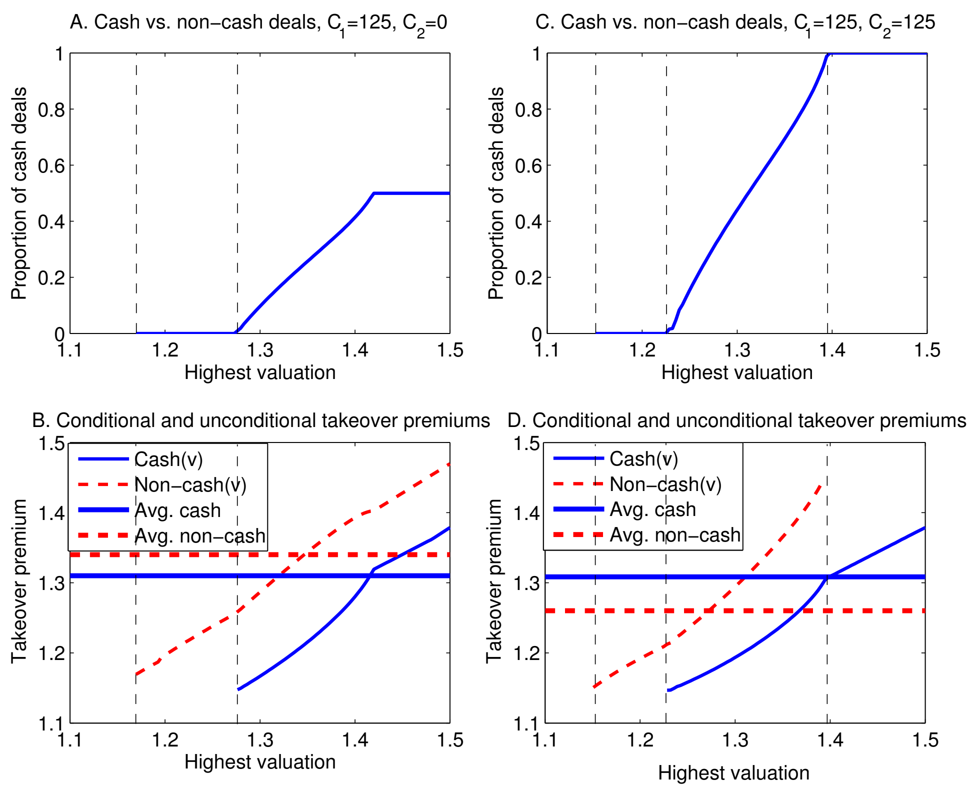

Figure 7: Conditional and unconditional takeover premiums in cash and non-cash deals. The figure corresponds to prediction A3. Panels A and C show, for the two cases: *(i)* $C_1 = 125$, $C_2 = 0$, *(ii)* $C_1 = 125, C_2 = 125$, the probability that a takeover contest is completed in cash as a function of the highest bidder valuation. Panels B and D show, for the same two cases, the average takeover premiums in cash and non-cash deals, both conditional on observing the highest bidder valuation (thick solid and dashed lines) and sample-wide unconditional (extra thick solid and dashed lines).

using a model of takeover auctions similar to ours. An implication of A3, then, is that conditional on the recovered valuation of the highest bidder, takeover premiums in cash deals should be lower than those in non-cash deals.

A4. *Stock bidders receive lower acquirer premiums than cash bidders.*

Not only do stock bidders give away a larger portion of their valuations, but also they have lower valuations, so the two effects complement each other.

A5. *The target premium and the target revenue is higher when the bidder has an access to outside funding.*

When the bidder with a low valuation can use its equity to finance the deal, it will approach the target later, pay a larger proportion of its valuation to the target, which also means paying more in absolute terms.

These properties are consistent with existing empirical evidence. The first property is consistent with the evidence that takeovers paid in cash are for smaller firms than those partially or fully paid in stock (e.g., Betton, Eckbo, and Thorburn, 2008). The third property is consistent with a number of studies (e.g., Franks, Harris, and Mayer, 1988; Eckbo and Langohr, 1989) that historically, offer premiums were greater in all-cash offers, even controlling for the differential tax impact. The fourth property is consistent with the findings by Eckbo, Giammarino, and Heinkel (1990) and Berkovitch and Narayanan (1990) that the larger the cash portion of the deal, the higher are acquirer abnormal announcement returns. The second property is the combination of the two described above. Finally, the fifth property is consistent with the evidence that both the initial and final offer premiums are higher when the bidder is a public company and thus likely has an access to equity markets (Betton, Eckbo, and Thorburn, 2008).

V.B Properties of Initiating and Winning Bidders

B1. *If the impact of an acquisition on the losing bidder is negative, bidders with sufficiently low valuations and cash constraints never initiate a contest.*

As a result, there is a non-zero probability that neither cash-constrained bidder initiates a contest and a valuable target continues as a stand-alone. Figure 8, region (1) shows that for the benchmark model parametrization and cash constraints $C_1 = 125$, $C_2 = 0$, the probability that the target is never acquired is approximately 4%. In the case of two stock bidders competing against each other, this probability is almost 9%.

B2. *In initiated contests, the distribution of participating bidders' valuations is determined endogenously and can be asymmetric.*

This result holds true even if the unconditional distribution of valuations is the same for the bidders. Figure 8, left-most dashed line shows valuations of bidders 1 and 2, v and w, at which they initiate contest at the same threshold, $\bar{X}_1(v) = \bar{X}_2(w)$. In contests initiated by any bidder, the highest possible valuation of the more constrained bidder is higher than that of the less constrained bidder; the less constrained bidder also faces a stronger competitor on average. Interestingly, in the sample of takeovers that differ only in valuations of participating bidders, this result is reversed: because more constrained bidders are less likely to initiate takeover contests in the first place, their average valuation across all initiated contests is lower than that of less constrained bidders. Figure 9 shows how average valuations of the bidders with cash constraints $C_1 = 125$, $C_2 = 0$ change with respect to the parameters that have the strongest effect on the probability that a contest is never initiated: the value of the losing bidder, P_0 and the cash constraint of one of the bidders, specifically, C_1. Lower P_0 and C_1 correspond to a larger gap between v_1^* and v_2^* and result in a larger difference between average valuations in the sample of similar takeovers.

B3. *Some initial bids of a less constrained bidder will be rejected in favor of a more constrained bidder. Under some parameterizations of the model, initial bids in cash have a smaller probability to be rejected compared to initial bids that include stock.*

The second prediction is consistent with empirical evidence (Betton, Eckbo, and Thorburn, 2009) while the first prediction is novel. The two predictions might seem contradictory at first. However, a less constrained bidder and a bidder who completes the deal in cash are not equivalent. The latter bidder is more likely to have both high cash balances and high valuation so that it approaches the target while the deal can still be sealed in cash. For the benchmark parametrization and $C_1 = 125$, $C_2 = 0$, Figure 8, regions (2) and (4) show contests initiated by the less constrained bidder 1 in which the initial bidder bids in combinations of cash and stock. Region (4) shows contests in which such bidder loses to bidder 2 who bids in stock. Region (6) shows contests initiated by bidder 2 who wins in stock. The conditional probability of the initiating non-cash bidder losing the contest is the area of region (4) divided by the combined areas of regions (2), (4), and (6) and is equal to approximately 10%. In contrast, regions (3) and (5) show contests initiated by the less constrained bidder 1 in which the initial bidder bids in cash. Region (5) shows contests in which such bidder loses to bidder 2 who bids in stock. The conditional provability of the initiating cash bidder losing the contest is the area of region (5) divided by the combined area of regions (3) and (5) and is equal to approximately 2.6%. Hence, for a given parametrization, cash bids by the initiating bidder indeed have a smaller probability to be rejected compared to non-cash bids. It is easy to construct an example in which the opposite is true: take $C_1 \to \infty$, $C_2 = 0$. In this case, there is zero correlation between cash bids and cash bidder valuations and only initial cash but not stock bids can be rejected.

V.C Target's Preference for Cash versus Stock Bids

An important result in the static security design literature (Hansen, 1985; DeMarzo, Kremer, and Skrzypacz, 2005) is that auctions in stock dominate contests in cash in terms of the expected revenues of the seller. As a result, if in a static setting the target can commit to accept only stock bids, it will do so. However, practical cases of such commitment in takeover contests are rare. An interesting question is to study whether the target would have incentives to commit to accept only stock bids in a dynamic setting, when bidders

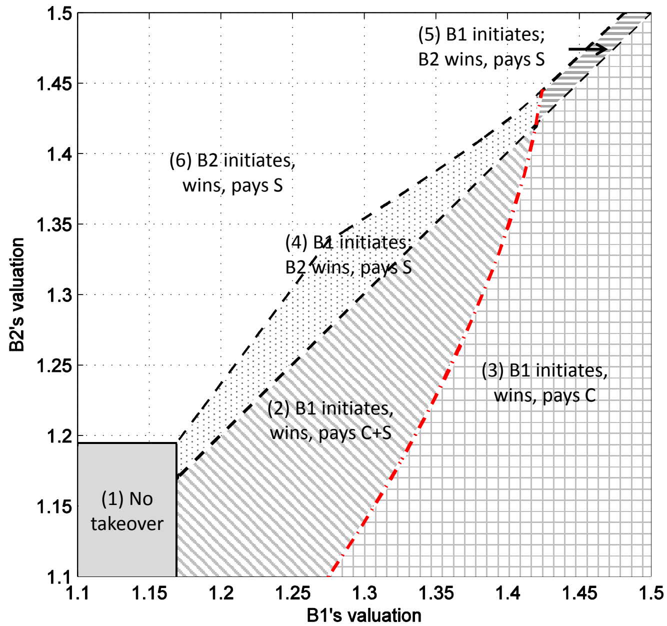

Figure 8: Initiation, acquisition and means of payment in takeover contests with cash constrained bidders. For the benchmark parametrization (Table I) and cash constraints of bidders 1 and 2 equal to $C_1 = 125$, $C_2 = 0$, the figure shows regions of valuations for which bidders initiate and win takeover contests, as well as the resulting type of the deal (cash, cash and stock, stock). The dash-dotted line separates the cases in which bidder 1 makes cash and non-cash final bids.

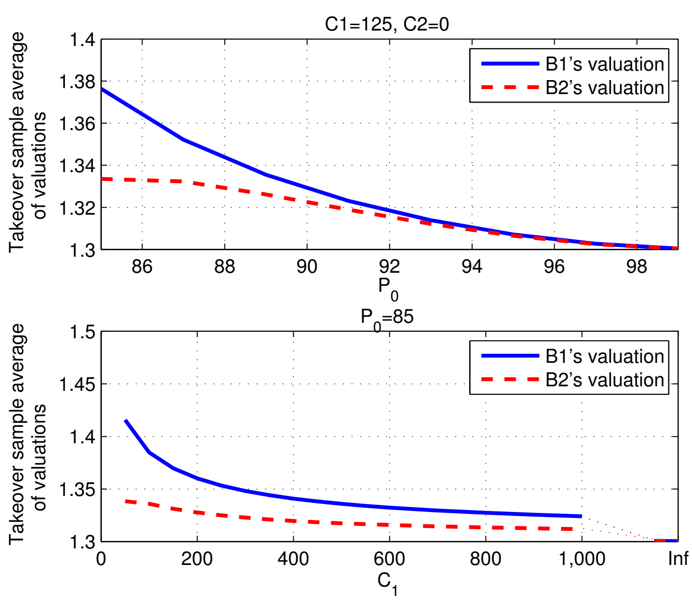

Figure 9: Average valuations of cash constrained bidders in initiated contests. The figure shows average valuations of cash constrained bidders for the benchmark parametrization (Table I) as a function of *(i)* the value of the losing bidder, P_0, assuming cash constraints $C_1 = 125, C_2 = 0$, and *(ii)* cash constraint of bidder 1, C_1, assuming $C_2 = 0$ and $P_0 = 85$. The solid (dashed) line is the average valuation of bidder 1 (2).

can time an acquisition. In this paper, we do not aim to provide a rigorous answer to this question. One of the complications that can arise is that the target, upon learning about bidder valuations from their initiation (and non-initiation) decisions, can change the preferred security design of the takeover contest dynamically. Instead, to provide a flavor of the more general case, we consider a simpler setting in which the target has to commit to the security design at time zero.[21] We also focus on the case in which both bidders are exogenously unconstrained: $C_1 \to \infty, C_2 \to \infty$. Our results in this section are related to Cong (2012), who shows that an auctioneer selling a real option, such as a lease to explore an oil well, can prefer the auction in cash over the auction in stock, because of the post-auction moral hazard that affects the timing of the option exercise. Our argument is different, because the timing of actions is reverse: a bidder exercises its option (approaches the target) before the auction takes place.

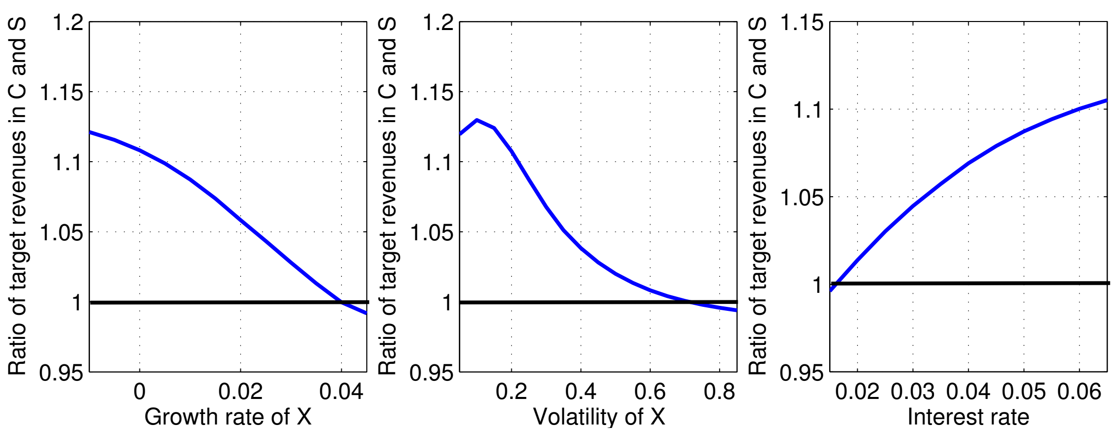

Figure 10: The ratio of the target revenue (present value) from contests in cash and in stock. For the benchmark parametrization (Table I), the figure shows the ratio of present values of target revenues in cash and stock deals as a function of *(i)* the growth rate of the target's assets, μ, *(ii)* the volatility of the target's assets, σ, *(iii)* the interest rate r.

Figure 10 shows the ratio of present values of target revenues in cash and stock contests as a function of μ, σ, and r. For realistic parameters, the target prefers not to commit

[21] The results remain the same for any X_0 below the lowest initiation threshold of the bidders: expected target revenue takes the form $\alpha + \gamma_j X_0^{\beta}$, where $j \in \{c, s\}$ corresponds to the case of cash and stock bids. This also means that as long as X_t stays below the lowest initiation threshold, the target does not have incentives to attempt and change the security design in this region.

to restricting bids to stock. When μ and σ are well above realistic parameters (or r is very low), contests in stock start to dominate contests in cash in terms of target revenue. Intuitively, if a target has a higher growth rate or higher volatility of assets (or interest rate is lower), the difference between initiation thresholds of cash and stock bidders is passed quicker (or affects the present value of target revenues less). As a result, the effect of extra delay is less important for the present value of high-growth targets, which leads to their preference for battles in stock.[22]

This result suggests that in a dynamic setting, most targets (including targets with "standard" characteristics that are similar to an average COMPUSTAT firm) can have aligned incentives with the bidders: both the bidders and the target can prefer cash deals. This is in line with the observation that there are very few (if any) practical cases in which the target attempts to restrict the type of bids. However, a small fraction of firms with either high growth or high volatility of assets can have misaligned incentives with the bidders. If there is any evidence regarding the target's attempts to restrict the type of bids in takeover contests, it is likely to be found among high-μ, high-σ targets.[23]

VI Concluding Remarks

This paper presents a theoretical analysis of the timing of acquisitions, takeover premiums, and means of payment in the setting in which firms' fortunes and ability to pay cash are affected by the technological change and cash constraints. Optimal choices of each bidder and, in turn, takeover outcomes are affected not only by synergies of this bidder with the target but also by financing constraints of both the bidder and its competitors. The results

[22] If the target can choose whether to restrict the type of bids at any point of time, learning about bidder valuations from X_t strengthens its incentives to commit to restricting bids. This is because, as the support of possible bidder valuations shrinks, stock bids extract an increasingly higher proportion of revenues from the bidders.

[23] In contests for growth targets and targets from hi-tech industries, it is common that target managers are major stockholders in their company and have much control over its decision making, including the ability to negotiate terms of a potential takeover. In many cases, they eventually become large stockholders of the combined company, which is consistent with our result.

of our general model are consistent with a variety of empirical findings and provide further implications. In particular, they provide an explanation, alternative to previous studies, why cash deals feature higher average takeover premiums than non-cash cash deals. In our model, high-synergy bidders approach their targets earlier, before they grow large enough to make bidders' cash constraints binding, and thus have enough cash to finalize the deal. We also propose several novel testable predictions that relate the timing of an acquisition and its outcomes to key characteristics of competing bidders. Importantly, these predictions explicitly recognize the effect of private information and selection on decision making.

A potential direction of future research is to understand targets' motives to initiate takeover contests by themselves. We abstract from this issue because our focus is on strategic acquisitions, and they are usually bidder-initiated (Fidrmuc et al., 2012). However, target-initiated deals are also common, especially among private equity deals. Another direction for future research is to test predictions of our model. In particular, it can be interesting to quantify the relative importance of our dynamic selection mechanism and other theories of means of payment (e.g., Fishman, 1989) on takeover outcomes.

Appendix A Proofs

Proof of Proposition 1. Taking the first-order condition (5) and dividing both sides by X_0^β yields

$$\begin{aligned} 0 \;=\; & -\beta \frac{1}{\bar{X}^{\beta+1}} \int_{\underline{v}}^{\bar{X}_c^{-1}(\bar{X})} \left(\bar{X} \max\{v-w,0\} - \Delta\right) dF(w) \\ & + \frac{1}{\bar{X}^{\beta}} \int_{\underline{v}}^{\bar{X}_c^{-1}(\bar{X})} \max\{v-w,0\}\, dF(w). \end{aligned} \tag{A1}$$

In equilibrium, the maximum is reached at $\bar{X}_c(v)$. Plugging in and multiplying both sides by $\bar{X}_c(v)^{\beta+1}$, we get

$$\bar{X}_c(v)(\beta-1)\int_{\underline{v}}^{v}(v-w)\,dF(w) = \beta\Delta F(v). \tag{A2}$$

Hence,

$$\bar{X}_c(v) = \frac{\beta}{\beta-1}\frac{\Delta}{v - \mathbb{E}[w|w \leq v]}. \tag{A3}$$

By assumption, $v - \mathbb{E}[w|w \leq v]$ is increasing in v. Therefore, $\bar{X}_c(v)$ is indeed decreasing in v.

Proof of Proposition 2. Taking the first-order condition (11) and dividing both sides by X_0^{β} yields

$$\begin{aligned} 0 &= -\beta\frac{1}{\bar{X}^{\beta+1}}\int_{\underline{v}}^{\bar{X}_s^{-1}(\bar{X})}\left(\Pi_o\frac{\Pi_b+\bar{X}v}{\Pi_b+\bar{X}w}-\Pi_b\right)dF(w) \\ &\quad +\frac{1}{\bar{X}^{\beta}}\int_{\underline{v}}^{\bar{X}_s^{-1}(\bar{X})}\Pi_o\left[\frac{\Pi_b+\bar{X}v}{\Pi_b+\bar{X}w}\right]'dF(w). \end{aligned} \tag{A4}$$

The derivative is equal to

$$\left[\frac{\Pi_b+\bar{X}v}{\Pi_b+\bar{X}w}\right]' = \frac{(v-w)\Pi_b}{\left(\Pi_b+\bar{X}w\right)^2}. \tag{A5}$$

Plugging it into (A4), dividing by $F(v)$, and using the fact that in equilibrium the maximum is reached at $\bar{X}_s(v)$, we obtain

$$\begin{aligned} 0 &= -\beta\Pi_o\mathbb{E}\left[\frac{\Pi_b+v\bar{X}_s(v)}{\Pi_b+w\bar{X}_s(v)}|w\leq v\right]+\beta\Pi_b \\ &\quad +\Pi_o\Pi_b\mathbb{E}\left[\frac{(v-w)\bar{X}_s(v)}{\left(\Pi_b+w\bar{X}_s(v)\right)}|w\leq v\right]. \end{aligned} \tag{A6}$$

Rewriting, we obtain (12).

Proof of Proposition 3. We need to compare

$$\mathbb{E}[v-w|w\leq v] \text{ and } \mathbb{E}\left[\frac{\Pi_o\left(\Pi_b+\frac{\beta}{\beta-1}w\bar{X}\right)}{\left(\Pi_b+w\bar{X}\right)^2}(v-w)|w\leq v\right]. \tag{A7}$$

Consider the following difference:

$$
\begin{aligned}
1-\frac{\Pi_o\left(\Pi_b+\frac{\beta}{\beta-1}w\bar{X}\right)}{\left(\Pi_b+w\bar{X}\right)^2} &= \frac{\Pi_b^2+2\Pi_b w\bar{X}+w^2\bar{X}^2-\Pi_o\Pi_b-\frac{\beta}{\beta-1}\Pi_o w\bar{X}}{\left(\Pi_b+w\bar{X}\right)^2} \\
&= \frac{\Pi_b\left(\Pi_b-\Pi_o\right)+\left(2\Pi_b-\frac{\beta}{\beta-1}\Pi_o\right)\bar{X}+w^2\bar{X}^2}{\left(\Pi_b+w\bar{X}\right)^2}.
\end{aligned} \tag{A8}
$$

The first term in the numerator is positive because $\Pi_b > \Pi_o$. The second term in the numerator is positive because of (16). Therefore, (A8) is positive for all w and $\bar{X}$. Consequently,

$$
\mathbb{E}\left[v-w|w\leq v\right] > \mathbb{E}\left[\frac{\Pi_o\left(\Pi_b+\frac{\beta}{\beta-1}w\bar{X}\right)}{\left(\Pi_b+w\bar{X}\right)^2}\left(v-w\right)|w\leq v\right]. \tag{A9}
$$

Because of this and monotonicity of the left-hand side of (12) with respect to $\bar{X}$, the unique solution of (12), $v > v^*$ is higher than the unique solution of (6).

Proof of Proposition 4. First, we maximize (20) with respect to threshold $\bar{X}$. Analogously to the proof of proposition 1, we obtain (21). Second, we maximize (22) with respect to threshold $\bar{X}$:

$$
\begin{aligned}
0 &= -\frac{\beta}{\bar{X}^{\beta+1}}\int_{\underline{v}}^{\bar{X}_1^{-1}(\bar{X})}\left(\Pi_o\frac{(v-w)\bar{X}}{\Pi_b+w\bar{X}}-\Delta\right)f(w)\,dw \\
&\quad +\frac{1}{\bar{X}^{\beta}}\int_{\underline{v}}^{\bar{X}_1^{-1}(\bar{X})}\Pi_o\left[\frac{(v-w)\bar{X}}{\Pi_b+w\bar{X}}\right]'f(w)\,dw.
\end{aligned} \tag{A10}
$$

Equivalently,

$$
\begin{aligned}
0 &= -\beta\int_{\underline{v}}^{\bar{X}_1^{-1}(\bar{X})}\Pi_o\frac{(v-w)\bar{X}}{\Pi_b+w\bar{X}}f(w)\,dw+\beta\Delta F\left(\bar{X}_1^{-1}\left(\bar{X}\right)\right) \\
&\quad +\bar{X}\int_{\underline{v}}^{\bar{X}_1^{-1}(\bar{X})}\Pi_o\frac{(v-w)\Pi_b}{\left(\Pi_b+w\bar{X}\right)^2}f(w)\,dw.
\end{aligned} \tag{A11}
$$

Dividing by $F\left(\bar{X}_1^{-1}\left(\bar{X}\right)\right)$:

$$\begin{aligned} 0 &= -\beta\Pi_o\mathbb{E}\left[\frac{(v-w)\,\bar{X}}{\Pi_b+w\bar{X}}|w\leq\bar{X}_1^{-1}\left(\bar{X}\right)\right]+\beta\Delta \\ &+\Pi_o\mathbb{E}\left[\frac{(v-w)\,\bar{X}\Pi_b}{\left(\Pi_b+w\bar{X}\right)^2}|w\leq\bar{X}_1^{-1}\left(\bar{X}\right)\right]. \end{aligned} \tag{A12}$$

Equivalently,

$$\mathbb{E}\left[\beta\frac{v-w}{\Pi_b+w\bar{X}}-\Pi_b\frac{v-w}{\left(\Pi_b+w\bar{X}\right)^2}|w\leq\bar{X}_1^{-1}\left(\bar{X}\right)\right]\bar{X}=\beta\frac{\Delta}{\Pi_o}. \tag{A13}$$

Rewriting yields (23). Finally, we need to determine valuation v^* such that bidder 2 never approaches the target if $v \leq v^*$. Consider $\bar{X}\to\infty$. Because $\bar{X}_1\left(\underline{v}\right)$ is finite as $\underline{v}>1$, $\bar{X}_1^{-1}\left(\bar{X}\right)=\underline{v}$. Therefore, the left-hand side of (A13) is

$$\mathbb{E}\left[\beta\frac{v-w}{w}|w\leq\underline{v}\right]=\beta\frac{v-\underline{v}}{\underline{v}}. \tag{A14}$$

Point v^* is such that

$$\beta\frac{v^*-\underline{v}}{\underline{v}}=\beta\frac{\Delta}{\Pi_o}, \tag{A15}$$

which yields

$$v^*=\frac{\Pi_b}{\Pi_o}\underline{v}. \tag{A16}$$

Proof of Proposition 5. Proposition 3 establishes that $\bar{X}_s(v) > \bar{X}_c(v)$ for all v when $\frac{\beta}{\beta-1} < 2\frac{\Pi_b}{\Pi_o}$. Suppose that $\bar{X}_1(\tilde{v}) = \bar{X}_2(\tilde{v})$ for some $\tilde{v}$. Then, $\Psi(\tilde{v}) = 1$, $\Omega(\tilde{v}) = v$. As a result, $\bar{X}_1(\tilde{v}) = \bar{X}_c(\tilde{v})$; $\bar{X}_2(\tilde{v}) = \bar{X}_s(\tilde{v})$ and, under the assumption $\bar{X}_1(\tilde{v}) = \bar{X}_2(\tilde{v})$, all four strategies have to be equal at $\tilde{v}$ – a contradiction with the result of Proposition 3. Hence $\bar{X}_2$ and $\bar{X}_1$ cannot cross.

Assume that $\bar{X}_1(\hat{v}) > \bar{X}_2(\hat{v})$ for some $\hat{v}$. From Proposition 4, as $v \downarrow v^*$, $\bar{X}_2(v) \to \infty$ while $\bar{X}_1(v)$ remains finite. Hence, there exists $\epsilon > 0$ such that $\bar{X}_2(v^*+\epsilon) > \bar{X}_1(v^*+\epsilon)$. This, together with the assumption $\bar{X}_1(\hat{v}) > \bar{X}_2(\hat{v})$ and continuity of both $\bar{X}_1(v)$ and $\bar{X}_2(v)$ in v, implies that $\bar{X}_1(\tilde{v}) = \bar{X}_2(\tilde{v})$ for some $\tilde{v} \in (v^*+\epsilon, \hat{v})$. By earlier proof, however, $\bar{X}_2$ and $\bar{X}_1$ cannot cross. Hence, $\bar{X}_2(v) > \bar{X}_1(v)$ for all v.

The final step is to show that $\bar{X}_s(v) > \bar{X}_2(v)$ and $\bar{X}_1(v) > \bar{X}_2(v)$ for all v. Both inequalities follow from the fact that, when $\bar{X}_2(v) > \bar{X}_1(v)$ for all v, then $\Psi(v) > 1$ and $\Omega(v) < v$.

Proof of Proposition 6. The first-order condition of (28) is

$$\begin{aligned} 0 &= -\frac{\beta}{\bar{X}^{\beta+1}} \int_{\underline{v}}^{\bar{X}_{-i}^{-1}(\bar{X})} \left(\min\left\{ \frac{\Pi_o + C_i}{\Pi_b + w\bar{X}}, 1 \right\} \bar{X} \max\{v - w, 0\} - \Delta \right) dF(w) \\ &\quad + \frac{1}{\bar{X}^{\beta}} \int_{\underline{v}}^{\bar{X}_{-i}^{-1}(\bar{X})} \left[\min\left\{ \frac{\Pi_o + C_i}{\Pi_b + w\bar{X}}, 1 \right\} \bar{X} \max\{v - w, 0\} \right]' dF(w). \end{aligned} \tag{A17}$$

Equivalently,

$$\begin{aligned} 0 &= -\beta \int_{\underline{v}}^{\bar{X}_{-i}^{-1}(\bar{X})} \left(\min\left\{ \frac{\Pi_o + C_i}{\Pi_b + w\bar{X}}, 1 \right\} \bar{X} \max\{v - w, 0\} \right) dF(w) \qquad \text{(A18)} \\ &\quad + \beta \Delta F\left(\bar{X}_{-i}^{-1}\left(\bar{X}\right)\right) + \bar{X} \int_{\underline{v}}^{\bar{X}_{-i}^{-1}(\bar{X})} \left[\min\left\{ \frac{\Pi_o + C_i}{\Pi_b + w\bar{X}}, 1 \right\} \bar{X} \max\{v - w, 0\} \right]' dF(w). \end{aligned}$$

Applying the equilibrium condition that the maximum is reached at $\bar{X}_i(v)$ and dividing by $F(\Omega(v))$ yields

$$\begin{aligned} &\mathbb{E}\left[\beta \min\left\{ \frac{\Pi_o + C_i}{\Pi_b + w\bar{X}_i(v)}, 1 \right\} (v - w) \,|\, w \le \Omega(v) \right] \bar{X}_i(v) \\ &- \mathbb{E}\left[\left[\min\left\{ \frac{\Pi_o + C_i}{\Pi_b + w\bar{X}_i(v)}, 1 \right\} (v - w) \bar{X}_i(v) \right]' \,|\, w \le \Omega(v) \right] \bar{X}_i(v) \\ = \; &\beta \Delta \Psi(v). \end{aligned} \tag{A19}$$

Let us decompose this expression into two intervals:

- if $w < (C_i - \Delta)/\bar{X}_i(v)$, then the expression under the expectation operator is

$$\beta(v - w) - \left[(v - w)\bar{X}_i(v)\right]' = (\beta - 1)(v - w); \tag{A20}$$

- if $w > (C_i - \Delta)/\bar{X}_i(v)$, then the expression under the expectation operator is

$$
\begin{aligned}
& (\Pi_o + C_i)\left(\frac{\beta(v-w)}{\Pi_b + w\bar{X}_i(v)} - \left[\frac{(v-w)\bar{X}_i(v)}{\Pi_b + w\bar{X}_i(v)}\right]'\right) \\
= \quad & (\Pi_o + C_i)\left(\frac{\beta(v-w)}{\Pi_b + w\bar{X}_i(v)} - \frac{(v-w)\Pi_b}{\left(\Pi_b + w\bar{X}_i(v)\right)^2}\right) \qquad \text{(A21)} \\
= \quad & (\beta - 1)\frac{(\Pi_o + C_i)(v-w)}{\Pi_b + w\bar{X}_i(v)} + (\beta - 1)\frac{(\Pi_o + C_i)(v-w)}{\Pi_b + w\bar{X}_i(v)}\frac{\frac{1}{\beta-1}w\bar{X}_i(v)}{\Pi_b + w\bar{X}_i(v)}.
\end{aligned}
$$

Hence, we can rewrite (A19) as (29).

Similar to Section II.B, equations (29) do not have solutions for low enough v. Let v_i^* be such that $\lim_{v \to v_i^*} \bar{X}_i(v) = \infty$. Rewriting (29) at this point yields

$$
\mathbb{E}\left[\frac{v_i^* - w}{w} | w \leq \Omega(v_i^*)\right] = \frac{\Delta\Psi(v_i^*)}{\Pi_o + C_i}. \qquad \text{(A22)}
$$

In the case of symmetric cash constraints, $C_1 = C_2 = C$ and $v_1^* = v_2^* = v^*$, given by

$$
\mathbb{E}\left[\frac{v^* - w}{w} | w \leq v^*\right] = \frac{\Delta}{\Pi_o + C_i}. \qquad \text{(A23)}
$$

It is easy to see that in the special cases of $C \to \infty$ and $C = 0$ and , we obtain $\underline{v}$ and v^* from Section II.B, respectively.

Proof of Proposition 7. Let $v_i(x) := \bar{X}_i^{-1}(x)$ be the type of bidder $i \in \{1, 2\}$ that approaches the target at threshold x. We can re-write (A19) in terms of $v_1(x)$ and $v_2(x)$:

$$
\begin{aligned}
& \mathbb{E}\left[\left(\beta \min\left\{\frac{\Pi_o + C_i}{\Pi_b + wx}, 1\right\} - \left[\min\left\{\frac{\Pi_o + C_i}{\Pi_b + wx}, 1\right\} x\right]'\right)(v_i(x) - w) | w \leq \min_{j \in \{1,2\}} v_j(x)\right] x \\
& -\beta\Delta\frac{F\left(\max_{j\in\{1,2\}} v_j(x)\right)}{F(v_i(x))} = 0. \qquad \text{(A24)}
\end{aligned}
$$

Denote the left-hand side by $\delta_i(x, v_i, v_{-i}, \Theta)$, where Θ is the set of comparative statics parameters, and where the suppress the dependence of v_i and v_{-i} on x for notational simplicity. The system of equations is thus $\delta_i(x, v_i(x), v_{-i}(x), \Theta) = 0$, $i \in \{1, 2\}$.

The following auxiliary result will be useful to prove the proposition.

Lemma 1. $\frac{\partial\delta_1}{\partial v_1}\frac{\partial\delta_2}{\partial v_2} - \frac{\partial\delta_1}{\partial v_2}\frac{\partial\delta_2}{\partial v_1} > 0$ *at the equilibrium.*

Proof of Lemma 1. Taking the full derivatives of these equations around the solution x everywhere where the derivatives exist yields

$$\begin{aligned} \frac{\partial\delta_1}{\partial x} + \frac{\partial\delta_1}{\partial v_1}v_1'(x) + \frac{\partial\delta_1}{\partial v_2}v_2'(x) &= 0, && \text{(A25)} \\ \frac{\partial\delta_2}{\partial x} + \frac{\partial\delta_2}{\partial v_2}v_2'(x) + \frac{\partial\delta_2}{\partial v_1}v_1'(x) &= 0. && \text{(A26)} \end{aligned}$$

Combining these equations, we obtain:

$$\left(\frac{\partial\delta_1}{\partial v_1}\frac{\partial\delta_2}{\partial v_2} - \frac{\partial\delta_1}{\partial v_2}\frac{\partial\delta_2}{\partial v_1}\right)v_i'(x) = \frac{\partial\delta_i}{\partial v_{-i}}\frac{\partial\delta_{-i}}{\partial x} - \frac{\partial\delta_i}{\partial x}\frac{\partial\delta_{-i}}{\partial v_{-i}}, \tag{A27}$$

where $i \in \{1,2\}$. Because $\bar{X}_i(v)$ maximizes the bidder's value function and not minimizes it, $\frac{\partial\delta_i(x,v_i,v_{-i},\Theta)}{\partial x} > 0$, $i \in \{1,2\}$.[24]

Fix x. Without loss of generality, assume $v_i(x) \geq v_{-i}(x)$. Then, $\min_{j\in\{1,2\}} v_j(x) = v_{-i}(x)$ and $\max_{j\in\{1,2\}} v_j(x) = v_i(x)$. First, consider bidder i. In the neighborhood of the equilibrium,

$$\delta_i(x,v_i,v_{-i},\Theta) = \mathbb{E}\left[\left(\beta\min\left\{\frac{\Pi_o+C_i}{\Pi_b+wx},1\right\} - \left[\min\left\{\frac{\Pi_o+C_i}{\Pi_b+wx},1\right\}x\right]'\right)(v_i-w)x - \beta\Delta | w \leq v_{-i}\right].$$

Hence,

$$\frac{\partial\delta_i}{\partial v_i} = \mathbb{E}\left[\left(\beta\min\left\{\frac{\Pi_o+C_i}{\Pi_b+wx},1\right\} - \left[\min\left\{\frac{\Pi_o+C_i}{\Pi_b+wx},1\right\}x\right]'\right) | w \leq v_{-i}\right]x > 0.$$

Let

$$d_i(x,v_i,w,\Theta) \equiv \left(\beta\min\left\{\frac{\Pi_o+C_i}{\Pi_b+wx},1\right\} - \left[\min\left\{\frac{\Pi_o+C_i}{\Pi_b+wx},1\right\}x\right]'\right)(v_i-w)x - \beta\Delta.$$

be the integrand under the expectation sign in $\delta_i(x,v_i,v_{-i},\Theta)$. Let us show that $d_i(x,v_i(x),v_{-i}(x),\Theta) < 0$. Consider $d_i(x,v_i,w,\Theta)$ as a function of w. Clearly, it is strictly decreasing in w in the range

[24] This follows from the second derivative of the bidder's value function with respect to the threshold at $\bar{X}_i(v)$ being $-\frac{\partial\delta(\bar{X}_i(v),v,v_{-i}(\bar{X}_i(v)),\Theta)}{\partial x}/\bar{X}(v)^{\beta+1}$. It must be negative for any v.

$w < (C_i - \Delta)/x$, as $d_i(x, v_i, w, \Theta) = (\beta - 1)(v_i - w)$. Consider $w > (C_i - \Delta)/x$. Differentiating with respect to w,

$$\begin{aligned}
\frac{\partial d_i}{\partial w}(x, v_i, w, \Theta) &= -\frac{(\Pi_o + C_i)x}{(\Pi_b + wx)^2}\left((\Pi_b + xv_i)\left(\beta - \frac{\Pi_b}{\Pi_b + wx}\right) - \frac{\Pi_b x (v_i - w)}{\Pi_b + wx}\right) \\
&< -\frac{(\Pi_o + C_i)x}{(\Pi_b + wx)^2}\left((\Pi_b + xv_i)\frac{\Pi_b + 2wx}{\Pi_b + wx} - \frac{\Pi_b x (v_i - w)}{\Pi_b + wx}\right) \\
&= -\frac{(\Pi_o + C_i)x}{(\Pi_b + wx)^3}\left(\Pi_b^2 + 3wx\Pi_b + 2wx^2 v_i\right) < 0,
\end{aligned}$$

where the intermediate inequality follows, because $\frac{\beta}{\beta-1} < 2\frac{\Pi_b}{\Pi_o}$ implies $\beta > 2$. Because either $C_i < \Delta$ or $C_i \to \infty$, $d_i(x, v_i, w, \Theta)$ never jumps from one region to the other as w changes. Therefore, $d_i(x, v_i, w, \Theta)$ is strictly decreasing in w. Thus, $\mathbb{E}[d_i(x, v_i(x), w, \Theta) | w \leq v_{-i}(x)] = 0$ implies $d_i(x, v_i(x), v_{-i}(x), \Theta) < 0$. Therefore,

$$\begin{aligned}
\frac{\partial \delta_i}{\partial v_{-i}} &= (d_i(x, v_i, v_{-i}, \Theta) - \delta_i(x, v_i, v_{-i}, \Theta))\frac{f(v_{-i})}{F(v_{-i})} \\
&= d_i(x, v_i, v_{-i}, \Theta)\frac{f(v_{-i})}{F(v_{-i})} < 0.
\end{aligned}$$

Second, consider bidder $-i$. In the neighborhood of the equilibrium,

$$\begin{aligned}
&\delta_{-i}(x, v_{-i}, v_i, \Theta) \\
= \;& \mathbb{E}\left[\left(\beta \min\left\{\frac{\Pi_o + C_{-i}}{\Pi_b + wx}, 1\right\} - \left[\min\left\{\frac{\Pi_o + C_{-i}}{\Pi_b + wx}, 1\right\} x\right]'\right)(v_{-i} - w)x | w \leq v_{-i}\right] - \beta\Delta\frac{F(v_i)}{F(v_{-i})}.
\end{aligned}$$

Hence,

$$\frac{\partial \delta_{-i}}{\partial v_i} = -\beta\Delta\frac{f(v_i)}{F(v_{-i})} < 0 \text{ for all } v_i \in [\underline{v}, \bar{v}];$$

$$\begin{aligned}
\frac{\partial \delta_{-i}}{\partial v_{-i}} &= \int_{\underline{v}}^{v_{-i}}\left(\beta \min\left\{\frac{\Pi_o + C_{-i}}{\Pi_b + wx}, 1\right\} - \left[\min\left\{\frac{\Pi_o + C_{-i}}{\Pi_b + wx}, 1\right\} x\right]'\right) x\frac{dF(w)}{F(v_{-i})} - \frac{f(v_{-i})}{F(v_{-i})}\delta_{-i}(x, v_{-i}, v_i, \Theta) \\
&= \int_{\underline{v}}^{v_{-i}}\left(\beta \min\left\{\frac{\Pi_o + C_{-i}}{\Pi_b + wx}, 1\right\} - \left[\min\left\{\frac{\Pi_o + C_{-i}}{\Pi_b + wx}, 1\right\} x\right]'\right) x\frac{f(w)}{F(v_{-i})}dw > 0.
\end{aligned}$$

Because in the neighborhood of the equilibrium $\partial\delta_i/\partial x > 0$, $\partial\delta_i/\partial v_i > 0$, and $\partial\delta_i/\partial v_{-i} < 0$, where $i \in \{1, 2\}$, the right-hand side of (A27) is negative. Because $v_i'(x) < 0$ in equilibrium with strictly decreasing strategies, $\frac{\partial\delta_1}{\partial v_1}\frac{\partial\delta_2}{\partial v_2} - \frac{\partial\delta_1}{\partial v_2}\frac{\partial\delta_2}{\partial v_1} > 0$ at the equilibrium.

Using this lemma, we can prove comparative statics. Consider the derivative of $\delta_i(x, v_i, v_{-i}, \Theta)$ with respect to $\theta \in \Theta$ at the equilibrium. Combining the equations for $i \in \{1, 2\}$, we obtain:

$$\left(\frac{\partial\delta_1}{\partial v_1}\frac{\partial\delta_2}{\partial v_2} - \frac{\partial\delta_1}{\partial v_2}\frac{\partial\delta_2}{\partial v_1}\right)\frac{\partial v_i}{\partial\theta} = \frac{\partial\delta_i}{\partial v_{-i}}\frac{\partial\delta_{-i}}{\partial\theta} - \frac{\partial\delta_i}{\partial\theta}\frac{\partial\delta_{-i}}{\partial v_{-i}}. \tag{A28}$$

Lemma 1 implies that the sign of $\partial v_i/\partial\theta$ coincides with the sign of the right-hand side of (A28). As shown above, $\frac{\partial\delta_i}{\partial v_{-i}} < 0$ and $\frac{\partial\delta_{-i}}{\partial v_{-i}} > 0$. In addition, because $v_i(x)$ is the inverse function of $\bar{X}_i(v)$ and $\bar{X}_i'(v) < 0$, the sign of $\partial v_i(x)/\partial\theta$ coincides with the sign of $\partial\bar{X}_i(v)/\partial\theta$. This can be seen from the full derivative of $\bar{X}_i(v)$ with respect to θ:

$$\bar{X}_i'(v)\frac{\partial v_i}{\partial\theta} + \frac{\partial\bar{X}_i(v)}{\partial\theta} = 0.$$

Therefore, a sufficient condition for $\partial\bar{X}_i(v)/\partial\theta$ to be positive (negative) is that $\partial\delta_i/\partial\theta < 0$ $(\partial\delta_i/\partial\theta > 0)$ for both $i \in \{1, 2\}$.

First, consider $\theta = \beta$:

$$\begin{aligned}\frac{\partial\delta_i(x, v_i, v_{-i}, \Theta)}{\partial\beta} &= \mathbb{E}\left[\min\left\{\frac{\Pi_o + C_i}{\Pi_b + wx}, 1\right\}(v_i - w)\,|w \leq \min_{j\in\{1,2\}} v_j(x)\right]x - \Delta\frac{F\left(\max_{j\in\{1,2\}} v_j(x)\right)}{F(v_i(x))} \\ &= \frac{1}{\beta}\mathbb{E}\left[\left[\min\left\{\frac{\Pi_o + C_i}{\Pi_b + wx}, 1\right\}x\right]'(v_i(x) - w)\,|w \leq \min_{j\in\{1,2\}} v_j(x)\right]x > 0,\end{aligned}$$

where the second equation sign holds by the first-order condition. Hence, $\partial\bar{X}_i(v)/\partial\beta < 0$. Because $\partial\beta/\partial\mu < 0$, $\partial\beta/\partial\sigma < 0$, and $\partial\beta/\partial r > 0$, we obtain $\partial\bar{X}_i/\partial\mu > 0$, $\partial\bar{X}_i/\partial\sigma > 0$, and $\partial\bar{X}_i/\partial r < 0$.

Second, consider $\theta = \Delta$, keeping Π_b fixed. If $C_i \to \infty$,

$$\frac{\partial\delta_i(x, v_i, v_{-i}, \Theta)}{\partial\Delta} = -\beta\frac{F\left(\max_{j\in\{1,2\}} v_j\right)}{F(v_i)} < 0.$$

If $C_i < \Delta$,

$$\frac{\partial\delta_i(x, v_i, v_{-i}, \Theta)}{\partial\Delta} = -\mathbb{E}\left[\frac{1}{\Pi_b + wx}\left(\beta - \frac{\Pi_b}{\Pi_b + wx}\right)(v_i - w)\,|w \leq \min_{j\in\{1,2\}} v_j\right]x - \beta\frac{F\left(\max_{j\in\{1,2\}} v_j\right)}{F(v_i)} < 0.$$

Hence, $\partial \bar{X}_i(v)/\partial\Delta > 0$.

Finally, consider $\theta = \Pi_b$, keeping Δ fixed. If $C_i \to \infty$, $\partial\delta_i(x, v_i, v_{-i}, \Theta)/\partial\Pi_b = 0$. If $C_i < \Delta$,

$$\begin{aligned}\frac{\partial\delta_i(x, v_i, v_{-i}, \Theta)}{\partial\Pi_b} &= \mathbb{E}\left[\frac{(wx+\Delta-C_i)(\beta(\Pi_b+wx)-\Pi_b) - wx(\Pi_b-\Delta+C_i)}{(\Pi_b+wx)^3}(v_i-w)\,|w \le \min_{j\in\{1,2\}} v_j\right] \\ &> \mathbb{E}\left[\frac{2w^2x^2+(\Delta-C_i)(\Pi_b+3wx)}{(\Pi_b+wx)^3}(v_i-w)\,|w \le \min_{j\in\{1,2\}} v_j\right] > 0,\end{aligned}$$

where the first inequality follows from $\beta > 2$. Hence, $\partial\bar{X}_i(v)/\partial\Pi_b \le 0$.

Appendix B Asymmetric Initiation: Numerical Procedure

For illustrative purposes, consider the case of cash versus stock bidder, $C_1 \to \infty$, $C_2 = 0$. The case of endogenous means of payment is numerically solved in the same fashion, using equations (A19). We use substitution of variables to express the first order conditions for the two asymmetrically constrained bidders in terms of $\bar{X}_1^{-1}(x)$, $\bar{X}_2^{-1}(x)$ for a given initiation threshold x. Specifically, let

$$\begin{aligned} x_1 &\equiv \bar{X}_1(v_1) \Rightarrow v_1 = \bar{X}_1^{-1}(x_1),\ \bar{X}_2^{-1}(\bar{X}_1(v_1)) = \bar{X}_2^{-1}(x_1); \\ x_2 &\equiv \bar{X}_2(v_1) \Rightarrow v_2 = \bar{X}_2^{-1}(x_2),\ \bar{X}_1^{-1}(\bar{X}_2(v_2)) = \bar{X}_1^{-1}(x_2). \end{aligned} \tag{B1}$$

Then, the system of equations (21), (23) becomes

$$x_1 = \frac{\beta}{\beta-1}\frac{\Delta}{\bar{X}_1^{-1}(x_1) - \int_{\underline{v}}^{\bar{X}_1^{-1}(x_1)} w\frac{f(w)}{F(\bar{X}_1^{-1}(x_1))}dw}\frac{F(\bar{X}_2^{-1}(x_1))}{F(\bar{X}_1^{-1}(x_1))}, \tag{B2}$$

$$x_2\int_{\underline{v}}^{\bar{X}_1^{-1}(x_2)} \frac{\Pi_o\left(\Pi_b + \frac{\beta}{\beta-1}wx_2\right)}{(\Pi_b+wx_2)^2}(\bar{X}_2^{-1}(x_2)-w)\frac{f(w)}{F(\bar{X}_1^{-1}(x_2))}dw = \frac{\beta}{\beta-1}\Delta. \tag{B3}$$

We have two equations and four different combinations of functions and arguments as unknowns. We consider the interior case ($\bar{X}_i^{-1}(x) \in (\underline{v}, \bar{v})$ for $i \in \{1,2\}$, $x \in \{x_1, x_2\}$). Assume that both boundaries are equal, $x_1 = x_2 = x$, for some $v = \bar{X}_1^{-1}(x), w = \bar{X}_2^{-1}(x)$. This allows to simplify the system to two non-linear equations and two functions of one argument as unknowns, which can be easily solved with a mathematical package.

Note that the above algorithm does not provide corner solution for $v > \tilde{v} = \bar{X}_1^{-1}(X_2(\bar{v}))$. Observe, however, that (B2) in this case can be rewritten as

$$x = \frac{\beta}{\beta - 1} \frac{\Delta}{\bar{X}_1^{-1}(x) - \int_{\underline{v}}^{\bar{X}_1^{-1}(x)} w \frac{f(w)}{F\left(\bar{X}_1^{-1}(x)\right)} dw} \frac{1}{F(\bar{X}_1^{-1}(x))}, \tag{B4}$$

and does not depend on $\bar{X}_2^{-1}(x)$. As a result, a single non-linear equation with a single unknown is easily solved numerically. Combinations $(\bar{X}_1^{-1}(x), x)$ and $(\bar{X}_2^{-1}(x), x)$ constitute pairs of valuations and equilibrium initiation strategies for the two bidders.

As an example, when bidder valuations are uniformly distributed on $[\underline{v}, \bar{v}]$, in the interior case

$$x = \frac{\beta}{\beta - 1} \frac{\Delta}{\left(\bar{X}_1^{-1}(x) - \underline{v}\right)/2} \frac{\bar{X}_2^{-1}(x) - \underline{v}}{\bar{X}_1^{-1}(x) - \underline{v}}, \tag{B5}$$

$$x \int_{\underline{v}}^{\bar{X}_1^{-1}(x)} \frac{\Pi_o \left(\Pi_b + \frac{\beta}{\beta - 1} wx\right)}{\left(\Pi_b + wx\right)^2} \frac{\bar{X}_2^{-1}(x) - w}{\bar{X}_1^{-1}(x) - w} dw = \frac{\beta}{\beta - 1}\Delta. \tag{B6}$$

The integral in (B6) has a closed form representation.

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Table I: Benchmark model parameters

This table reports the benchmark parametrization of the model.

Variable	**Description**	**Value**
r	Risk-free rate	0.05
μ	Growth rate of target value	0.01
σ	Volatility of growth rate of target value	0.25
Π_b	Initial value of bidders	100
Π_o	Post-takeover value of the losing bidder	95
Δ	Value loss of the losing bidder	5
$\underline{v}$	Lowest value of the acquired target	110%
$\bar{v}$	Highest value of the acquired target	150%
$F(v)$	Distribution of valuations	Uniform
$D(v)$	Dispersion of valuations*	11.55%

$*$ Note: Dispersion of valuations for the uniform distribution is $D(v) = \sqrt{(\bar{v} - \underline{v})^2/12}$.