A Markov Switching Unobserved Component Analysis of the CDX Index Term Premium

Giovanni Calice*[1], Christos Ioannidis**, RongHui Miao***

Abstract

Using a Markov switching unobserved component model we decompose the term premium of the North American CDX investment grade index (CDX-IG) into a permanent and a stationary component. We explain the evolution of the two components in relating them to monetary policy and stock market variables. We establish that the inversion of the CDX index term premium is induced by sudden changes in the unobserved stationary component, which represents the evolution of the fundamentals underpinning the probability of default in the economy. We find strong evidence that the unprecedented monetary policy response from the Fed during the 2008-2009 financial crisis period was effective in reducing market uncertainty and helped to steepen the term structure of the index thereby mitigating systemic risk concerns. The impact of stock market volatility, as captured by the VIX index, in flattening the term premium was substantially more robust in the crisis period. We also show that equity returns make a substantial contribution to the term premium over the entire sample period.

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*Corresponding author. University of Birmingham, Department of Economics, Edgbaston, Birmingham, B152TT. E-mail: G.Calice@bham.ac.uk
Tel: +44 (0) 121 414 4956. Fax: +44 (0) 121 414 7377.
** University of Bath, Claverton Down, BA2 7AY, Bath. E-mail: c.ioannidis@bath.ac.uk.
*** Aviva plc., St. Helen's, 1 Undershaft, London, EC3P3DQ. E-mail: ronghuimiao@gmail.com.

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1. Introduction

The sub-prime mortgage crisis, unveiled in July 2007, has caused billions of dollars of losses in the credit markets as systemically important financial institutions had been forced to write off mortgages and related securities linked to credit derivatives instruments, like credit default swaps (CDSs) and collateralized debt obligations (CDOs). Great uncertainties filled almost every corner of the financial markets, which seriously interrupted its normal functioning (see Taylor and Williams (2008)).

It has been argued (see, amongst the others, Longstaff (2010), Calice (2011)) that the 2007/2008 sub-prime crisis was amplified through structured credit products-tranches trading. Consequently, if on the one hand, these instruments seem to have enriched the scope of investment strategies, on the other hand, their increased complexity depth have unduly induced instability in financial markets.

CDS indices are simply portfolios of single name default swaps, serving both as trading vehicles and as barometers of credit market conditions. Users of the most popular indices (the Dow Jones CDX North American investment grade and the iTraxx Europe investment grade) include those who want to hedge against credit defaults of pooled entities and those who want to speculate.

These indices are responsible for the increased liquidity and popularity of tranching of credit risk. By buying protection on an index, an investor is protected against defaults in the underlying portfolio and makes quarterly premium payments to the protection seller. If there is a default, the protection seller pays par to the protection buyer.

The term premium of the CDX index which, in this paper, is measured as the difference between the CDX 10-year and the CDX 5-year maturities, can be viewed as representing the uncertainty regarding corporate default over a 5 years time horizon. Therefore, the CDX term premium can be interpreted as an early warning market indicator of improvement or deterioration in macroeconomic conditions for the next 5 years.

If an investor perceives the difference between the 5-year index premia and the 10-year index premia too steep, in other words, that the implied probability of default between 5 and 10 years is higher than that implied from fundamentals, but he/she expects the slope to flatten, then this investor could buy 5-year protection and sell 10-year protection on the CDX index. Finance theory suggests that the credit curves of companies with high credit quality should be upward sloping, whereas those of companies having very poor credit quality do exhibit negative slopes. For example, the credit risk of an AAA rated corporate bond should in general be positively correlated with its maturity, and hence the required yields slope upwards against its maturity. In cross-sectional space, the likelihood of credit quality deterioration should increase as rating lowers, which is to say that the required average yield should increase with the downgrading of corporate bonds. However, the credit curve for a company on the brink of default (or with foreseeable immediate downgrade) would invert to trend negatively to reflect higher credit risk in the near future. As a result, the yield for such bond is very high for short maturities but relatively lower for longer maturities, which reflects investors' view that it is still possible for this company to improve its credit quality for longer term maturities.

CDX curve trading has assumed enormous importance over the latest very turbulent period. Clearly, curves tend to flatten in periods of imminent higher default rates, and tend to be steep in periods of economic expansion.

Index curve trading is generally motivated on one or more of the following[2]

a) As a way of expressing market direction views with different risk-reward profiles.

b) Carry and roll-down reasons.

c) Hedging purposes – both cash and CDS underlying portfolios.

Many opportunities for trading curves on single-name CDS occur around forecasted or announced specific corporate actions. Such events change the perception of a company's creditworthiness and the shape of the CDS curve also evolves. Curve trades can be more attractive than outright positions around events, thanks to the variation in the available payoff profiles.

In addition, macroeconomic conditions can trigger default events that affect the curves of not only specific entities but also of entire industries. Changes in consumer preferences, the monetary policy stance, and developments in the housing market are critical industry-wide events and market sentiment often transcend worries about profitability and focus instead on viability and the possibility of default of a specific firm. In this case, recovery expectations, following a higher default rate regime, become progressively important in determining the curve shape of the index, as it clearly tends to flatten.

In this paper, we investigate the dynamic behaviour of the CDX index term premium, the difference between the 10-year and 5-year maturities, through time by using a Markov Switching Unobserved Component (MS-UC) model. In the econometric literature, several approaches have been proposed on the decomposition of univariate time series. A well-

[2] For more details, see Barclays Capital Research (2008), "CDS Curve Trading Handbook 2008".

established methodology is the *unobserved components approach*, postulated in separate contributions by Harvey (1985), Watson (1986) and Clark (1987), respectively. It seems natural to consider an economic time series in terms of permanent and stationary components. The decomposition of a univariate time series into these two components is a primary tool for analyzing business cycles, with these two components often used as measurements of unobserved trend and cycle. Traditionally researchers also adopted unobserved component models to study mean reversion in stock prices. Fama and French (1988) find a stationary mean reverting component in addition to a permanent component in the US stock price dynamics. Porterba and Summers (1988) test the existence of a stationary component although they do not perform a formal decomposition of the stock prices in stationary and permanent components. Others like Lo and McKinlay (1988) and Kim et al. (1991) use variance ratio tests to detect mean reversion in stock prices. Although the evidence of mean reversion in stock prices is mixed, as Summers (1986) argues, statistical tests used in testing the random walk (RW) hypothesis have usually low power against the alternative of mean reversion.

In formulating an unobserved components model for econometric analysis, we depart from others working on the observable determinants of CDS indices. Alexander and Kaeck (2008) and Byström (2006), for example, relate the CDS/CDX premia to several observed variables (such as the slope of the yield curve, stock market returns and stock market volatility), and analyze the significance of each observable variable in determining the CDS iTraxx Europe premia using single-equation regression. Our interest in this paper, however, is to study how the factors themselves (not the factor loadings) drive the dynamics of the term premium. Since the CDX index measures the economy-wide default probabilities (the higher the index value, the higher the probability of default on firms included in the index), the macroeconomic conditions, which can be encompassed by those fundamental factors, will be closely related to the CDX index value and its term premium.

To characterize the observed patterns of volatility jumps on the CDX index term premium, we allow on the innovation terms a regime switching process, following two distinct first-order Markov chain variables.

This paper contributes to the rapidly growing literature on structured credit in its attempt to understand the evolution of the term premium of the CDS index market and its link to observed macroeconomic and financial information.

Our paper has two main contributions. First, we present a readily implementable new approach to modeling CDS index dynamics, by conducting a regime dependent factor analysis of the evolution of the CDX index. Second, we provide insights into how the fundamental and volatility components of the CDX index are determined by daily observed monetary policy and stock market variables, over a sample period surrounding the 2007-2009 global financial crisis.

The current literature on CDS is primarily limited to the pricing with a large strand of it revolving around the key determinants of these contracts. First, there is an extensive literature on the driving forces of CDS premia ranging from the model of Hull, Predescu and White (2004), Aunon-Nerin, Cossin, Hricko and Huang (2002), which examine the relationship between CDS premia and credit spreads, to more elaborate analysis by – amongst the others – Zhu (2006), Longstaff, Mithal and Neis (2005) and Blanco, Brennan and Marsh (2005) which include also bond and equity markets measures.

Much of the research on credit markets has focused on corporate bond spreads and single-name CDS premia. Despite a sizeable literature on credit risk empirical studies on CDS that involve the modeling of the entire credit curve are uncommon. A major reason for this is that data on the CDS premia for a wide range of maturities have only recently become available. Consequently there is a paucity of empirical works regarding CDS indices, with studies focused mainly on the North America CDX investment grade index (CDX.NA.IG).

Our work is also closely related to two recent studies by Pan and Singleton (2008) and Zhang (2008), who attempt to estimate default risk using the entire credit curve of sovereign CDS premia. Byström (2005, 2006) and Alexander and Kaeck (2008) are the early studies on CDS indices. In a correlation study of a sample of European CDS iTraxx indices for different industrial sectors, Byström (2005) finds a tendency for iTraxx premia to narrow when stock prices rise, and vice versa. Furthermore, he finds that the stock market reacts quicker than the iTraxx market to firm-specific information and the stock price volatility is significantly and positively related to the volatility of CDS premia. Alexander and Kaeck (2008) use a Markov switching model to examine the determinants of the European CDS iTraxx index in two different regimes. Their results show that the CDS market is sensitive to stock returns under 'ordinary' market conditions but extremely sensitive to stock volatility during turbulent periods. One recent paper by Bhar, Colwell and Wang (2008), which is mostly related to our paper, decomposes three European CDS iTraxx indices premia into persistent and stationary components using the Kalman filter. The authors investigate these dynamics for two different maturities (5 and 10 years) and find that the stationary component is affected largely by stock market volatility whereas the persistent component is more sensitive to illiquidity. However, their sample period does not include the recent sub-prime mortgage crisis. Therefore, the dynamic behavior of these two components during crisis times remains still unexplained.

Blanco, Brennan and Marsh (2005) analyze the relationship between investment grade bonds and CDS, and explore the determinants of CDS premia. They find that the theoretical relationship linking credit spreads and CDS premia holds reasonably well for most of the investment grade reference entities. In addition, they report that increases in interest rates and equity prices reduce CDS premia whilst a steeper-sloping yield curve has the opposite effect.

The paper's main results are as follows. First, the inversion of the CDX term premium is induced by sudden changes in the stationary component, which represents the evolution of the fundamentals underpinning the probability of default in the economy. Equally notable is that our findings show that the non-stationary component, which represents increases in volatility, spikes quite dramatically around the occurrence of tail risk events (e.g. Bear Sterns bailout and Lehman Brothers bankruptcy).

Second, the empirical evidence strongly suggests that the direct impacts of monetary policy rates and the slope of the yield curve on the term premium of the CDX index are time varying and business cycle dependent. Credit risk modeling that ignores this regime dependent feature would bias the pricing of credit contracts. Developments in both the first and second moments of the equity market have a lasting influence on both components, with more pronounced effects in volatile market conditions.

The paper is organized as follows. Section 2 discusses the possible economic determinants of the term premium and suggests its decomposition into two unobserved components allowing for regime switching. Section 3 presents and discusses the data used in the estimation. The results are reported in Section 4 and Section 5 concludes.

2. Motivation and Methodology

The econometric methodology employed in this paper is based on the statistical approach developed initially by Nervole, Grether and Carlvalho (1979) and developed further by Harvey (1989) and Harvey and Shephard (1993). The essential element of this methodology is to estimate a model which considers the observed time series as being the sum of permanent and stationary components. These components capture the salient features of the series that may be unobserved and are useful in explaining and predicting its time evolution. The Kalman filter is employed, in linear models, as the most efficient means of updating the state as new information becomes available.

Although the latent variable model is an effective tool in decomposing macro/financial variables into a number of unobservable components, the usefulness of the model is however still limited if we are unable to link the components to a set of observable economic variables. To overcome this problem, one may model the unobserved components and observed variables together in a macro-finance setting (as suggested, for example, by Ang and Piazzesi (2003)). Our analytical approach here is instead as follows. We begin by filtering out the unobserved components and then in a second step, we empirically estimate the relationship between the unobserved components and a set of variables observed at the same frequency.

Our aim is to test for the economically meaningful relationship between the unobserved components and a set of observed information that is available to both market participants and policy makers. Such link, if established, will add predictive ability to the model as the evolution of the components will be conditional on the underlying data and will enhance the model's analytical appeal.

At a conceptual level, the US Federal Fund Rate (*FFR*) is the standard monetary policy tool available to the Fed to influence the short end of the yield curve and hence, in turn, affects investors' expectations on the movements of long-term interest rates. An increase in *FFR* signals the Fed's reaction against the risk of rising inflation in the near future and will aggravate the external financing position of companies that rely heavily on short-term financing. The impact of monetary policy on the term premium will be conditional on the state of the economy. Under "normal" conditions a tightening of monetary policy will indicate future inflationary pressures due to expanding demand. In this case, increases in the policy rate may be consistent with mitigating insolvency risks and thus with lower 5-year CDX premia. However in periods of crisis when expectations of future demand are gloomy, the same rate increases will enhance the probability of imminent default as the companies' abilities to secure funds at reasonable rates are reduced, resulting in widening 5-year CDX premia and a flattening of the CDX index term premium.

The slope of the yield curve reflects simply a forward expectation of how the short-term interest rate is expected to fluctuate over a long-term horizon and is largely driven by the market-wide expectations about the future path of monetary policy. Once more the impact of changes in the yield curve will depend upon the prevailing market conditions. Under stable market conditions increases in the long-rate imply future rises of the short-rate. Such predicted evolution will impact positively on both the 5 and the 10-year CDX premia, rendering ambiguous its effect on the term premium. During episodes of generalized stress, the 'steepening' due to decreases in the short-rate curve will reduce the 5-year premia relatively to the 10-year, widening the term premium of the CDX index, as the reduction in the short-rate reduces the probability of imminent default.

Companies' borrowing depends largely on the market value of their net worth (financial and tangible assets). Asymmetric information between borrowers and lenders, would prompt lenders to set forth the abilities of borrowers to repay the debt, which will take the form of collateralizing their financial assets. Falling asset prices erode the value of collateral, tightening credit and depressing demand. Through the so-called "credit channel", the level of economic activity and the aggregate output will eventually shrink. If an adverse shock to the macro-economy is amplified by credit rationing, conditions in the real economy and in financial markets mutually reinforce each other, giving rise to a feedback loop which may lead to a deep recession. This self-reinforcing process, known as the "*financial accelerator*" (a term coined by Bernanke, Gertler and Gilchrist (1981, 1983, 1989, 1996)), operates in reverse during a downturn. Increases of the equity index return will always result in reduction in the 5-year CDX premia as the firm's collateral increases in value and enables them to secure funding.

The natural logarithm of the VIX index is a measure of forward uncertainty in the value of the firm's assets. Increasing uncertainty hinders the ability of the markets to assess the intrinsic probability of default. As a result, risk adverse investors will demand "excessive" 5-year premia, thus raising the term premium of the CDX index.

Additional features in our model are the interrelation between the stochastic elements of each component and the endogenous shift of their volatility between regimes. This feature enables us to capture the occasional and recurrent endogenous regime switches of volatilities in time series. To understand the rationale for assuming regime shifts in the two components' disturbance terms consider the evidence on the standard deviations for different time periods of the term premium presented in Table 1[3]. For the period 2004-2007 there is a modest change in the standard deviation of the term premium prior to November 2007 as new observations are added, and since August 2004 the mean of the term premium remains around 23. However, once interest rates began to rise and housing prices started to drop in 2006-2007 in many parts of the US, the refinancing of mortgages (especially the sub-prime

[3] Details of the data used in this paper are described in section 3.

mortgages) became extremely difficult. Defaults and foreclosure on those mortgages increased dramatically, which brought the sub-prime mortgage industry to the edge of collapse, and hence generated considerable uncertainty in financial markets. The standard deviation of the CDX term premium for the November 2007-July 2009 sub-sample jumps to 18.804, which is more than 6 times higher compared to the August 2004-November 2007's estimate. Concurrently, the mean of the term premium falls to -16.913, suggesting that the later sub-sample might be experiencing a different regime in terms of both mean and volatility. By allowing for regime switches in volatility (and in mean) to take place endogenously, we do not explicitly set a switching threshold value but we allow for the data to decide endogenously when to switch to a different regime.

TABLE 1: MEAN AND STANDARD DEVIATION OF THE CDX 5-YEAR, CDX 10-YEAR AND TERM PREMIUM FOR DIFFERENT SAMPLE PERIODS

		CDX 5-year		CDX 10-year		Term Premium	
Sample Period		**Mean**	**Std. Dev**	**Mean**	**Std. Dev**	**Mean**	**Std. Dev**
03Aug04~09 Nov05		52.648	6.814	76.072	7.003	23.424	2.249
03Aug04~29 Nov06		46.896	8.071	69.764	8.603	22.868	1.769
03Aug04~27 Nov07		46.832	11.379	69.836	10.528	23.004	3.098
	28Nov07~27Jul09	159.322	50.501	142.401	34.372	-16.913	18.804
03Aug04~17 Nov08		68.402	42.749	84.499	30.615	16.097	12.942
	18Nov08_27Jul09	197.643	43.071	162.763	31.488	-34.880	15.056
03Aug04~27Jul09		68.402	42.749	84.499	30.614	16.097	12.942

This regime switching attribute in the unobserved components space allows us to generate probabilities that each component of the term premium experiences either high or low volatility regimes through time. Note that although this specification complicates the estimation procedures - since additional filters must be employed to make inference on the hidden Markov chain process - allowing the two components to depend on different states of the economy provide us with an alternative approach to deal with the potential heteroskedastic variance in the daily CDX index series. The more conventional way of testing for financial time series heteroskedasticity is to consider ARCH-type volatility models, which allow constant unconditional volatility but time-varying conditional volatility. However, neglecting possible regime shifts in the unconditional variance, as shown in Lamoureux and Lastrapes (1990), would overestimate the persistence of the variance of a time series.

The remaining subsections present our stylized model of analysis. We first show how to construct the two components that drive the evolution of the CDX term premium. We outline next the state space representation of the system and our extension of modeling Markov switching disturbance terms.

2.1 Stationary and Random Walk Components in State Space Representation

Let $X_{1,t}$ represents the stationary component that drives the term premium, and assume that $X_{1,t}$ is an Ornstein-Uhlenbeck process, whose dynamic evolution can be described by the stochastic differential equation

$$dX_{1,t} = k\left(\delta - X_{1,t}\right)dt + \tilde{\sigma}_1 dZ_{1,t}$$

EQUATION 1

where δ is the target equilibrium or mean value supported by fundamentals; $\tilde{\sigma}_1 > 0$ is the scale of volatility that the exogenous shocks can transmit to the dynamics of $X_{1,t}$; $dZ_{1,t}$ is the standard Brownian motion with zero mean and unity variance that generate random exogenous shocks; $k > 0$ is the rate by which these shocks dissipate and the variable, $X_{1,t}$, reverts back to its mean. The Ornstein-Uhlenbeck process is an example of a Gaussian process that admits a stationary probability distribution and has a bounded variance. In contrast to the Brownian motion process that has constant drift term, the former allows for a drift term that is dependent on the current value of the process. If the current value of the process is lower than its long-term mean value, the drift term will be positive in order to bring the process back to its long-term mean value. If, on the other hand, the current value of the process is greater than its long-term mean value, the drift term will be negative in order to drag down the process back to its long-term mean value. In other words, this is a *mean-reverting process*. Setting $f\left(X_{1,t}, t\right) = X_{1,t}e^{kt}$ and applying the Ito's Lemma to this function, this leads to

$$df\left(X_{1,t}, t\right) = k\delta e^{kt}dt + \tilde{\sigma}_1 e^{kt}dZ_{1,t}$$

EQUATION 2

Integrating both sides of Equation 2, we obtain

$$X_{1,t} = X_{1,0}e^{-kt} + \delta\left(1-e^{-kt}\right) + \tilde{\sigma}_1\int_0^t e^{-k(t-s)}dZ_{1,s}\ ,\ 0 \le s \le t$$

EQUATION 3

where $X_{1,0}$ is the initial value of the process and the first and the second moments are given by

$$E\left(X_{1,t}\right) = X_{1,0}e^{-kt} + \delta\left(1-e^{-kt}\right)$$
$$Var\left(X_{1,t}\right) = \frac{\tilde{\sigma}_1^2\left(1-e^{-2kt}\right)}{2k}$$

EQUATION 4

The Ornstein-Uhlenbeck process is one of several widely used approaches to model stochastically interest rates, exchanges rates and stock prices. The advantages of its simple and tractable solutions, under continuous-time framework, have been embodied in many empirical asset pricing models. The econometric modeling, however, emphasizes the discrete-time representation of stochastic processes. Consequently, the exact discrete time model corresponding to Equation 1 is given by the following AR(1) process

$$X_{1,t} = \delta\left(1-e^{-k\Delta t}\right) + e^{-k\Delta t}X_{1,t} + \sigma_1\Delta Z_{1,t}$$

EQUATION 5

where $\Delta t = 1/250$ is the sampling interval and $\sigma_1 = \tilde{\sigma}_1\sqrt{\frac{\left(1-e^{-k\Delta t}\right)}{2k}}$. From this expression, it is immediately clear that $k > 0$ implies $e^{-k\Delta t} < 1$ and hence stationarity, $k \to 0$ or $\Delta t \to 0$ implies $e^{-k\Delta t} \to 1$ and the model converges to a unit root model.

Now, let $X_{2,t}$ be the second component that drives the term premium. We assume that it follows a driftless RW process as shown in Equation 6

$$dX_{2,t} = \sigma_2 dZ_{2,t}$$

EQUATION 6

where σ_2 is the scaled volatility parameter and $dZ_{2,t}$ is the standard Brownian motion that can be assumed to be either dependent or independent of $dZ_{1,t}$ .The discrete time version of Equation 6 yields

$$X_{2,t} = X_{2,t-1} + \sigma_2 \Delta Z_{2,t}$$

EQUATION 7

The RW process has long been a popular choice for modeling the price dynamics of financial assets. In continuous time financial models, the price of stocks and stock indices are modeled as geometric Brownian motions. It is relatively straightforward to show that the geometric Brownian motion of the price dynamic is equivalent to a RW path followed by the logarithm of the price in discrete time. The efficient market hypothesis in fact states that a financial asset's price follows a RW process, which literally assumes that the asset's price at time t is determined by its previous time period price and the instantaneous impact of the new flow of information. Although a RW process, like the one described in Equation 7, has infinite unconditional mean and variance, the conditional mean and variance can be measured as

$$E_t\left(X_{2,t}\right) = X_{2,t-1}$$
$$Var_t\left(X_{2,t}\right) = \sigma_2^2$$

EQUATION 8

where the conditional expectation of the process at current time t depends only on the observation at previous time period.

Given the two unobserved components, constructed using Equation 1 to Equation 7, we estimate the parameter space , as given by the system in Equation 9, with the dynamics of the two components in a Bayesian updating manner, namely the Kalman filter algorithm based on a state space system. State space representation is usually applied on dynamic time series models that involve unobserved variables (see, e.g., Engle and Watson (1981), Hamilton (1994), Kim and Nelson (1989)). In our modeling, the fact that the two driving forces of the CDX index term premium - stationary and RW components - are assumed to be unobserved state variables leads to the justification of using the state space representation. A typical state space model consists of two equations. One is a state equation that describes the dynamics of unobserved variables, which is shown below in Equation 9; and the other one is the measurement equation that describes the relation between measured variables and the unobserved state variables, as shown in Equation 10.

$$
\begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} = \begin{bmatrix} \delta\left(1-e^{-k\Delta t}\right) \\ 0 \end{bmatrix} + \begin{bmatrix} e^{-k\Delta t} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_{1,t-1} \\ X_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix},
$$

$$
\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \sim \mathrm{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho_{12} \\ \sigma_2\sigma_1\rho_{21} & \sigma_2^2 \end{bmatrix} \Delta t \right)
$$

EQUATION 9

$$
Y_t = X_{1,t} + X_{2,t}
$$

EQUATION 10

In Equation 9, the covariance terms $\sigma_1\sigma_2\rho_{12}$ and $\sigma_2\sigma_1\rho_{21}$ will be zero under the assumption of independence between the two disturbance terms (the correlation between the two disturbance terms - ρ_{12} - is zero).

In compact form, Equation 9 can be rewritten as

$$
\begin{aligned} X_t &= C + FX_{t-1} + \Sigma_t, \\ \Sigma_t &\sim \mathrm{N}(0, Q) \end{aligned}
$$

EQUATION 11

where $X_t = \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix}$, $C = \begin{bmatrix} \delta\left(1-e^{-k\Delta t}\right) \\ 0 \end{bmatrix}$, $F = \begin{bmatrix} e^{-k\Delta t} & 0 \\ 0 & 1 \end{bmatrix}$, $\Sigma_t = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$ and $Q = \begin{bmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho_{12} \\ \sigma_2\sigma_1\rho_{21} & \sigma_2^2 \end{bmatrix} \Delta t$.

The measurement equation, as described by Equation 10, links linearly the term premium of the CDX index to the stationary and RW components. Rewriting this expression in a compact form, Equation 10 reduces further to yield

$$
Y_t = HX_t
$$

EQUATION 12

where Y_t is the term premium series and $H = \begin{bmatrix} 1 & 1 \end{bmatrix}$ represents the weights of the two components in the term premium.

2.2 State Space Model with Markov Switching Disturbances

An additional feature of our model is to allow each component's disturbance term to depend on different states of the economy. In practice, we let the volatilities of the disturbance terms to switch between high and low volatility regimes. Formally, we assume that σ_1^2 and σ_2^2 in Equation 9 are driven by two discrete-valued, independent unobserved first-order Markov chain processes $S_{1,t} = \{0,1\}$ and $S_{2,t} = \{0,1\}$ given by

$$\sigma_1^2 = (1 - S_{1,t})\sigma_{1H}^2 + S_{1,t}\sigma_{1L}^2 \ , \ \sigma_{1H}^2 > \sigma_{1L}^2$$
$$\sigma_2^2 = (1 - S_{2,t})\sigma_{2H}^2 + S_{2,t}\sigma_{2L}^2 \ , \ \sigma_{2H}^2 > \sigma_{2L}^2$$

EQUATION 13

When both $S_{1,t}$ and $S_{2,t}$ are zeros, the two components will be in the high volatility state as $\sigma_1^2 = \sigma_{1H}^2$ and $\sigma_2^2 = \sigma_{2H}^2$; similarly if both $S_{1,t}$ and $S_{2,t}$ equal 1, the two components will be in the low volatility state since $\sigma_1^2 = \sigma_{1L}^2$ and $\sigma_2^2 = \sigma_{2L}^2$. The two remaining scenarios then categorize situations where the first component is in the high volatility state while the second is in the low ($S_{1,t} = 0, S_{2,t} = 1$) and where the first component is in the low volatility state while the second is in the high ($S_{1,t} = 1, S_{2,t} = 0$). This is a Markovian chain process which means that the current value of the process at time t depends only on its previous value at time $t-1$. The likelihood for the process to remain at the previous value or change to the alternative depends on the transition probabilities from one state to the other, which are shown below as

$$p_{1,00} = \Pr\left[S_{1,t} = 0 \mid S_{1,t-1} = 0\right]$$
$$p_{1,11} = \Pr\left[S_{1,t} = 1 \mid S_{1,t-1} = 1\right]$$
$$p_{2,00} = \Pr\left[S_{2,t} = 0 \mid S_{2,t-1} = 0\right]$$
$$p_{2,11} = \Pr\left[S_{2,t} = 1 \mid S_{2,t-1} = 1\right]$$

EQUATION 14

Equivalently, the two transition probability matrices for each disturbance term can be written as

$$p_1 = \begin{bmatrix} p_{1,00} & p_{1,01} \\ p_{1,10} & p_{1,11} \end{bmatrix}, \ p_2 = \begin{bmatrix} p_{2,00} & p_{2,01} \\ p_{2,10} & p_{2,11} \end{bmatrix}$$

EQUATION 15

where $p_{q,ij} = \Pr\left[S_{q,t} = j \mid S_{q,t-1} = i\right]$ with $\sum_{j=1}^{2} p_{ij} = 1$, $\forall\ i$ and $q = \{1,2\}$.

The estimation of the transition probabilities as shown above requires the choice of the appropriate functional forms of the probability functions that govern the Markov chain variables. Since the transition probabilities have to be bounded within $[0,1]$, the usual choice is the adoption of the logistic transformation on the probability terms that are given by

$$
\begin{aligned}
p_{1,00} &= \Pr\left[S_{1,t} = 0 \mid S_{1,t-1} = 0\right] = \frac{\exp\left(d_{1,0}\right)}{1+\exp\left(d_{1,0}\right)} \\
p_{1,01} &= 1 - p_{1,00} \\
p_{1,11} &= \Pr\left[S_{1,t} = 1 \mid S_{1,t-1} = 1\right] = \frac{\exp\left(d_{1,1}\right)}{1+\exp\left(d_{1,1}\right)} \\
p_{1,10} &= 1 - p_{1,11} \\
p_{2,00} &= \Pr\left[S_{2,t} = 0 \mid S_{2,t-1} = 0\right] = \frac{\exp\left(d_{2,0}\right)}{1+\exp\left(d_{2,0}\right)} \\
p_{2,01} &= 1 - p_{2,00} \\
p_{2,11} &= \Pr\left[S_{2,t} = 1 \mid S_{2,t-1} = 1\right] = \frac{\exp\left(d_{2,1}\right)}{1+\exp\left(d_{2,1}\right)} \\
p_{2,10} &= 1 - p_{2,11}
\end{aligned}
$$

EQUATION 16

where $d_{1,0}$, $d_{1,1}$, $d_{2,0}$ and $d_{2,1}$ are the unconstrained parameters. Appendix A contains a detailed account of the estimation procedures used in this model.

3. Data

In this section, we describe the relevant CDs and the data series used in the study.

CDS contracts are by nature over-the-counter. As a result, the availability and quality of the data are not as dependable as those of exchange-based transactions. CDS data are mainly collected by large investment banks which only record their own entering transactions. Although professional data vendors are the primary source of CDS research data, the data suffer the following pitfalls: (1) as in Zhu (2004), data prior to 1999 are very limited; (2) the frequency of CDS transaction data is low, therefore, the usual instantaneous lead-lag analysis which use much higher frequency data may not be robust; (3) CDS prices are usually obtained as "quoted" prices which may not reflect the actual information contained in trading prices; (4) CDS data are truncated in the way that the majority contracts have a maturity of 5 years with nominal amount of $5 million or $10 million; (5) CDS data are usually unevenly spaced with many spurious observations in time series[4].

To circumvent the above mentioned limitations, we restrict our sample of analysis to the CDS tranche index market, specifically to the North American CDX investment-grade indices. In these indices, all 125 single-name credits have equal weights in the portfolio. The Dow Jones CDX IG five-year index is a basket of CDSs on 125 names for the U.S. investment-grade market. Each reference entity has a weight of 0.8%. We use a representative dataset of daily CDS prices for the North American CDX tranche index and focus our analysis on the most liquid segments of the CDX index market, which are the 5-year and 10-year maturities. The analysis is based on daily data spanning from 2004 to 2009. The main source of CDX data is Markit.

In our sample period (August, 5. 2004 to July, 23. 2009), as shown in Table 2 and Figure 1, the average premium is 87.3161 basis points for the CDX 5-year index and 95.9463 for the CDX 10-year index. The CDX 5-year index reaches its maximum spread (283 basis points) on September, 16 2008, which is the day after the announcement of Lehman Brothers default. Similarly, the CDX 10-year index reaches its maximum premium (251 basis points) on the same day, as it is reflected in the negative slope of the credit curve. Not only the volatility of the CDS indices increased dramatically as of July 2007, but also the levels of these indices started to rise significantly at the onset of the crisis. The unprecedented financial turmoil directly raised investors' expectations on imminent defaults of firms' debts, especially of those financial firms heavily exposed to sub-prime-mortgage lending. From the second panel of Figure 1, we can see that the term premium of the CDX 10-year – 5-year index becomes negative at the start of 2008, suggesting an increase of investors' concerns over short-term default risk. Nevertheless, market sentiment remains unchanged over long-term horizons.

Daily Federal Fund Rates (FFR) are obtained from the St. Louis FRED database. Daily slope of the yield curve (SLOPE101), calculated using 10-year and 1-year U.S. Treasury bond yields, are from Thomson Reuters© Datastream. S&P 500 index daily observations are obtained from Thomson Reuters© Datastream. VIX index data are from the Chicago Options Mercantile Exchange (CBOE). The descriptive statistics of monetary policy

[4] An in-depth discussion of CDS data quality can be found in: Blanco, Brennan and Marsh (2005).

and equity market condition variables are also presented in Table 2 and Figure 2 depicts their evolution over the sample period.

TABLE 2: DESCRIPTIVE STATISTICS OF THE DATA SERIES

	CDX5	CDX10	Term Premium	FFR	SLOPE101	SP500RTN	VIX
Mean	87.3161	95.9463	8.6302	3.289400	1.268461	-0.0001	21.2356
Median	53.0000	77.0000	22.0000	3.620000	1.045000	0.0007	15.6300
Maximum	283.3708	251.3626	33.0000	5.410000	4.010000	0.1042	80.8600
Minimum	29.0000	54.0000	-53.3332	0.080000	-0.780000	-0.0947	9.8900
Std. Dev.	62.6127	41.3712	22.3793	1.810837	1.232534	0.0156	12.8773
Skewness	1.1973	1.1911	-1.3317	-0.491720	0.277599	-0.2528	1.8976
Kurtosis	3.2020	3.4167	3.6070	1.884992	1.911517	12.3573	6.5427
Jarque-Bera	281.2800	284.8754	363.4610	105.9148	71.54148	4277.3390	1312.9130
Probability	0.0000	0.0000	0.0000	0.000000	0.000000	0.0000	0.0000
Observations	1169	1169	1169	1169	1169	1169	1169

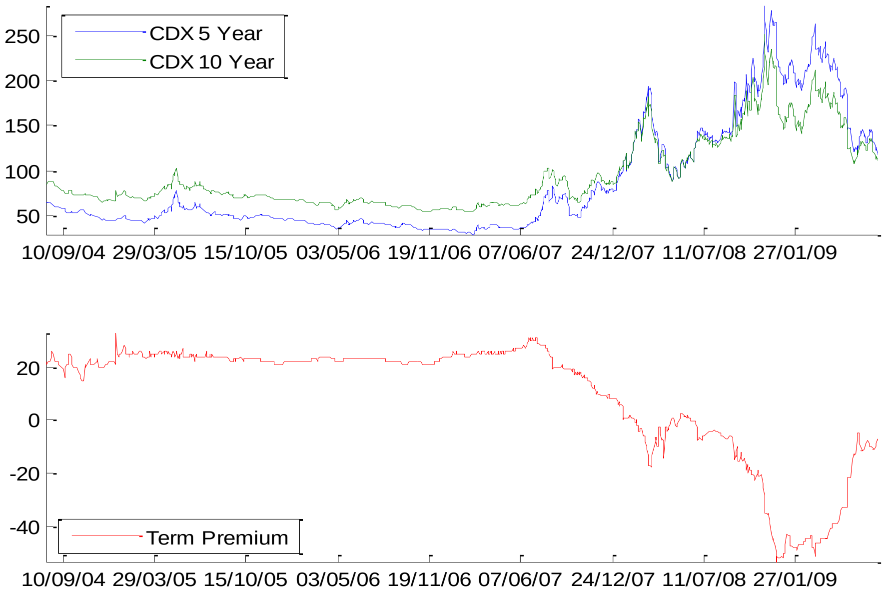

FIGURE 1: CDX-5 YEAR, CDX-10 YEAR AND CDX TERM PREMIUM

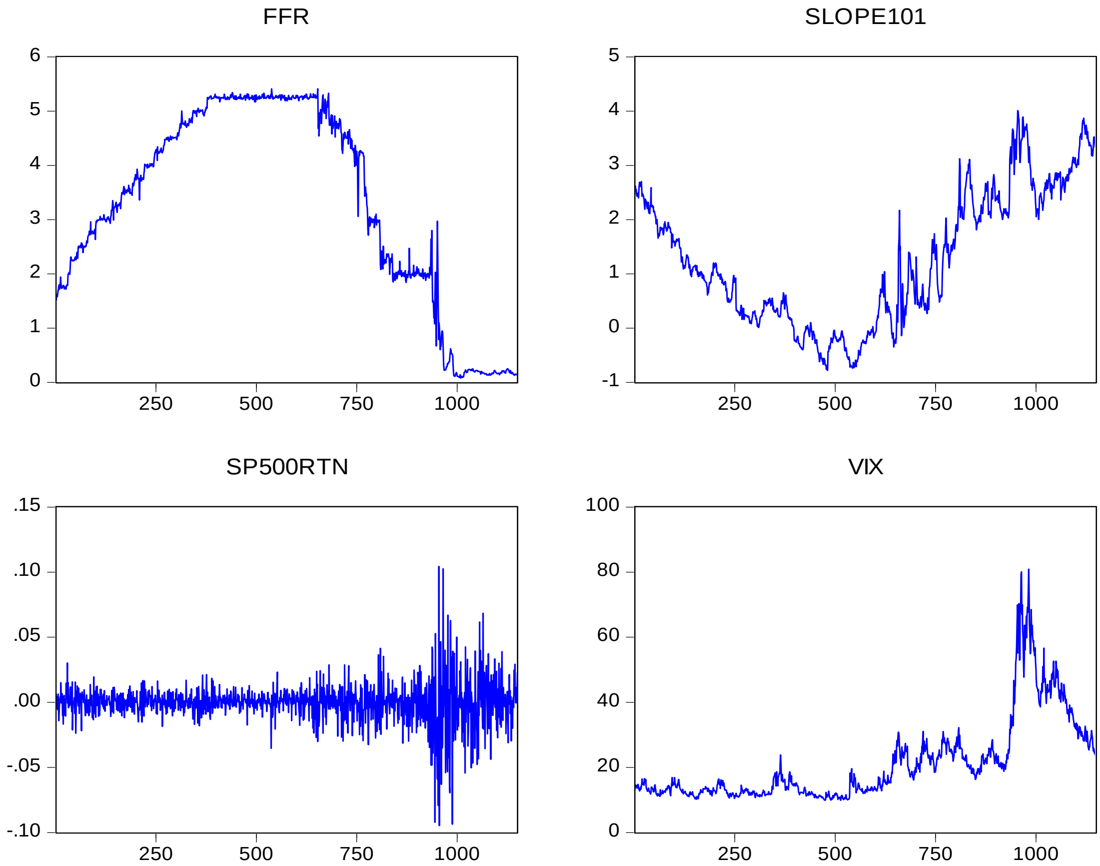

FIGURE 2: ACTUAL PLOTS OF: FFR, SLOPE101, S&P 500 RETURNS AND VIX INDEX

4. Empirical Results

Using the methodology described in section 2, we estimate a series of nested Markov-switching unobserved component models, followed by a battery of tests on model specification to determine the preferred model that will be used in the empirical analysis.

4.1 Model selection tests

It is well known that for Markov-switching models the standard likelihood ratio test of the null hypothesis of linearity does not have the usual χ^2 distribution. The reason is that there are nuisance parameters which cannot be identified under the null hypothesis. As a result, the scores evaluated at the null hypothesis are identically zero. Hansen (1992) and Garcia (1998) introduce alternative tests of the linearity against regime switching. In this paper, we use Hansen (1992) procedure, which provides an upper bound of the $p-$value for linearity, to determine the significance of improvement for allowing Markov-switching disturbance terms in the two components. In addition, we also consider more conventional ways of selecting models based on the Akaike Information Criterion (AIC) and Schwarz Bayesian Information Criterion (BIC). Finally, we verify our model selection results by running a series of residual diagnostic tests to see if the selected model could capture serial correlation and heteroskedasticity in the data series.

To implement Hansen's procedure (1992), we need to evaluate the constrained likelihood under the null hypothesis over a grid of values for the nuisance parameters. Defining the restricted model under the null hypothesis of no regime switching of the two components' disturbance terms as represented in equations 9-10 with $\rho_{12} = \rho_{21} = 0$, and the alternative model under the assumption of Markov-switching disturbance terms as involving equations 13-16, we have nuisance parameters $\{\sigma_{1H}, \sigma_{2H}, p_{1,00}, p_{1,11}, p_{2,00}, p_{2,11}\}$. The grids that we use for σ_{1H} and σ_{2H} are $[1, 2.5]$ and $[5, 10.5]$, respectively, each in increment step of 0.5. The grids for $\{p_{1,00}, p_{2,00}\}$ varies from 0.7 to 0.9 in increment step of 0.05 and for $\{p_{1,11}, p_{2,11}\}$ from 0.8 to 0.9 in increment step of 0.05. The Hansen test applied as described above, yields a conservative $p-\text{value}$ of 0.001, which clearly indicates a strong rejection of linearity in favoring of Markov-switching disturbance terms.

TABLE 3 and Table 4 contain the selection criteria results for models enabling correlated disturbance terms and models which are zeros correlations restricted. While AIC and BIC suggest that correlated models may perform better, likelihood ratio tests show that Model 1 and Model 2 better fit the data than Model 5 and 6, respectively. However, note that the improvement in likelihood value when allowing correlations and all parameters to switch between regimes (Model 8) is remarkable. In Table 5 we also report the likelihood ratio tests within each nested group and we can see that the most flexible model (Model 8) significantly outperforms all the models. We verify this result with the residual diagnostic tests in Table 6, where we test the overall randomness of the residuals of the models (summation of the disturbance terms of the two components) with the null hypothesis of assuming randomness. We report two Ljung-Box Q statistics for each model: one is the autocorrelation Q statistics based on the standardized residuals up to 20 lags; the other one is the ARCH effect Q statistics based on the squared standardized residuals up to 20 lags. From Table 6, the Q statistics suggest Model 8 as the best performing model, since it captures both the autocorrelation and ARCH effects in the residuals.

TABLE 3: MODEL SELECTION RESULTS

Model Specifications	Number of parameters	AIC	BIC	$\ln L$
Model 0: $\rho_{12} = \rho_{21} = 0$, 1 regime	4	3.1346	3.1521	-1798.3900
Model 1: $\rho_{12} = \rho_{21} = 0$	10	2.8000	2.8439	-1600.0100
Model 2: $\rho_{12} = \rho_{21} = 0$, k_2	11	2.7772	2.8255	-1585.9164
Model 3: $\rho_{12} = \rho_{21} = 0$, δ_2	11	2.7625	2.8108	-1577.4354
Model 4: $\rho_{12} = \rho_{21} = 0$, k_2, δ_2	12	2.6900	2.7426	-1534.7273
Model 5: $\rho_{12} = \rho_{21} \neq 0$	14	2.8012	2.8627	-1596.6954
Model 6: $\rho_{12} = \rho_{21} \neq 0$, k_2	15	2.7822	2.8480	-1584.7574
Model 7: $\rho_{12} = \rho_{21} \neq 0$, δ_2	15	2.7622	2.8281	-1573.2809
Model 8: $\rho_{12} = \rho_{21} \neq 0$, k_2, δ_2	16	2.6710	2.7412	-1519.8149

Note: Model 0 refers to system of equation 9-10 assuming $\rho_{12} = \rho_{21} = 0$. Model 1 builds on Model 0 with Markov-switching variances defined in equations 13-16; Model 2 builds on Model 1 but allow k to switch regimes; Model 3 builds on Model 1 but allow δ to switch regimes; Model 4 builds on Model 1 but allow both k and δ to switch regimes; Models 5-8 differ from 1-4 for allowing correlations between the two components' disturbance terms. $\ln L$ denotes the natural logarithm of likelihood value. AIC denotes the Akaike Information Criterion and BIC denotes the Schwarz Bayesian Information Criterion. A smaller statistic of AIC or BIC corresponds to smaller estimated Kullback-Leibler distance from the true model.

TABLE 4: LIKELIHOOD RATIO TESTS ON CONSTRAINT $\rho_{12} = \rho_{21} = 0$

Constraint: $\rho_{12} = \rho_{21} = 0$	Likelihood ratio	p-value
Model 1 to Model 5	6.6292	0.1568
Model 2 to Model 6	2.3180	0.6775
Model 3 to Model 7	8.3089	0.0809
Model 4 to Model 8	29.8248	5.31336E-06

TABLE 5: LIKELIHOOD RATIO TESTS WITHIN GROUPS

Constraints	Likelihood ratio	p-value
Group of models apply $\rho_{12} = \rho_{21} = 0$		
Model 1 to Model 2: $k_2 = 0$	28.1871412	0.0000
Model 1 to Model 3: $\delta_2 = 0$	45.1492514	1.82575E-11
Model 1 to Model 4: $k_2 = \delta_2 = 0$	130.565418	4.44713E-29
Group of models apply $\rho_{12} = \rho_{21} \neq 0$		
Model 5 to Model 6: $k_2 = 0$	23.8759758	1.02746E-06
Model 5 to Model 7: $\delta_2 = 0$	46.8289503	7.74606E-12

Model 5 to Model 8: $k_2 = \delta_2 = 0$	153.761032	4.08523E-34

TABLE 6: RESIDUAL DIAGNOSTIC TESTS

Lags	Autocorrelation		ARCH		Autocorrelation		ARCH	
	Q-stats	p-value	Q-stats	p-value	Q-stats	p-value	Q-stats	p-value
	Model 0							
1	3.3651	0.0666	55.4375	1.00E-13				
5	6.4061	0.2687	56.5142	6.37E-11				
10	21.4859	0.0179	84.9913	5.00E-14				
20	51.2066	0.0001	128.0163	0.00E+00				
	Model 1				Model 5			
1	5.8659	0.0154	12.4945	0.0004	0.002	0.9643	1.4605	0.2268
5	38.7978	0	77.4846	0	21.916	0.0005	12.6867	0.0265
10	46.7586	0	98.3647	0	39.9475	0	21.0755	0.0206
20	127.264	0	157.386	0	91.3123	0	42.9094	0.0021
	Model 2				Model 6			
1	10.665	0.0011	2.0368	0.1535	12.7731	3.52E-04	2.2811	0.131
5	64.4022	0	6.0608	0.3003	69.3635	0.00E+00	5.6359	0.3433
10	129.4365	0	38.6211	0	140.9098	0.00E+00	48.6666	0
20	244.9473	0	59.528	0	256.8626	0.00E+00	67.5147	0
	Model 3				Model 7			
1	34.5333	4.19E-09	0.3524	0.5528	14.6746	1.28E-04	0.0017	0.9673
5	73.5529	0.00E+00	13.7145	0.0175	22.747	3.77E-04	0.0029	1
10	94.9219	0.00E+00	37.0741	0.0001	50.5479	2.00E-07	1.4622	0.999
20	114.7697	0.00E+00	97.5292	0	76.9609	0.00E+00	1.4862	1
	Model 4				Model 8			
1	9.6918	0.0019	0.0337	0.8544	1.0875	0.297	0.0044	0.9468
5	19.6283	0.0015	1.3234	0.9325	4.0601	0.5408	0.0107	1
10	37.2164	0.0001	13.1973	0.2128	9.7284	0.4646	0.0267	1
20	56.5368	0	16.2277	0.7024	22.5042	0.3138	0.1733	1

4.2 Estimates of the Markov-switching unobserved component model

Table 7 reports the maximum likelihood estimates of Model 8, the most flexible and best performing model suggested by the model selection procedures considered above. The first noticeable result is the two regime dependent long term equilibriums of the stationary component: 8.4516 as in the low volatility regime and -0.5633 in the high regime. As can be seen, during the calm period (from 05 August 2004 to 31 December 2007), the slope along the credit curve is positive since the term premium is the compensation for default risk in 5 years' time. However, since the outbreak of the crisis (August 2007), severe strains in financial markets, banks' assets writedowns and diminishing liquidity in funding markets raised the level of uncertainty about corporate default risk. The inversion of the credit curve, as embedded in a negative CDX index term premium, vividly capture this deteriorating outlook.

TABLE 7: ESTIMATION RESULTS OF MODEL 8

Parameters	Estimate	Std. Error
δ_L	8.451611479	0.19272889
δ_H	-0.56331625	0.22729646
k_L	465.0780475	30.5714495
k_H	23.94429862	3.79151988
$\sigma_{1,L}$	0.006555485	0.01754211
$\sigma_{1,H}$	7.238340818	0.74796998
$\sigma_{2,L}$	0.000709143	0.00671695
$\sigma_{2,H}$	4.115509141	0.23609131
$\rho_{1L,2L}$	0.882338837	2.69775828
$\rho_{1H,2L}$	-0.8868773	23.1607869
$\rho_{1L,2H}$	-0.62831343	4.08740951
$\rho_{1H,2H}$	-0.72940729	0.21734431
$p_{1,LL}$	0.995606738	0.00148892
$p_{1,HH}$	0.999997253	1.68E-06
$p_{2,LL}$	0.964267884	0.00110252
$p_{2,HH}$	0.97346475	0.00067856
$\ln L$	-1519.81491	

Note: Subscript L denotes Low volatility regime and H denotes High volatility regime. Subscript T denotes Stationary component and R denotes Random walk component.

The second remarkable result is the regime dependent mean reverting speed for the stationary component. During non-crisis periods, asset prices are less likely to stay high or low period-to-period but mean revert quickly to their long-term equilibrium values. In other words, mean reverting assets prices imply a low probability of ending up in the tail of the distribution. Our estimation of the measure of mean reverting speed (k) is 465.07 in a low volatility regime, which translates to a first-order autocorrelation of 0.1556. The speed in the high volatility regime, on the other hand, falls to 23.94 or 0.9087 in terms of first-order autocorrelation, which suggests a very persistent behavior of the stationary component in the high volatility regime.

Next, we look at the volatilities of the stationary and RW components in each regime. In terms of stationary shocks, the estimated volatility in the low volatility regime is positive but statistically insignificant. This is primarily due to the very high mean reverting speed in the low volatility regime. Therefore, the variations in the low volatility regime are barely zero. It is more clearly shown in Figure 3 that the stationary component's variation in low volatility regime is tight at around 8.45 and the largest change is only 1.4 basis points. On the other hand, the scale of the variation for the stationary component in the high volatility regime is by far higher than in the non-crisis period, with an estimated annual volatility of 7.238.

The distinguishing feature of our model is that it allows us to decompose the term premium into two correlated driving components. The filtered RW and the stationary components with the associated probabilities of switching regimes are displayed in Figure 4 and Figure 5. In the first part of the sample period, temporary volatile movements in the RW and stationary components are induced by severe shocks such as the GM and Ford downgrade events of May 2005. In the subsequent period, the credit market enjoyed a rapid growth in terms of both trading volumes and products innovation. During this time period, both components stay in the low volatility regime, as reflected by a flat CDX credit curve. The frequent regime changes of the RW component start taking place in the aftermath of Countrywide's bankruptcy. In fact, on August, 15 2007, Countrywide Financial, the largest mortgage lender in the United States, announced that the foreclosure and mortgage delinquencies had risen to their highest level since early 2002. Since then, because of that episode and of the events around the onset of the subprime mortgage crisis, the CDX index term premium exhibits a downward trend. Ironically, that event occurred just one month after the DJIA index hit its historical record level of 14,000 (on July, 19 2007).

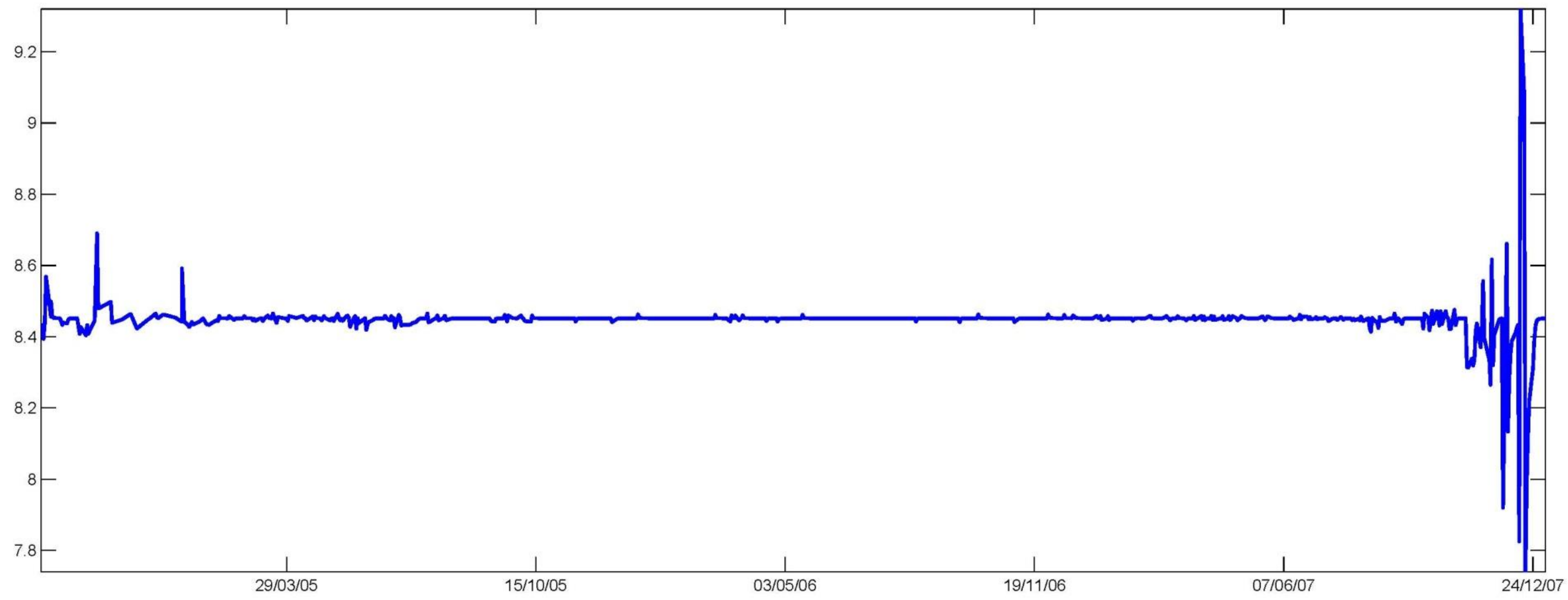

FIGURE 3: SNAPSHOT OF THE STATIONARY COMPONENT IN THE LOW VOLATILITY REGIME

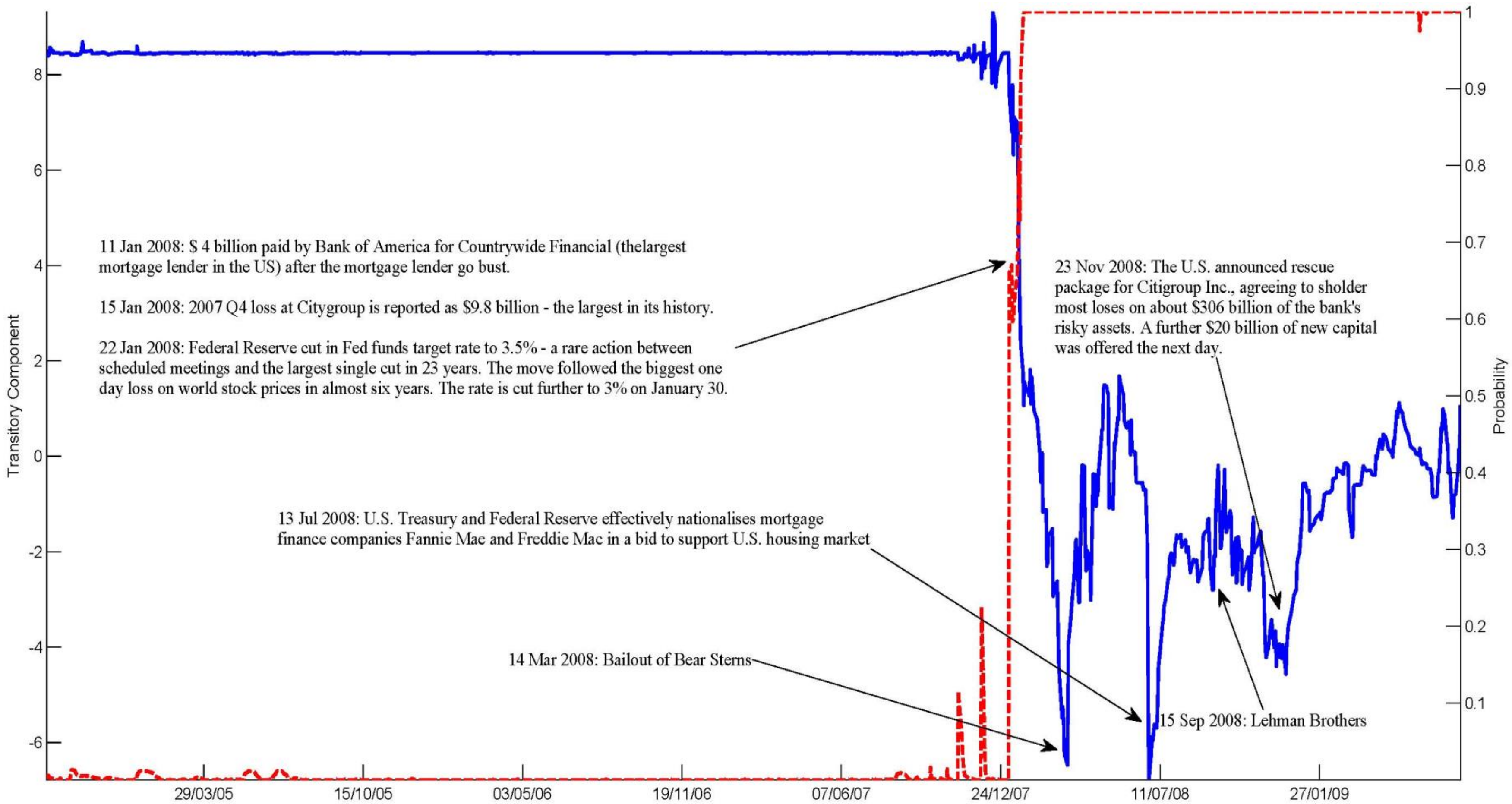

FIGURE 4: STATIONARY COMPONENT AND ITS PROBABILITIES TO SWITCH TO THE HIGH VOLATILITY REGIME

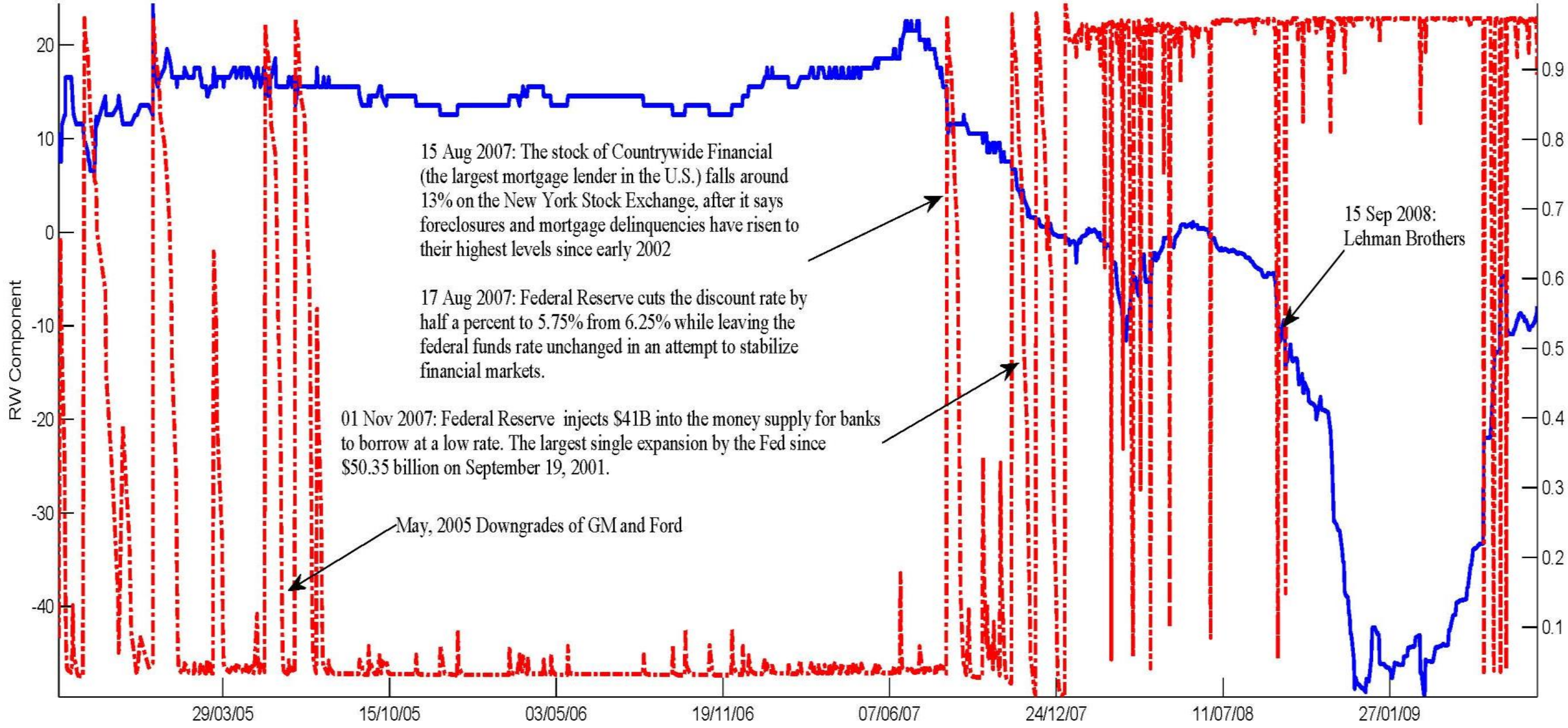

FIGURE 5: RANDOM WALK COMPONENT AND ITS PROBABILITIES TO SWITCH TO THE HIGH VOLATILITY REGIME

On the other hand, as shown in Figure 4, the stationary component enters a high volatility period from the beginning of 2008. Increased credit-related writedowns for individual banks (e.g. Citigroup) owe to a further deterioration in the corporate debt and prime residential mortgage markets, as the crisis originally centered in subprime mortgages spilled over to adversely affect economic prospects more broadly. On January, 22 2008, the Federal Reserve cut the Fed funds target rate to 3.5% - an unprecedented decision, taken between scheduled meetings, and the largest single cut in 23 years. The move followed the biggest one day loss on world stock exchanges in almost six years. The rate is reduced further to 3% on January, 30 2008. On March, 14 2008, Bear Sterns' demise brought about a dramatic increase in stock market volatility and liquidity shortages in funding markets. As illustrated in Figure 4, the resulting decline of the stationary component on that day captures investor confidence-induced downward spirals. The subsequent abrupt jumps occur on July, 13 2008 and November, 23 2008 when the US authorities announced the nationalization of Fannie Mae and Freddie Mac and a rescue package of Citigroup.

Although the probability of the stationary component to switch to the high volatility regime accurately signal greater market volatility since early 2008, it is quite evident from inspection of this data that the probability for the RW component to switch into the high volatility regime is close to zero at the occurrence of extreme credit events, such as the Bear Stearn’s and Lehman Brothers’ default announcements. At first sight, this counter-intuitive result may be difficult to understand. It conveys the message that credit market uncertainties, as measured by the conditional variance of the term premium, decline significantly with the abrupt unveiling of tail risk events.

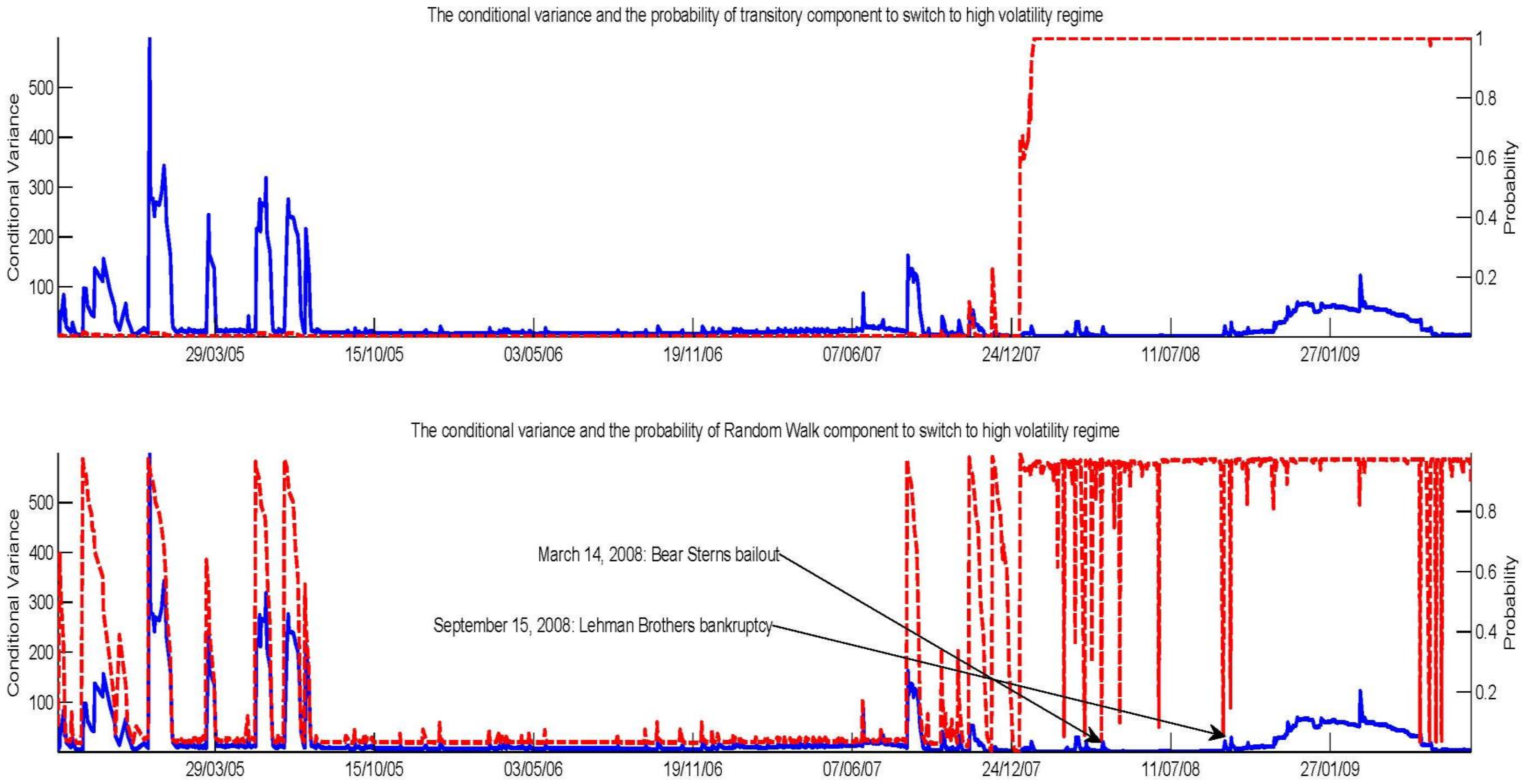

FIGURE 6: CONDITIONAL VARIANCE OF THE TERM PREMIUM WITH EACH COMPONENTS' PROBABILITIES TO SWITCHING TO THE HIGH VOLATILITY REGIME

Figure 6 plots the conditional variance derived from model 8. Using the definition of Kalman filter provided in the previous section, the conditional forecast error variance is given by

$$f_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} = HP_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}H'$$

EQUATION 17

which can be regarded as the implied conditional variance of the CDX term premium. Since this conditional variance depends on the two Markov chain processes, combining the filtered probabilities of states $\sum_{S_{1,t-1}=0}^{1}\sum_{S_{1,t}=0}^{1}\sum_{S_{2,t-1}=0}^{1}\sum_{S_{2,t}=0}^{1}\Pr\left(S_{1,t-1}=i, S_{1,t}=j, S_{2,t-1}=i, S_{2,t}=j \mid I_{t-1}\right)$ with Equation 17, we can calculate the conditional variance as a product of these two equations, based on the available information at time $t-1$. From Figure 6 we can observe that the conditional variance, in the immediate aftermath of Bear Stern's bailout (March, 14 2008), remains at low levels for a few days. What is particularly striking is that the probability for the RW component to switch into the high volatility regime falls back to a near-zero value at the outbreak of the crisis. In the sub-sample period surrounding Lehman Brothers' default, the conditional variance of the CDX term premium is much bigger than in the Bear Stern's bailout period. Consequently, the probability of the RW component to switch into the high volatility regime initially falls back to a near-zero value, but rebounds very rapidly in the subsequent days as investors begin to worry about the stability of other systemically important financial institutions. Our results show that the persistent co-movement between the high conditional variance of the CDX term premium and the probability of the RW component switching into the high volatility regime disappears during the financial crisis period. This suggests that the RW and the stationary components may behave differently depending on whether the financial system is experiencing a systemic crisis or not.

4.3 VAR Analysis

We now test for the impact of observed economic and financial variables on the unobserved stationary and RW components, in the context of the following VAR (4) model[5]

[5] The order of the VAR is established using the SBC criterion.

$$Y_t = c + \Pi_1 Y_{t-1} + \Pi_2 Y_{t-2} + \Pi_3 Y_{t-3} + \Pi_4 Y_{t-4} + \varepsilon_t$$

EQUATION 18

where Y_t includes the stationary (stat) and changes in the RW components (difrw), the observed level of the effective US Federal Fund Rate (FFR), the slope of the yield curve (calculated as the difference between the 10-year and the 1-year US treasury bond yields), the Standard & Poor's 500 index return (SP500RTN), and the implied volatility of the Standard & Poor's 500 index (LogVIX), $Y_t = [FFR_t, SLOPE101_t, SP500RTN_t, LOGVIX_t, DIFRW_t, STAT_t]'$, c is a (6×1) vector of constants, Π_i are (6×6) coefficient matrices and ε_t is an (6×1) unobservable zero mean white noise vector process with invariant covariance matrix Σ. Given the structural break occurring in January 2008, we divide the sample into two sub-periods depending on the two volatility regimes of the stationary component. The first sub-sample spans from September 13, 2004 through January 3, 2008 whilst the second sub-sample starts on January 4, 2008 and ends on July 23, 2009.

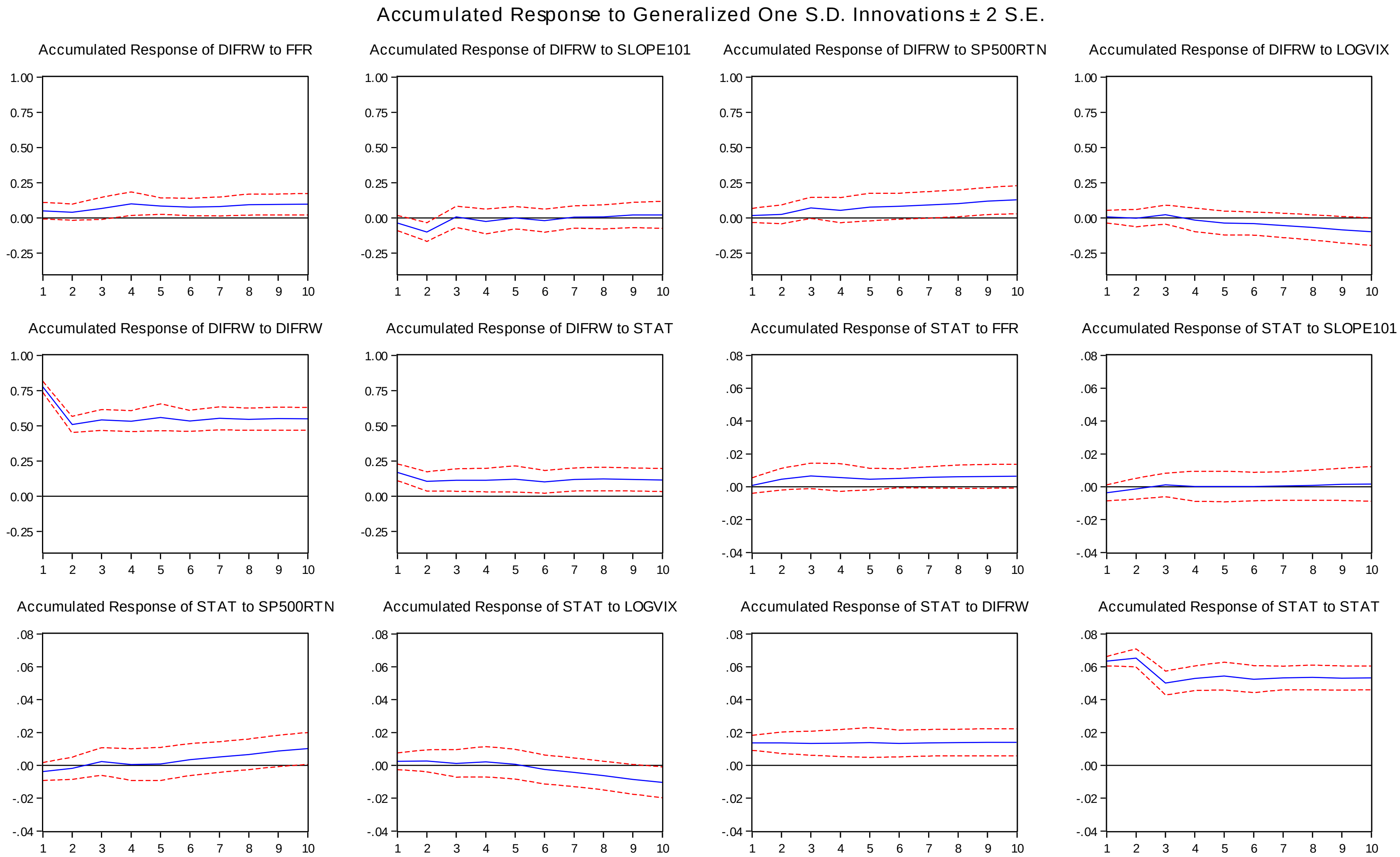

FIGURE 7: ACCUMULATIVE GERNERALIZED IMPULSE RESPONSE FUNCTIONS OF THE VAR MODEL IN THE PRE-FINANCIAL CRISIS PERIOD (013/09/04 TO 03/01/08)

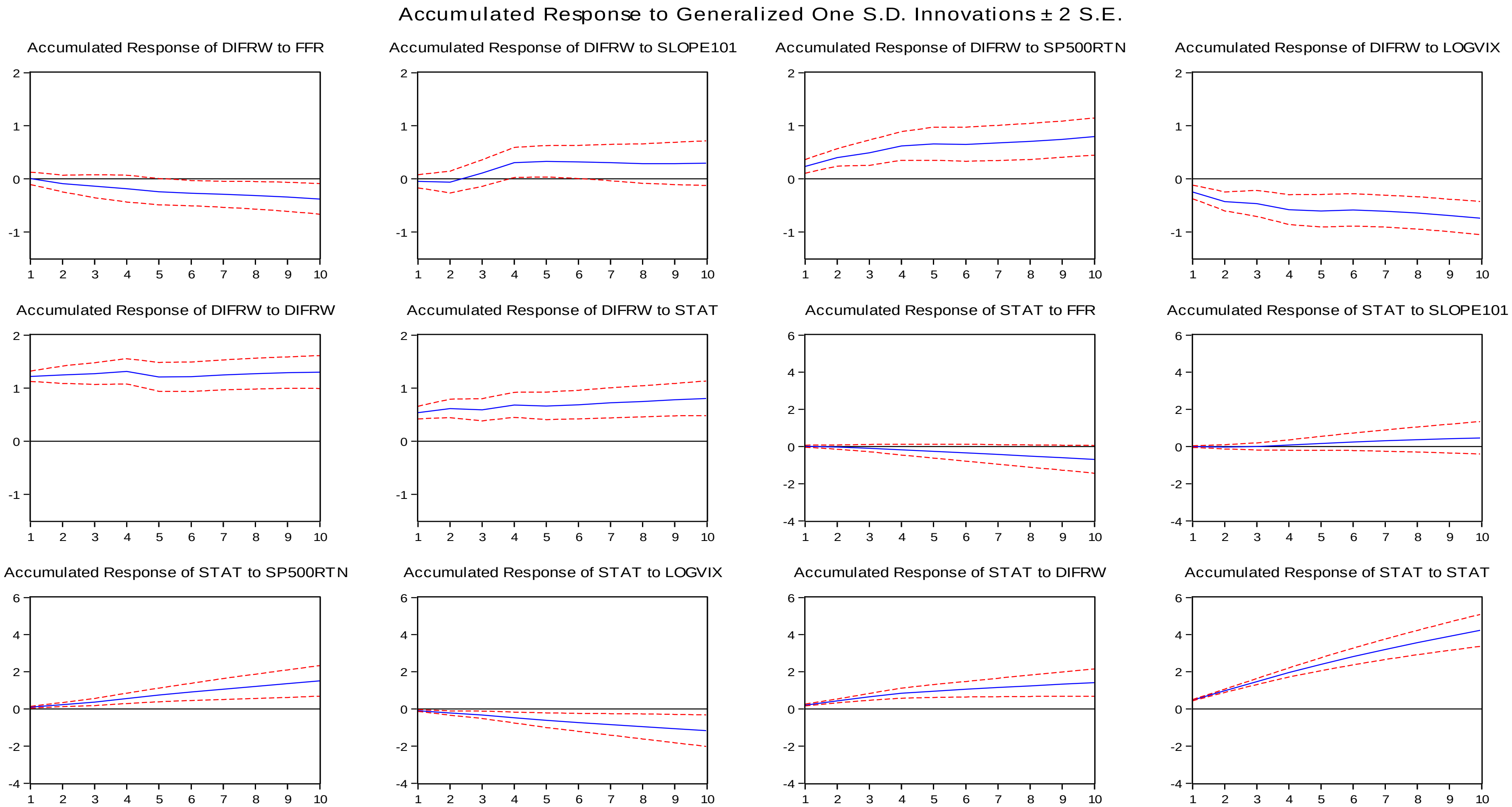

FIGURE 8: ACCUMULATIVE GENERALIZED IMPULSE RESPONSE FUNCTIONS OF THE VAR MODEL IN THE POST-FINANCIAL CRISIS PERIOD (04/01/08 TO 23/07/09)

To evaluate the relationships between the two components and monetary policy and stock market variables, we compute the accumulated generalized impulse response functions, which trace out the responsiveness of the dependent variables to one unit generalized shock to each of the variables in the VAR system. These impulse response functions provide useful insights into the dynamic properties of the system.

As forecasting variables of the variation in the term premium of the CDX index, these monetary policy and stock market variables appear to impact differently on the two components, before and after the onset of the crisis. Figure 7 shows the accumulated generalized impulse response functions of the VAR model in the pre-crisis period. The responses of DIFRW and STAT to FFR are significantly positive, which is consistent with the findings of Longstaff and Schwartz (1995) who document that corporate yield spreads vary inversely with the benchmark short-term treasury yield. Since an increase in the monetary policy rate would translate into an increase in the level of interest rate in normal periods, a higher interest rate level will decrease the present value of future cash flows and hence the value of default protection. This would lead to a tightening of the credit spread. If the tightening of the spread is more severe for the shorter CDS maturities, the term premium in the credit curve would increase as represented by a rise in the slope of the credit curve. This result, as illustrated in Figure 7, is consistent with Longstaff and Schwartz (1995) who suggests that the relationship between the CDX index spread and the risk-free interest rate depends on the time horizon. Compared to the negative responses of DIFRW and STAT in Figure 8, an increase in the monetary policy rate in the crisis-period would sharply reduce liquidity, increasing the probability of imminent default. The obvious effect would be widening the 5-year CDX premia and flattening the term premium of the CDX index.

The responses of the two components to SLOPE101 are negative but insignificant during the pre-crisis period. Bedendo et al. (2007) report a negative relationship between the slope of the US treasury yield curve and credit spreads. The reason is that when a positively sloped yield curve is the outcome of an expansionary monetary policy, which will increase future firm value and reduce default risk, the term premium decreases due to a decline in the 10-year premia. At the same time a steeper yield curve in normal time periods may indicate an increase in the future short rate through the expectations channel. If the inflation risk premium on longer term interest rates is very low (or even negative, see Kim and Wright (2005)), the latter would change little in response to a continuous rise in short term rates (Smith and Taylor (2009)). This effect, known as the "*conundrum*", may make the shorter maturity default protection contract more costly than those with longer maturities, and hence increasing the 5-year credit risk premia. This will, in turn, reduce the term premium of the CDX index. In contrast, during the crisis time period, any increase in the slope of the yield curve caused by a reduction in the short rate would be regarded as a signal of liquidity injections provided by central banks to contain systemic risks. Default protection premia (5-year) will fall and this would subsequently lead to an increase in the term premium. The positive responses of both DIFRW and STAT to an increase in the slope of the yield curve in crisis period (Figure 8) lend support to this hypothesis.

In the pre-crisis period the initial responses to equity market returns and the implied volatility index, have the opposite signs on DIFRW and STAT at the outset. Yet, at longer lags, both components exhibit a positive (negative) relationship with the equity returns (equity volatility). During the post-crisis period this initial divergence dissipates and both factors strongly respond positively to equity returns and negatively to the VIX. Such finding is consistent with the evidence in Bystrom (2006) who reports that on-the-run single-name CDS spread is significantly negatively related to the equity returns for the period 2004-2006. Scheicher (2006) also demonstrates that there is a significant contemporaneous link between the CDS market and the stock market. The inverse relationship between the two components and equity return volatility is broadly consistent with previous econometric evidence, as illustrated by Campbell and Taksler (2003), Alexander and Kaeck (2008) and Zhang et al. (2008). In the theoretical framework of Merton (1974), higher equity volatility means higher probability of hitting the default barrier, which induces a higher compensation on holding the bond in the form of larger credit spread. Although the positive (negative) responses to equity return (equity return volatility) hold both in non-crisis and crisis periods, the magnitude of responses in crisis period is far more pronounced.

To quantify the impact of all the observed variables on the unobserved factor we compute the generalized variance decomposition over the two periods. The results are presented in Tables 8 and 9. The key finding is the presence of strong effects attributable to stock market variables on both components before and after the crisis. Initially, in the pre-crisis period the impact of all the variables on both components is modest (Table 8). Stock market returns and their volatility appear to be the variables exerting some influence on DIFRW and STAT. Collectively they account for 7.5% of the DIFRW variability's and 3.51% of the STAT's variance.

In the post-crisis period of our sample (Table 9), approximately 13.5% of the variation in the DIFRW can be attributed to a combination of stock market variables, the monetary policy stance and the slope of the yield curve. More specifically, a substantial part of this variation is primarily explained by the stock market variables (7.68%). For the stationary component the same information set explains 16.44% of the variance, that is a nearly fivefold increase compared to the previous period. The impact of monetary policy alone is 4.2% compared to a previous value of 0.5%.

Importantly, our results indicate that during the crisis period, information pertinent to the immediate valuation of firm's assets, such as the stock market benchmark index and monetary policy exerted relatively strong influence on the CDX term premium, compared to factors such as the VIX and the slope of the yield curve.

Within the confines of the model, the crisis period is indicative of a significant change to the fundamentals that determine the term premium, as captured by the stationary component. The evidence from the RW component is indicative of increased volatility that can be only partly explained by the set of the observed macro and financial variables included in our analysis. In summary, these results do provide evidence that the inversion of the term premium since the onset of the crisis is primarily attributable to the evolution of the stock market and monetary policy. In particular, the drastic and immediate response of the Fed played an important role in avoiding dramatic increases in the short (5-year) premia and therefore helped stabilizing the short-term segment of the CDS market.

TABLE 8: GENERALISED VARIANCE DECOMPOSITION OF DIFRW AND STAT IN THE PRE-FINANCIAL CRISIS PERIOD (SEPTEMBER 13, 2004 TO JANUARY 03, 2008)

Variance Decomposition of DIFRW:							
Period	S.E.	FFR	SLOPE101	SP500RTN	LOGVIX	DIFRW	STAT
⋮							
10	0.838	0.695	2.746	0.446	3.284	92.716	0.113
15	0.839	0.698	2.758	0.505	3.307	92.615	0.116
Variance Decomposition of STAT:							
⋮							
10	0.065	0.506	0.579	1.209	0.875	4.320	92.512
15	0.065	0.509	0.579	1.445	0.973	4.307	92.186

TABLE 9: GENERALISED VARIANCE DECOMPOSITION OF DIFRW AND STAT IN THE POST-FINANCIAL CRISIS PERIOD (JANUARY 04, 2008 TO JULY 23, 2009)

Variance Decomposition of DIFRW:							
Period	S.E.	FFR	SLOPE101	SP500RTN	LOGVIX	DIFRW	STAT
⁞							
10	1.249	1.284	4.203	6.314	1.090	86.421	0.688
15	1.254	1.577	4.174	6.556	1.127	85.794	0.773
Variance Decomposition of STAT:							
⁞							
10	1.381	2.837	1.213	9.592	0.210	8.664	77.485
15	1.538	4.258	1.171	10.837	0.173	7.502	76.059

5. Conclusion

In this article, we estimate a Markov switching unobserved component model to explain the evolution of the term premium of the most liquid CDS maturities for the North American CDX index.

We consider an appropriately specified *Markov Switching Unobserved Components* model as a reliable measure of volatility dynamics of the CDX index spread curve and investigate the presence and significance of both monetary policy adjustments and stock market returns for the US economy over the sample period September 2004 – July 2009.

To the best of our knowledge, this is the first direct empirically based evidence that is brought on the evolution of the term premium of the CDS index market and its observed macroeconomic and financial determinants.

To capture the magnitude of uncertainty in the CRT market, we decompose the level of the CDX index term premium into two components. The first, the *RW component* is assumed to capture the changes in volatility driving the term premium whereas the second, a *stationary AR(1) process*, represents the fundamentals. Furthermore, we formulate a model with time-varying regime-switching probabilities and regime dependent components.

Our results suggest that the inversion of the curve around September 2008 is largely driven by abrupt moves in the stationary component, representing the evolution of the fundamentals underpinning the probability of default in the economy. The component enters the high volatility regime after a prolonged period of remarkable stability. Notably, by the end of 2007, the stationary component exhibits slight turbulence in the low volatility regime, but of a very different order of magnitude from the subsequent evolution of the component in 2008. The decline of the term premium accelerates sharply throughout the end of 2007 when it partially reverses its trend. However, it remains in the high volatility regime in the final part of the sample period. Interestingly, although the RW component appears to evolve in a very stable and predictable manner from 2004 to 2008, it fluctuates somewhat intensively between the low and high volatility regime over short periods of time during 2005 and towards the end of 2007. From the beginning of the sub-prime crisis (August 2007) the component exhibits downward movement but does not enter decisively the high volatility regime. Over the last part of the sample, the component enters more frequently the low-volatility regime but in a rather unpredictable manner, indicating that the uncertainty surrounding asset values still remain elevated.

Remarkably, the inclusion of observed economic and financial variables to predict the evolution of the unobserved components does a relatively good job only during the ‘crisis’ period. These variables are found to make a statistically significant contribution that is consistent with economic theory. Indeed, we find robust evidence that the unprecedented monetary policy response, of sharp rate reductions by the Fed during the crisis period, was effective in reducing market uncertainty and helped to steepen the curve of the index thereby mitigating systemic risk concerns. The impact of market volatility in flattening the curve and exerting comparatively higher upward pressure on the 5-year CDX is substantially more robust in the crisis period, as both components are significantly affected by the VIX measure. It also appears that equity returns are important drivers of the term premium during both periods. This impact results in a steepening of the curve as the current value of the underlying ‘collateral’ increases. Our results also suggest that in the pre-crisis period the RW component associated with increased volatility displays a low reaction to the stock market. Additionally, as expected the impact on the stationary component albeit positive is not significant. Yet, from January 01, 2008 both components respond immediately and significantly to stock market fluctuations. We demonstrate that the unprecedented stock market collapse is a very important contributory factor to the inversion of the CDX index term premium.

Overall, this evidence implies that credit risk modeling that ignores this regime dependent feature would bias the pricing of credit contracts. Developments in both the first and second moments of the equity market have a lasting influence on both components, with more pronounced effects during volatile market conditions.

The evolution of the CDX index in all maturities is an important signal of the ‘health’ of the economy over the short and long run. Sudden inversions indicate sharp deterioration of the current economic conditions and increased probability of default. Such movements are triggered by both the evolving stance of monetary policy and developments in the equity markets that make a significant albeit modest contribution to their predictability.

This article is only a first step toward the development of a fully fledged consistent framework to gain greater insight in the dynamics of the CDX curve indices across different parts of the credit cycle and in the relationship between the shape of the term structure and macro/financial variables fluctuations. Further research is warranted. Interesting possibilities for further research include the consideration of an extended number of maturities and of other index tranches, such the high-yield segment of the market. These extensions along with a complementing examination of liquidity risks and the risk of spillovers will enhance our understanding of the dynamics of such important markets, primarily from a systemic viewpoint.

APPENDIX A

Estimation Procedure

To estimate the state space Markov switching model, described in detail in the previous subsections, we use Kim's filter (Kim (1994)), which is a numerical algorithm that combine the Kalman filter in estimating state space models and the Hamilton filter in estimating Markov switching models. In the conventional derivation of the Kalman filter for an invariant parameter state space model, the goal is to make predictions of the unobserved state variables based on the current information set, denoted $X_{t|t-1} = \mathrm{E}\left(X_t \mid I_{t-1}\right)$, where I_{t-1} represents all observed variables available at time $t-1$. The mean squared error of the prediction, denoted as $P_{t|t-1}$, is $P_{t|t-1} = \mathrm{E}\left(\left(X_t - X_{t|t-1}\right)\left(X_t - X_{t|t-1}\right)' \mid I_{t-1}\right)$. The Kalman filter algorithm then implements a sequence of Bayesian updating on the unobserved variable X_t and the mean square error P_t when observing a new data entry. The updated unobserved variable $X_{t|t}$, given the observation of the information set at time t, is formed as a weighted average of $X_{t|t-1}$ and new information contained in the prediction error, where the weight assigned to this new information is called *Kalman gain*. This prediction and updating process evolves over time and is conditional on the correctly estimated parameters of the model. As a result, Kalman filter will need to be initialized in the first place with some carefully chosen initial values. Then, the prediction errors and their variances, as the by-products of the prediction process, will be used to construct the log-likelihood function

$$L(\theta) = -\frac{1}{2}\sum \ln\left[\left(2\pi\right)^n \left|\omega_{t|t-1}\right|\right] - \frac{1}{2}\sum \psi_{t|t-1}^{'}\omega_{t|t-1}^{-1}\psi_{t|t-1}$$

EQUATION 19

where $\psi_{t|t-1}$ is the prediction error and $\omega_{t|t-1}$ is its conditional variance.

For our model, however, the Markov variables $S_{1,t}$ and $S_{2,t}$, as the additional unobserved variables in the state space system, would undoubtedly complicate the estimation procedures. The prediction and updating processes in the Markov switching state space system now will additionally depend on both the previous and current values of the Markov variables. Since we have two independent Markov chain processes in our model, for given realizations of the two Markov variables at times t and $t-1$ ($S_{1,t-1}=i$, $S_{1,t}=j$, $S_{2,t-1}=i$ and $S_{2,t}=j$, where $i=\{0,1\}$, $j=\{0,1\}$), the Kalman filter equations can then be represented as follows

$$
\begin{aligned}
X_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} &= C + FX_{t-1|t-1}^{S_{1,t-1}S_{2,t-1}} \\
P_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} &= FX_{t-1|t-1}^{S_{1,t-1}S_{2,t-1}}F' + \Sigma^{S_{1,t}S_{2,t}} \\
\eta_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} &= Y_t - HX_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} \\
f_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} &= HP_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}H' \\
X_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} &= X_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} + \frac{P_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}H'}{f_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}}\eta_{t|t-1}^{S_{1,t-1}S_{2,t-1}} \\
P_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} &= P_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} - \frac{P_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}H'HP_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}}{f_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}}
\end{aligned}
$$

EQUATION 20

where $X_{t-1|t-1}^{S_{1,t-1}S_{2,t-1}}$ is the value of X_{t-1} based on the information up to time $t-1$, given that $S_{1,t-1}=i$ and $S_{2,t-1}=i$; $X_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ is the updated value of X_t based on the information up to time $t-1$, given that $S_{1,t-1}=i$, $S_{1,t}=j$, $S_{2,t-1}=i$ and $S_{2,t}=j$; $P_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ is the mean squared error of the unobserved $X_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ given $S_{1,t-1}=i$, $S_{1,t}=j$, $S_{2,t-1}=i$ and $S_{2,t}=j$; $\eta_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ is the prediction error of Y_t in the measurement equation, given the updated forecast of X_t as $X_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ conditional on $S_{1,t-1}=i$, $S_{1,t}=j$, $S_{2,t-1}=i$ and $S_{2,t}=j$ based on the information up to time $t-1$; $f_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ is the conditional variance of the forecast error $\eta_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$; $X_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ and $P_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ are the updated X_t and P_t based on the information up to time t, given that $S_{1,t-1}=i$, $S_{1,t}=j$, $S_{2,t-1}=i$ and $S_{2,t}=j$.

Since each iteration of the Kalman filter produces a 4-fold increase in the number of cases to consider[6], we reduce the 16 one-period posteriors $X_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ and $P_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ into 4 by taking appropriate approximations at the end of each iteration. This is computed through Kim's approximation procedures

[6] We have 4 cases to consider in each iteration of the Kalman filter: (1) both stationary and RW components are in high volatility regime; (2) stationary component is in the high volatility regime while the RW component is in the low volatility regime; (3) stationary component is in the low volatility regime while the RW component is in the high volatility regime; (4) both stationary and RW components are in low volatility regime. Therefore, every new iteration, the first order dependence of the current Markov chain variable on its previous value leads to a 4-fold increase in the number of cases to consider.

$$X_{t|t}^{S_{1,t}S_{2,t}} = \frac{\sum_{S_{1,t-1}=0}^{1}\sum_{S_{2,t-1}=0}^{1} \Pr\left(S_{1,t-1}=i, S_{1,t}=j, S_{2,t-1}=i, S_{2,t}=j \mid I_t\right) X_{t|t}^{S_{1,t}S_{1,t-1}S_{2,t}S_{2,t-1}}}{\Pr\left(S_{1,t}=j, S_{2,t}=j \mid I_t\right)}$$

EQUATION 21

$$P_{t|t}^{S_{1,t}S_{2,t}} = \sum_{S_{1,t}=0}^{1}\sum_{S_{2,t}=0}^{1}\left(P_{t|t}^{S_{1,t}S_{1,t-1}S_{2,t}S_{2,t-1}} + \left(X_{t|t}^{S_{1,t}S_{2,t}} - X_{t|t}^{S_{1,t}S_{1,t-1}S_{2,t}S_{2,t-1}}\right)\left(X_{t|t}^{S_{1,t}S_{2,t}} - X_{t|t}^{S_{1,t}S_{1,t-1}S_{2,t}S_{2,t-1}}\right)'\right)$$
$$\times \frac{\Pr\left(S_{1,t-1}=i, S_{1,t}=j, S_{2,t-1}=i, S_{2,t}=j \mid I_t\right)}{\Pr\left(S_{1,t}=j, S_{2,t}=j \mid I_t\right)}$$

EQUATION 22

where the probability terms in the above two equations are obtained from Hamilton's filter as

$$\Pr\left(S_{1,t-1}=i, S_{1,t}=j, S_{2,t-1}=i, S_{2,t}=j \mid I_t\right)$$
$$= \frac{\Pr\left(Y_t \mid S_{1,t-1}=i, S_{1,t}=j, S_{2,t-1}=i, S_{2,t}=j, I_{t-1}\right)\Pr\left(S_{1,t-1}=i, S_{1,t}=j, S_{2,t-1}=i, S_{2,t}=j \mid I_{t-1}\right)}{\Pr\left(Y_t \mid I_{t-1}\right)}$$

with

$$\Pr\left(Y_t \mid S_{1,t-1}=i, S_{1,t}=j, S_{2,t-1}=i, S_{2,t}=j, I_{t-1}\right)$$
$$= \frac{1}{\sqrt{(2\pi)^N \left| f_{t|t-1}^{S_{1,t}S_{1,t-1},S_{2,t}S_{2,t-1}} \right|}} \exp\left(-\frac{1}{2}\frac{\left(\eta_{t|t-1}^{S_{1,t-1}S_{2,t-1}}\right)'\left(\eta_{t|t-1}^{S_{1,t-1}S_{2,t-1}}\right)}{f_{t|t-1}^{S_{1,t}S_{1,t-1},S_{2,t}S_{2,t-1}}}\right),$$

$$\Pr\left(Y_t \mid I_{t-1}\right) = \sum_{S_{1,t-1}=0}^{1}\sum_{S_{1,t}=0}^{1}\sum_{S_{2,t-1}=0}^{1}\sum_{S_{2,t}=0}^{1} \Pr\left(Y_t, S_{1,t-1}=i, S_{1,t}=j, S_{2,t-1}=i, S_{2,t}=j \mid I_{t-1}\right)$$

and

$$\Pr\left(S_{1,t-1}=i, S_{1,t}=j, S_{2,t-1}=i, S_{2,t}=j \mid I_{t-1}\right)$$
$$= \Pr\left(S_{1,t}=j \mid S_{1,t-1}=i\right)\Pr\left(S_{2,t}=j \mid S_{2,t-1}=i\right)\Pr\left(S_{1,t-1}=i, S_{2,t-1}=i \mid I_{t-1}\right)$$

with

$$\Pr\left(S_{1,t-1}=i, S_{2,t-1}=i \mid I_{t-1}\right) = \sum_{S_{1,t-2}=0}^{1}\sum_{S_{2,t-2}=0}^{1} \Pr\left(S_{1,t-2}=i, S_{1,t-1}=i, S_{2,t-2}=i, S_{2,t-1}=i \mid I_{t-1}\right)$$

At the end of each iteration, Equation 21 and Equation 22 are used to collapse 16 one-period posteriors ($X_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ and $P_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$) into 4 ($X_{t|t}^{S_{1,t}S_{2,t}}$ and $P_{t|t}^{S_{1,t}S_{2,t}}$). As a by-product of the Hamilton filter, the approximate log likelihood function is given by

$$L(\theta)=\sum_{t=1}^{T}\ln f\left(Y_t \mid I_{t-1}\right)$$

that will be maximized with respect to the parameter vector space $\Theta=\left\{p_{1,00}, p_{1,11}, p_{2,00}, p_{2,11}, \delta, k, a, \sigma_{1,H}, \sigma_{1,L}, \sigma_{2,H}, \sigma_{2,L}, \rho_{12}\right\}$.

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