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Abstract

We compute the response of output to a monetary shock in a general equilibrium model in which firms set prices subject to a menu cost as well a costly observation of the state. We consider economies that are observationally equivalent with respect to the average frequency and size of price adjustments, and show that these economies respond differently to monetary shocks, depending on the size of the menu cost relative to the observation cost. A calibration on US data requires both costs to be present and predicts real effects that are more persistent than in the corresponding menu-cost model, but smaller than in the observation-cost model. The presence of the observation cost injects a time dependent component in the firms' decision rule which makes the impulse response quasi linear in the size of the shock.

JEL Classification Numbers: E5

Key Words: sticky prices, inattentiveness, monetary shocks, impulse responses.

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1 Introduction

Sticky price models are a cornerstone of macroeconomics which witnessed substantive advancements in the last two decades. New studies based on finely disaggregated micro data unveiled robust patterns on price setting behavior which now serve as a litmus test for state of the art macroeconomic models (see Klenow and Malin (2010) for a survey of the micro evidence). A new generation of models was developed to analyze the monetary transmission mechanism in settings that can reproduce key features of the micro data. Common to the new models is a prominent role for firm-level idiosyncratic shocks, which are essential to replicate the frequent and sizable price increases and decreases that appear in the cross sectional data. A main difference between the leading models is the nature of the friction that gives rise to the sticky prices. Some models focussed on the physical cost of price changes (such as menu or managerial costs), as in Golosov and Lucas (2007); Midrigan (2011); Kehoe and Midrigan (2010); Nakamura and Steinsson (2010). Other models were developed assuming that the key friction for sticky prices lies in the firm's imperfect information concerning state variables or information that others have, as in Woodford (2001, 2009); Reis (2006); Angeletos and La'O (2009); Mackowiak and Wiederholt (2009).

The nature of the friction is critical to the macroeconomic outcomes: it is known since Caplin and Spulber (1987) that the link between the micro economic stickiness (a given pattern of behavior at the firm-level) and the aggregate stickiness (the response of the aggregate CPI to shocks) is subtle. Indeed, we show that models that are observationally equivalent in terms of two popular statistics summarizing pricing behavior, frequency and size of price changes, produce very different predictions about the effects of monetary shocks depending on which friction is behind the sticky prices.[1]

[1]Golosov and Lucas (2007) first showed that real effects of monetary shocks are small and short lived in menu cost models parametrized to fit the micro data. Others, such as Mankiw and Reis (2006); Mackowiak, Moench, and Wiederholt (2009), have shown that information frictions (underlying sticky prices) are more successful at generating a large output response to monetary shocks. See also Midrigan (2011), Kehoe and Midrigan (2010), Nakamura and Steinsson (2010), Gertler and Leahy (2008) for versions of the menu cost model that can generate substantial non-neutrality and Mankiw and Reis (2010) for a review of imperfect information models and the Phillips curve.

This paper proposes a general equilibrium model in which firms face two frictions for price setting: a menu-cost and a specific information friction, namely an observation-cost.[2] As usual, the menu cost generates an sS type behavior, inducing price adjustments only when the markup is further away from the optimal one. Instead, the observation costs induces the firm to gather infrequently the information which is necessary to optimize the posted prices. The simultaneous presence of observation and menu costs produces complementarities that change the predictions of simpler models featuring only one cost in a non-trivial way. For instance infrequent information gathering, an activity we label "price-review", may reflect a high menu cost rather than a high observation cost. It is essential for the quantitative analysis that observation and menu costs are both identified. In Alvarez, Lippi, and Paciello (2011) we analytically characterized the single-firm decision problem and we studied a theoretical mapping to quantify each of these costs using statistics on price adjustments and price reviews. This paper features two main novelties compared to that contribution: first, we quantify the role of each friction using a micro dataset on the firms' price setting activity which distinguishes between the frequency of price adjustments and the frequency of price reviews. Second, we nest the firm's decision problem into a general equilibrium framework, solve for aggregation, and study (numerically) the economy's response to monetary shocks. Different parametrizations of our model span from the pure menu cost model of Golosov and Lucas (2007) to the pure observation cost model of Reis (2006), and the continuum in between.[3] These models imply very different "Phillips curves" (correlation between nominal wage changes and output changes in the model). A calibrated version of the model is used to select between these models and to quantify the real effects of monetary shocks.

There are two main results in the paper. First, economies that are observationally equivalent with respect to the average frequency and size of price adjustments are characterized by

[2] This specification of the information friction closely relates to the *sticky information* literature initiated by Caballero (1989); Reis (2006). Mathematically this amounts to assume that the decision maker cannot condition his choices on the state variables unless a fixed cost is paid. The economic nature of this friction captures the costs of acquiring, gathering and processing information, as in Reis (2006).

[3] For instance, a special parametrization of our model can also nest the model by Bonomo and Carvalho (2004) where each price review coincides with an adjustment.

different responses to nominal shocks, depending on the ratio of observation to menu costs. The larger the observation cost relative to the menu cost, the larger and the more persistent the output response to a monetary shock. This is an instance of the subtle linkages between micro stickiness and macro stickiness mentioned above which originates from the different responsiveness to news of the optimal decision rules in the different models. Calibrating the model to reproduce frequency and size of price adjustments for the US requires observation costs that are about three times larger than menu costs. The resulting output response to an unexpected (small) increase of the money supply is above the impulse response of the menu cost model, but smaller than the impulse response of the observation cost model. Second the shape of the impulse response is not just a "simple average" of the shapes in the models with menu cost only and observation cost only: over a wide range of values for observation and menu costs, the shape largely resembles the profile of the impulse response in a model with observation cost only. At our baseline parametrization, the time-dependent component of the adjustment rule dominates the state-dependent component of the adjustment rule in determining the shape of the output response to the monetary shock, so that the shape of the impulse response is roughly linear. This linearity, akin to the one in *Taylor* type price adjustments, implies that the effects on output are much more persistent than in the pure menu cost model. In particular, the presence of a significant time-dependent component of the decision rule implies that the size of the real effects of monetary shocks is roughly proportional to the size of the shock. In this sense the model behaves very differently from a pure menu-cost model (see Figure 4).[4]

Our paper relates to a rich strand of literature that studies the role of information frictions in the propagation of aggregate shocks.[5] In our framework firms pay attention to the state only infrequently and, when they do, they receive a perfect signal on the relevant state of the price setting decision. Our approach differs from the *rational inattention* literature

[4]The purely state-dependent nature of the decision rules in the menu cost model makes the economy response highly non-linear in the size of monetary shocks.

[5]Early contributions include Phelps (1969), Lucas (1972), Barro (1976) and Townsend (1983).

that followed Sims (2003), where agents can process a flow of new information every period, as we aim to obtain infrequent price reviews and relate them to infrequent price changes. Our technology of information acquisition does not allow firms to allocate their attention across different types of shocks that impact their price setting decision, as in Mackowiak and Wiederholt (2009). We also abstract from strategic interactions in the price setting and information acquisition decisions which, as in Hellwig and Veldkamp (2009), can create a wedge between the precision of available information about the nominal shock and the degree of price inertia.

Our paper also relates to a growing literature studying the propagation of aggregate shocks in economies that feature both sticky prices and information frictions. Several authors combine nominal rigidities with informational frictions (generally different from the specific one we chose). Klenow and Willis (2007) allow for exogenously different frequencies of review of idiosyncratic and aggregate shocks. Angeletos and La'O (2009) highlight the distinct role of higher-order beliefs in price setting (originating from strategic complementarity). Hellwig and Venkateswaran (2009, 2011) assume that firms perfectly observe the endogenous transaction prices. Dupor, Kitamura, and Tsuruga (2010) combine exogenously random times of observation as in Mankiw and Reis (2002) with sticky prices as in Calvo (1983). A closely related paper in this literature is Demery (2012) who, building on the results of Alvarez, Lippi, and Paciello (2011), studies how the real effects of monetary shocks depend on the relative size of observation and menu costs, as we do here. He finds that the output effect of monetary shocks is largest when, for given size and frequency of price changes, both the observation and the menu cost are positive, whereas we find that it is largest when the menu cost is zero. When calibrated to match the average size and frequency of price changes in the US, he concludes that the output effect of monetary shocks is similar to that of a model with *Calvo* pricing. We instead find that the output effect is substantially smaller, i.e. smaller than that of a model with *Taylor* pricing. We argue that the different results do not stem from different assumptions: Demery's conclusion is flawed by the solution method he used, and it is internally inconsistent with other results of his own paper.[6]

The rest of the paper is organized as follows. The next section reviews the evidence on the price setting activities using firm level evidence that is useful to parametrize the model and identify its frictions. In Section 3 we describe the model, and characterize the firm's optimal decision rules as well as the general equilibrium. Section 4 discusses the choice of parameters and the calibration to the US economy. Section 5 computes the output response to a monetary shock using the model calibration for the US economy. Section 6 briefly reviews the scope and robustness of the results and some avenues for future research.

2 Evidence on price adjustments vs. price reviews

This section uses survey data to document that firms review and adjust their price infrequently. A price review is an activity related to the firm's information gathering and processing that is necessary to evaluate the current price policy. A robust finding of this section is that the frequency of price reviews is larger than the frequency of price adjustments. In Section 4 we will use these data to calibrate our model.

Table 1: Number of price-reviews and price-adjustments per year

	AT	BE	FR	GE	IT	NL	PT	SP	EURO	CAN	U.K.	U.S.
	Medians											
Review	4	1	4	3	1	4	2	1	2.7	12	4	2
Adjust	1	1	1	1	1	1	1	1	1	4	2	1.4
	Percentage of firms with at least 4 price reviews or adjustments											
Review	54	12	53	47	43	56	28	14	43	78	52	40
Adjust	11	8	9	21	11	11	12	14	14	44	35	15

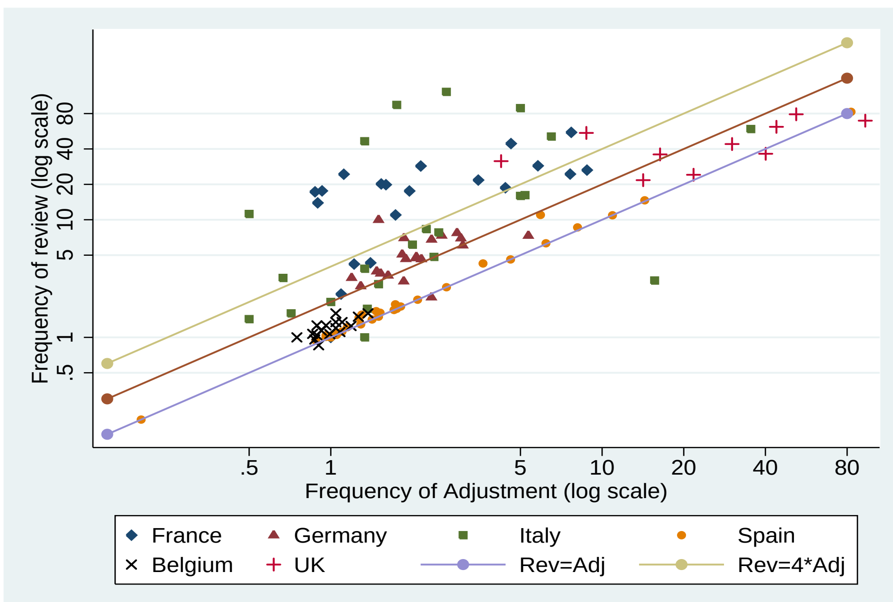
Note: Medians are computed over number of price adjustments and reviews per year. Sources: Fabiani et al. (2007) for the Euro area; Amirault, Kwan, and Wilkinson (2006) for Canada; Greenslade and Parker (2008) for the U.K.; Blinder et al. (1998) for the U.S.

The upper panel of Table 1 reports the median yearly frequencies of price reviews and adjustments across all firms in surveys taken from various countries. The typical survey

[6] See Appendix D for a more detailed comparison of results and a critical review.

question asks firms: "In general, how often do you review the price of your main product (without necessarily changing it)?"; with possible choices being yearly, semi-yearly, quarterly, monthly, weekly and daily. The same surveys contain questions on the frequency of price changes, allowing a comparison to the frequency of reviews. At the median, a firm in the Euro area reviews its price a bit less than three times a year, but changes its price only about once a year. The U.K., the U.S. and Canada have higher frequency of price changes than the Euro area, but also higher frequency of reviews, so that on average firms review more frequently than they adjust their price.

Figure 1: Price-reviews vs. price-changes across industries



Note: data for each dot are the mean number of price changes (or price reviews) in a given industry (NACE 2 digits) and country. Source: our calculations based on the individual firm data described in Loupias and Ricart (2004), Stahl (2009), Fabiani, Gattulli, and Sabbatini (2004), Alvarez and Hernando (2005), Greenslade and Parker (2008) and Aucremanne and Druant (2005).

We notice that the comparison between the median frequencies of adjustments and reviews may be subject to measurement error because firms are often asked to choose the frequency of reviews among a discrete set of alternatives (e.g. daily, weekly, etc.), whereas they are asked to report a number with no restriction for the frequency of price adjustments.[7] As a

[7] See Appendix A for a discussion on the measurement error in the survey data.

robustness, the bottom panel of Table 1 reports the fraction of firms reviewing the price, and the fraction of firms changing the price, at least four times a year. It shows that the mass of firms reviewing prices at least four times a year is substantially larger than the corresponding one for price changes, across all countries.

Next, we document that the frequency of price reviews is consistently higher than the frequency of price adjustments also at the industry and firm level. Figure 1 documents this fact across a number of industries (2 digits NACE classification) in six OECD countries. Using firm level data Table 2 classifies the answers of each firm in a sample of 4 countries in three mutually exclusive categories: (1) firms for which the frequency of price changes is greater than the frequency of price reviews; (2) firms for which the two frequencies are equal, and (3) firms that change prices less frequently than they review them. The table shows that for the large majority of firms in the sample the frequency of price reviews is greater than the frequency of price adjustment.

Table 2: Relative frequency of Price Changes and Price Reviews (Firm Level data)

	Belgium	France	Germany	Italy	Spain*
Percentage of Firms with:					
(1) Change > Review	3	5	19	16	0
(2) Change = Review	80	38	11	38	89
(3) Change < Review	17	57	70	46	11
N. of firms	890	1,126	835	141	194

Each column reports the percentage of firm-level records for which the frequency of price changes is greater, equal or smaller, than the frequency of price reviews. Sources: Table 17 in Aucremanne and Druant (2005) for Belgium, and our calculations based on the individual firm data described in Loupias and Ricart (2004), Stahl (2009), and Fabiani, Gattulli, and Sabbatini (2004) for France, Germany, and Italy respectively; Section 4.4 of Alvarez and Hernando (2005) for Spain. *For Spain we only report statistics for firms that review four or more times a year.

The evidence in this section indicates that firms review the level of their prices more often than they adjust them. This information is useful for modeling purposes: the data display very little presence of price changes in the absence of a review of information, a behavior the literature refers to as “price plans” or “indexation”.

3 The model

This section describes the model, the general equilibrium and the monetary shock. We consider firms that set prices under two frictions: a standard fixed cost of adjusting the price, inducing infrequent price adjustments, and a fixed cost of observing the state, inducing infrequent information acquisition. In the model each firm plans about two related choices: observing the state and adjusting the price. Our model is a general equilibrium version of the price setting problem studied in Alvarez, Lippi, and Paciello (2011), and embeds as special cases the "menu cost" model (e.g. Barro (1972); Dixit (1991)) as well as the "observation cost" model (e.g. Caballero (1989); Bonomo and Carvalho (2004); Reis (2006)). The menu cost model aggregates similarly to Golosov and Lucas (2007) and provides a useful benchmark of comparison since the predictions of this model have been extensively studied in the literature. The observation cost model is a general equilibrium version of Reis's (2006) inattentive producers model which, with a constant fixed cost of observing the state, features reviews at approximately uniformly distributed times, and therefore behaves similarly to Taylor's (1980) staggered price model. We consider on an economy where money grows at the constant rate μ, and study the output effects of a one time unexpected permanent increase in money supply produced by models with different combinations of observation and menu costs, including the two special cases where one of the costs is zero.

There are two types of agents in this economy, a representative household and a unit mass of monopolistically competitive firms, each producing a different variety of consumption good. Firm i's output at time t is given by $Y_{i,t} = z_{i,t}\, l_{i,t}$, where $l_{i,t}$ is the labor employed by firm i in production, and $z_{i,t}$ is an idiosyncratic productivity evolving according to

$$d\,log(z_{i,t}) = \gamma\, dt + \sigma\, dB_{i,t} \ , \tag{1}$$

where $B_{i,t}$ is a standard brownian motion with zero drift and unit variance, the realizations of which are independent across firms.

3.1 The household problem

We assume that (real) aggregate consumption c_t is given by the Spence-Dixit-Stiglitz consumption aggregate

$$c_t = \left[\int_0^1 (A_{i,t}\, C_{i,t}))^{(\eta-1)/\eta}\, di \right]^{\eta/(\eta-1)} \quad \text{with} \quad \eta > 1 \ , \tag{2}$$

where $C_{i,t}$ denotes the consumption of variety i at time t. There is a preference shock $A_{i,t}$ associated to good i at time t, which acts as a multiplicative shifter of the demand of good i. We assume that $A_{i,t} = 1/z_{i,t}$, so the (log) of the marginal cost and the demand shock are perfectly correlated.[8]

Household's preferences over time are given by

$$\int_0^\infty e^{-\rho t} \left[\frac{c_t^{1-\epsilon}}{1-\epsilon} - \xi\, L_t + \log\left(\frac{\hat{m}_t}{P_t}\right) \right] dt \quad \text{with} \quad \rho > 0 \ , \tag{3}$$

where period t utility depends on consumption, c_t, labor supply, L_t, and cash holdings $\hat{m}_t$ deflated by the price index $P_t = \left[\int_0^1 \left(A_{i,t}^{-1}\, p_{i,t} \right)^{(1-\eta)} di \right]^{1/(1-\eta)}$. The household has perfect foresight on the path of money, nominal wages, nominal interest rates, nominal lump-sum subsidies and aggregate nominal profits. Financial markets are complete, in the sense that all profits of firms are held in a diversified mutual fund. Since all aggregate quantities are deterministic, the budget constraint of the representative agent is:

$$\hat{m}_0 \ \ \geq \ \ \int_0^\infty Q_t \left[\int_0^1 p_{i,t}\, C_{i,t}\, di + R_t \hat{m}_t - \mu m_t - W_t L_t - D_t \right] dt \ , \tag{4}$$

[8] We introduce this assumption for several reasons. First, the cross-section distribution of outputs will be stationary, and the maximum static profit of the firms is constant across the different productivity levels. Second, this version of the model with correlated demand and cost shocks has been analyzed in the literature by several authors (see Woodford (2009), Bonomo, Carvalho, and Garcia (2010) Midrigan (2011), Alvarez and Lippi (2014)), so it makes the results for our benchmark case comparable to the existing literature. Nevertheless in the Online Appendix G we solve the model without preference shocks, i.e. $A_{i,t} = 1$ for all i and t, and conclude that the assumption on preference shocks is irrelevant for the quantitative predictions of our benchmark economy.

where $Q_t = \exp\left(-\int_0^t R_s\, ds\right)$ is the time zero price of a dollar delivered at time t, R_t is the instantaneous risk-free net nominal interest rate (and hence the opportunity cost of holding money), m_t is the stock of money supply, W_t is the nominal wage, and D_t is aggregate nominal net profits rebated from all firms to households. The household chooses the buying strategy, $C_{i,t}$, labor supply, L_t, and money-holding, $\hat{m}_t$, so to maximize equation (3), subject to equation (4), and taking prices Q_t, P_t, R_t, W_t, and initial money holdings, $\hat{m}_0$, as given.

Using the equilibrium condition in the money market, $\hat{m}_t = m_t$, the first order condition for money holdings reads

$$e^{-\rho t}/m_t = \zeta\, Q_t R_t \ , \tag{5}$$

where ζ is the Lagrange multiplier of equation (4). The first order conditions for consumption and labor supply are given by

$$e^{-\rho t}\, c_t^{1/\eta-\epsilon}\, C_{i,t}^{-1/\eta}\, z_{i,t}^{-1+1/\eta} = \zeta\, Q_t\, p_{i,t} \ , \tag{6}$$

$$e^{-\rho t}\xi = \zeta\, Q_t\, W_t \ . \tag{7}$$

Taking logs and differentiating w.r.t. time equation (5) one obtains the following o.d.e., $\dot{R}_t = R_t\,(R_t - \mu - \rho)$, which has two steady states, zero and $\rho + \mu > 0$. The steady state $\rho + \mu$ is unstable: if $0 < R(0) < \rho + \mu$ then it converges to zero, and if $R(0) > \rho + \mu$ it diverges to $+\infty$. Thus, there exists an equilibrium where regardless of the sequence of firm prices, $p_{i,t}$, we have:

$$R_t = R = \rho + \mu \quad , \quad W_t = \xi\, R\, m_t \ , \quad \text{and} \quad \zeta = \frac{1}{m_0 R} \ . \tag{8}$$

An important property of the equilibrium conditions in equation (8) is that the equilibrium wage is proportional to the money supply, and thus its dynamics are exogenously determined. This property simplifies the solution of the model substantially, as there are no general equilibrium feedback effect through nominal wages into firms' pricing decisions.

3.2 The firm problem

In this section we analyze the price setting problem of the firm. We first define the firm profits, then we describe the information structure and the price adjustment technology, and finally present the dynamic programming problem of the firm.

The firm's per period nominal profit, scaled by the economy money supply m_t, is

$$\Pi_{i,t} = c_t^{1-\epsilon\eta} z_{i,t}^{1-\eta} R \left(\frac{p_{i,t}}{R\, m_t}\right)^{-\eta} \left(\frac{p_{i,t}}{R\, m_t} - \frac{\xi}{z_{i,t}}\right), \tag{9}$$

where we used equations (6)-(7) to obtain an expression for the firms' demand $C_{i,t}$, and the equilibrium condition in equation (8) to obtain an expression for the nominal wage. The price that maximizes the firm's profit in equation (9) is given by

$$p_{i,t}^* \equiv \frac{\eta}{\eta - 1} \frac{R m_t}{z_{i,t}} \xi \ . \tag{10}$$

Next we substitute $p_{i,t}^*$ into equation (9) to express the firm's profit as

$$\Pi\left(\frac{p_{i,t}}{p_{i,t}^*}, c_t\right) \equiv c_t^{1-\epsilon\eta} F\left(\frac{p_{i,t}}{p_{i,t}^*}\right) , \tag{11}$$

where the period profit only depends on two state variables, and the function $F(\cdot)$ is

$$F(x) \equiv R\, \xi^{1-\eta} \left(x \frac{\eta}{\eta - 1}\right)^{-\eta} \left(x \frac{\eta}{\eta - 1} - 1\right) \ .$$

The firm profit only depends on the ratio $p_{i,t}/p_{i,t}^*$, and on the aggregate level of consumption. We will refer to (the log of) the ratio $p_{i,t}/p_{i,t}^*$ as to the "price gap", and denote it by $g_{i,t} \equiv \log(p_{i,t}/p_{i,t}^*)$, so that $g = 0$ is the gap that maximizes the period profits. It follows from the definition of g, and from the laws of motion of W and z, that the dynamics of g for any firm

i, when firm i is not adjusting the price, are given by

$$dg_{i,t} = (\gamma - \mu)\, dt + \sigma dB_{i,t}\ . \tag{12}$$

We notice that the function $F(x)$ has a unique maximum at $x = 1$, so that $\Pi^*(c_t) \equiv \Pi(1, c_t)$ is the maximum profit per period. The maximum profit $\Pi^*(c_t)$ is independent of the firm's idiosyncratic state z, but only vary if the aggregate consumption vary. This is because, at the profit maximizing price, the idiosyncratic demand shock faced by each firm exactly offsets the effect on profit of an idiosyncratic shock to productivity. Finally, we denote by $\bar{\Pi}$ the profit evaluated at the steady state consumption $\bar{c}$, i.e. $\bar{\Pi} \equiv \Pi^*(\bar{c})$.

3.2.1 The costs of price adjustment and price reviews

Each firm faces two frictions. First, we assume that paying attention to economic variables that are relevant for the price setting decision is costly. Second, the firm has to pay a menu cost anytime it changes its price. We model this framework along the lines of Alvarez, Lippi, and Paciello (2011). In particular, we assume that firms do not observe their productivity $z_{i,t}$, or other variables informative about the firms' relevant state, unless they decide to undertake a costly action, which we refer to as a review. After paying the observation cost the firm learns perfectly the current value of z. Firms have no information on the realizations of idiosyncratic productivity shocks until the next review. A review requires a fixed amount of labor. Given that the cost of labor scaled by the money supply is a constant, we can express the value of the observation cost as a fraction of the steady state profit: $\theta\, \bar{\Pi}$, where $\theta > 0$ is a parameter. Similarly to the observation cost, each price change requires a fixed amount of labor. We express the value of this cost as a fraction of steady state frictionless profits: $\psi\, \bar{\Pi}$, where $\psi > 0$ is a parameter.

3.2.2 The firm recursive problem

Under our assumptions no new information arrives between review dates. In principle, even absent new information, the firm could implement some price changes between two review dates, e.g. to keep track of predictable changes in the price gaps, such as those due to its drift. We showed in Alvarez, Lippi, and Paciello (2011) that as long as the drift is "small" relative to its variance the firms will find it optimal to adjust their price only upon review of the state, so that "price plans" will not be implemented.[9] Notice that this assumption is consistent with the empirical evidence on the average frequency of price reviews and adjustments, discussed in the previous section. Thus, to ease notation, we set up the firms' problem so that no price adjustment occurs between review dates.

Let $\{\tau_{i,n}\}$ denote the dates where the subsequent reviews will take place. The subindex n denotes the n^{th} review date, while the subindex i denotes a firm. These stopping times satisfy $\tau_{i,n-1} \leq \tau_{i,n} \leq \tau_{i,n+1}$. Thus, upon reviewing the state in period t, the value of a firm i with price gap g is given by $V_t(g) = \max\{\hat{V}_t\,,\, \bar{V}_t(g)\}$, where $\hat{V}_t$ is the value of the firm conditional on adjusting the price,

$$\hat{V}_t = -(\theta + \psi)\,\bar{\Pi} + \max_{T,\hat{g}} \int_0^T e^{-r\,s}\,\mathbb{E}\left[\Pi(e^{g_{i,t+s}}, c_{t+s}) \mid g_{i,t} = \hat{g}\right] ds + e^{-r\,T}\,\mathbb{E}\left[V_{t+T}(g_{i,t+T}) \,|\, g_{i,t} = \hat{g}\right] \;,$$

and $\bar{V}_t(g)$ is the value conditional on g and not adjusting the price,

$$\bar{V}_t(g) = -\theta\,\bar{\Pi} + \max_{T} \int_0^T e^{-r\,s}\,\mathbb{E}\left[\Pi(e^{g_{i,t+s}}, c_{t+s}) \,|\, g_{i,t} = g\right] ds + e^{-r\,T}\,\mathbb{E}\left[V_{t+T}(g_{i,t+T}) \,|\, g_{i,t} = g\right] \;,$$

where r is the equilibrium real discount rate which, from the household first order conditions, is equal to ρ. The firm value depends on the expected discounted sum of firm's profits which, from equation (11), depend on the path of the price gap $\{g_{i,t+s}\}$ and on the path of aggregate

[9]See Section III of that paper. We also showed that for a wide range of parameters that are consistent with low inflation economies such as those discussed in Section 2, *price plans* or *indexation* would not be optimal. See Appendix C for a mapping from the model of this paper to the framework of Alvarez, Lippi, and Paciello (2011).

consumption $\{c_{t+s}\}$. We notice that the value function $V_t(g)$ depends on time t *only* because of the effect of $\{c_{t+s}\}$: given perfect foresight, the current time t is enough to infer the future dynamics of the aggregate state. In steady state, i.e. when $c_t = \bar{c}$, the solution of the stationary value function characterizes completely the firm problem.[10]

In the next proposition we use the process of the price gap $g_{i,t}$ in equation (12) and the profit in equation (11) to express the firm problem as a function of the structural parameters. The homogeneity of the value functions with respect to the level of the steady state profits is used to normalize the value functions and reduce the state space.

Proposition 1. Consider the problem of firm i evaluated at a review date $t = \tau_{i,n}$, for a given path of aggregate consumption $\{c_{t+s}\}_{s\geq 0}$. Let $v_t(g) \equiv V_t(g)/\bar{\Pi}$, $\bar{v}_t(g) \equiv \bar{V}_t(g)/\bar{\Pi}$ and $\hat{v}_t(g) \equiv \hat{V}_t(g)/\bar{\Pi}$. The firm maximizes the value function $v_t(g) = \max\{\hat{v}_t\ ,\ \bar{v}_t(g)\}$, where:

$$\begin{aligned} \hat{v}_t &= -\theta - \psi + \max_{T,\hat{g}} \int_0^T e^{-rs}\left(\frac{c_{t+s}}{\bar{c}}\right)^{1-\epsilon\eta} f(\hat{g},s)ds + \\ &+ e^{-rT}\int_{-\infty}^{\infty} v_{t+T}\left(\hat{g} + (\gamma-\mu)T + \sigma\sqrt{T}x\right) dN(x)\ , \end{aligned} \tag{13}$$

$$\begin{aligned} \bar{v}_t(g) &= -\theta + \max_{T} \int_0^T e^{-r\,s}\left(\frac{c_{t+s}}{\bar{c}}\right)^{1-\epsilon\eta} f(g,s)ds\ + \\ &+ e^{-rT}\int_{-\infty}^{\infty} v_{t+T}\left(g + (\gamma-\mu)T\ + \sigma\sqrt{T}\,x\right)\, dN(x)\ , \end{aligned} \tag{14}$$

and

$$f(g,s) \equiv \eta\, e^{(\eta-1)\left(\left(\mu-\gamma+\frac{\sigma^2}{2}(\eta-1)\right)s-g\right)} - (\eta-1)e^{\eta\left(\left(\mu-\gamma+\frac{\sigma^2}{2}\eta\right)s-g\right)},$$

while $N(\cdot)$ is the CDF of a standard normal.

The proof of the proposition follows immediately from the recursive firm problem described above. The term involving $\{c_{t+s}/\bar{c}\}$ reflects the impact of aggregate consumption on discounted profits. At standard parameter values we have $\epsilon\eta > 1$ which implies that higher

[10] We study a version of the steady-state problem of equation (13) in Alvarez, Lippi, and Paciello (2011). In that paper the period profit was assumed to be a quadratic function of g, which can be derived as a second order approximation to $\Pi(\cdot,\bar{c})$ of equation (11).

expected growth in aggregate consumption is associated to lower expected discounted profits, as the firm discounts states of the world with higher aggregate consumption more, as can be seen from equation (11).

The function $f(g,s) = \mathbb{E}\left[\Pi(e^{g_{t+s}}, \bar{c})/\Pi^*(\bar{c}) \,|\, g_t = g\right]$ is a measure of the expected growth of profits s periods ahead, conditional on inaction and constant aggregate consumption. The first term of $f(g,s)$ depends on the expected growth in real revenues. The second term of $f(g,s)$ depends on the expected growth in real marginal cost. We notice that $f(g,0)$ is maximized at $g = 0$ where it takes the value of 1. In choosing whether to adjust the price, the firm trades-off higher expected profits from changing the price against the menu cost ψ, giving rise to an Ss type of adjustment rule. The optimal decision rule for each review time is described by three values for the price gap, $\underline{g}_t < \hat{g}_t < \bar{g}_t$, and a function $\mathrm{T}_t(g)$, where the decision rules may vary over time because of variation in the aggregate state c_t. After observing its price gap g at t, the firm leaves its price unchanged if $g \in (\underline{g}_t\,,\, \bar{g}_t)$. Otherwise the firm changes its price gap to $\hat{g}_t$. The function $\mathrm{T}_t(g)$ gives the (optimally chosen) time the firm will wait until the next review as a function of the price gap after the adjustment decision. In Alvarez, Lippi, and Paciello (2011) we provide an analytical characterization for these decision rules in the steady state where $c_t = \bar{c}$.

3.3 Equilibrium

The equilibrium is such that the household supplies labor L_t to satisfy the demand from all the firms, and each firm i supplies goods so that its output satisfies demand, i.e. $C_{i,t} = Y_{i,t}$ for each i, t. As discussed above the equilibrium nominal wages and interest rates are given by equation (8), whereas the equilibrium prices $p_{i,t}$ of the different varieties are determined by the solution to the firm problem in Proposition 1 and depend on the path for the aggregate consumption $\{c_t\}$. In turn, using equation (2) and the household's first order conditions in equations (5)-(6) for optimal demand $C_{i,t}$, the equilibrium aggregate consumption c_t depends on the equilibrium prices $p_{i,t}$ of the different varieties. Thus we have a fixed point problem:

finding that aggregate consumption sequence $\{c_t\}$ that generates optimal pricing decisions $p_{i,t}$ that are consistent with $\{c_t\}$.

We are now ready to describe the consistency condition implied by the equilibrium, i.e. a mapping from policies to a path of aggregate consumption $\{c_t\}$. As a consequence of the first order conditions in equations (5)-(6), and of the definition of c_t, the path of consumption has to satisfy

$$c_t = \left(\int \left(\xi \, \frac{\eta}{\eta - 1} \, e^{g} \right)^{1-\eta} \phi_t(dg) \right)^{\frac{1}{\epsilon\,(\eta-1)}} ,$$

where $\phi_t(\cdot)$ is the cross-sectional distribution of price gaps, $g_{i,t}$, in period t. The cross-sectional distribution of $g_{i,t}$ at any $t > t_0$ is determined by an initial condition $\phi_{t_0}(\cdot)$ and the equilibrium law of motion for $g_{i,t}$. We notice that the dynamics of $g_{i,t}$ are given by equation (12) in the inaction region, while they depend on the firm policy $\{\underline{g}_t, \hat{g}_t, \bar{g}_t, \mathrm{T}_t(\cdot)\}$, at firm i's review dates, which in turn depend on the path of aggregate consumption. An equilibrium consists of a fixed point in the sequence $\{c_t\} \rightarrow$ policy rules $\rightarrow \{c'_t\}$ such that $\{c'_t\} = \{c_t\}$. In particular, a steady state equilibrium is characterized by an invariant distribution $\phi_t(\cdot) = \bar{\phi}(\cdot)$ for each t, so that $c_t = \bar{c}$.

As noticed by Golosov and Lucas (2007), we remark that the cross-sectional distribution $\phi_t(\cdot)$ enters the firm problem in Proposition 1 only as a determinant of aggregate consumption c_t. This simplifies the numerical solution of the problem as firms do not need to form expectations based on the law of motion for the cross-sectional distribution, but only on the path of a scalar: $\{c_t\}$. Moreover, while our definition of equilibrium and our numerical solution take this general equilibrium feedback fully into consideration, its effect on the decision rules is very small for realistic monetary shocks. The result that the general equilibrium effects are negligible for small monetary shocks is formally established in closely related set-ups in Gertler and Leahy (2008) and Alvarez and Lippi (2014).[11]

[11] See proposition 1 in Gertler and Leahy (2008) and proposition 7 in Alvarez and Lippi (2014).

3.4 The monetary shock

Here we describe the policy experiment we will perform in the paper. We start the economy in steady state at some $t = t_0$ where economic agents expect $c_t = \bar{c}$ and $\phi_t(g) = \bar{\phi}(g)$ for all $g \in (-\infty, +\infty)$ and all $t \geq t_0$. The monetary shock takes the form of an unforeseen, one time, permanent increase in the stock of money supply so that

$$\log(m_t) = \begin{cases} \log(m_{t-T}) + \mu\,(t-T) + \delta & \text{for all } t \geq t_0 \text{ and } T > 0 \\ \log(m_{t-T}) + \mu\,(t-T) & \text{for all } t < t_0 \text{ and } T > 0 \end{cases},$$

where δ is the log-difference in money supply upon the realization of the shock. We assume that firms only learn about the realization of the monetary shock after their first review. This is equivalent to assume that, in the spirit of the *rational inattentiveness* literature, firms don't pay attention to the changes in these variables, or in their own profits, unless they pay the observation cost.[12] Using equation (8), we notice that the shock to the level of money supply causes a proportional change in the nominal wage on impact, after which the nominal wage grows at the rate of growth of money supply, μ. As the profit-maximizing price in equation (10) is a constant markup over nominal marginal cost, the monetary shock causes a parallel shift of size δ in the distribution of price gaps $\phi_{t_0}(g)$. For instance, an increase in money supply of δ log-points causes a decrease on impact of size δ to the log-price gap for all i at $t = t_0$. In solving for the response of the economy to the monetary shock, we will compute the dynamics of the distribution of price gaps $\phi_t(g)$, and the associated path for c_t, for all $t \geq t_0$ until the economy converges back to the steady state.

As mentioned, firms are not aware of the monetary shock until their first review after the shock has occurred. Because of this, the time of their first review after the monetary shock is unaffected, so that the firm will take no action before then. Upon the first review after t_0,

[12] In Appendix F we consider the case where firms learn about the exact realization of the aggregate shock to m_t immediately at the time of the monetary shock and that have perfect foresight as households with respect to aggregate variables.

the firm learns about the aggregate shock and revises its beliefs so that the firm problem in Proposition 1 at any review date $\tau_{i,t} \geq t_0$ is based on perfect foresight of the path of c_t. In Appendix F we consider an alternative setup in which all firms are perfectly informed about the realization and size of the unexpected monetary shock: for small monetary shocks the results are essentially identical.

4 A calibration for the US economy

This section presents a calibration of the model fundamental parameters that matches some key statistics on price setting behavior from the U.S. economy. We use this calibrated model in the next section to study how the aggregate economy responds to a monetary shock. We set $\eta = 4$ so that the average price markup is roughly one third, i.e. between the values used by Midrigan (2011) and Golosov and Lucas (2007). Following Golosov and Lucas (2007), we set $\epsilon = 2$ so to have an intertemporal elasticity of substitution of $1/2$, and $\xi = 6$ so that households allocate approximately $1/3$ of the unit time endowment to work in steady state. We set the yearly discount rate to $\rho = 0.02$.

We next discuss the calibration of μ and γ. The growth rate of the money supply is chosen to target an yearly inflation (of the price index P_t) equal to 2%, implying $\mu = 0.02$. For a given value of μ, the value of γ determines the incentives of firms to use *price plans* between consecutive review dates. As extensively discussed in Alvarez, Lippi, and Paciello (2011), at the baseline calibration of the menu cost ψ, price adjustments occur only upon reviews for a large and empirically reasonable range of values of γ and μ, as implicitly conjectured in the firm problem of Proposition 1: at $\mu = 0.02$, γ should be larger than 10% for a price plan to be optimal. Moreover, the frequency of price adjustments and reviews has a near zero elasticity with respect to μ and γ in that range.[13] Thus the qualitative and quantitative results of the baseline model with observation and menu cost are not affected by the choice of γ, so long

[13] In Table I of Alvarez, Lippi, and Paciello (2011) we show that the frequency of price adjustments and reviews has a near zero elasticity with respect to the drift (inflation) for values of the drift smaller than 10%.

such value is not extremely large. As we noticed in Section 2, this property of the model appears consistent with the evidence (on the scant evidence on price plans) in several low inflation economies. Given the ample range of values of γ consistent with no price plans, we decided to set $\gamma = \mu + (2\eta - 1)\sigma^2/2$ so that *price plans* are not optimal in our model *even* if the menu cost ψ is arbitrarily small.[14] This choice has the advantage of making the polar case with positive observation cost and zero menu cost able to reproduce the same frequency of price changes of our baseline model.

We choose the remaining parameters, θ, ψ and σ, so that the steady steady moments from our model match some key U.S. statistics about the frequency and size of price adjustments, as well as on the frequency of price reviews. An analytical result in Alvarez, Lippi, and Paciello (2011) shows that the ratio between the frequency of price reviews and adjustments identifies the ratio of menu to observation costs: the larger the ratio of the frequency of price reviews to adjustments, the larger the ratio ψ/θ. For a given value of ψ/θ, the frequency and average size of price adjustments identify the *levels* of θ and ψ, as well as the volatility of the state, σ. Thus, we target the average number of price adjustments (denoted by n_a) and reviews (denoted by n_r) per year implied by the estimates of Blinder et al. (1998) for a sample of U.S. firms reported in Table 1, i.e. $n_a = 1.4$ and $n_r = 2$. The target for the average size of price changes, measured by their mean absolute value, is given by the estimates of Nakamura and Steinsson (2008) on U.S. data and it is equal to $e_{\Delta p} = 0.085$.[15] This procedure gives $\theta = 0.75\%$, $\psi = 0.27\%$ which are percentages of year profits, and a volatility of productivity shocks given by $\sigma = 0.11$. The value of $\psi = 0.27\%$ implies that the yearly cost of price adjustments is about 0.1% of revenues, which is comparable to the menu cost estimated directly by Levy et al. (1997) on retailer data.[16] The estimated value of

[14]When $\gamma = \mu + (2\eta - 1)\sigma^2/2$ expected profits are invariant to the time elapsed between consecutive review dates up to a second order approximation. As no new information arrives between review dates, the optimal price is constant during this period. See Appendix C for more details.

[15]This statistic is computed as $e_{\Delta p} \equiv \mathbb{E}\left[|\Delta \log(p)| \;\middle|\; \Delta \log(p) \neq 0\right]$.

[16]Revenues are measured in steady state and at the profit-maximizing price in absence of frictions in price setting, i.e. revenues are equal to $\eta\ \bar{\Pi}$. The yearly flow costs of adjustments and reviews are obtained by multiplying the cost of each adjustment and review with the average frequency of adjustments and reviews respectively, i.e. $\psi\, n_a$ and $\theta\, n_r$.

θ implies that the yearly cost of reviews is about four times larger than that of the physical menu cost of price adjustment. This finding is consistent with estimates by Zbaracki et al. (2004) for a large U.S. manufacturer, who find that managerial (information processing costs) are about 6 times larger than physical menu cost.

Finally, we notice that a concern of our calibration exercise relates to the measurement error with which the statistics about the frequency of reviews and adjustments reported in Table 1 have been computed. We address this concern in two ways. First, we notice that the estimate of the U.S. frequency of price adjustments in Table 1 is similar to the estimate obtained by Nakamura and Steinsson (2008) when excluding sales in a much larger sample of price changes. Second, we exploit another result of Alvarez, Lippi, and Paciello (2011) where we showed that the ratio ψ/θ can also be identified from moments of the distribution of price changes. In fact, the ratio ψ/θ is over-identified. We can therefore use moments from the distribution of price changes to assess the reliability of our estimate of ψ/θ. For instance, Eichenbaum et al. (2014) find that the fraction of price changes smaller than 5% in absolute value is equal to 24.4% of all price changes (see their Table 1). Our model with parameters chosen to match $n_r/n_a = 1.4$ predicts the fraction of price changes smaller than 5% in absolute value to be equal to 25% of all price changes. We interpret these results as a sign of robustness of our baseline choice of parameters.

4.1 The calibration of the menu-cost and observation-cost cases

Our model nests the menu cost and the observation cost model, each with only one friction, as particular cases. These models have been studied extensively in the literature, and therefore offer an interesting benchmark of comparison against our baseline model where the two frictions coexist. As a disciplining device in comparing the different economies, we calibrate the parameters governing the frequency and size of price adjustments for each model to match the same average frequency and size of price changes of our baseline parametrization, i.e. $n_a = 1.4$ and $e_{\Delta p} = 0.085$ respectively. We find this interesting because it allows us to compare the predictions of models that are observationally equivalent in terms of two popular statistics summarizing pricing behavior, frequency and size of price changes, but which can yet deliver very different predictions about the effects of monetary shocks due to the different nature of the friction behind sticky prices.

The case of menu cost only is obtained when $\psi > 0$ and $\theta = 0$ so that firms observe the state continuously. The firm's problem in this case has been analyzed in the seminal papers by Barro (1972) and Dixit (1991), and its aggregate consequences in Danziger (1999) and Golosov and Lucas (2007) among others. In this case the posted price is adjusted whenever it is far enough from the optimal price, so that the price adjustment rule is a state-dependent one.

The observation cost only model is obtained when $\theta > 0$ and $\psi = 0$. If $\psi = 0$, firms adjust prices continuously, even between two reviews, as long as they expect a drift in the nominal marginal cost. Under the assumption that $\gamma = \mu + (2\eta - 1)\sigma^2/2$, the expected drift in inflation and productivity offset each other, so that prices adjust only in response to shocks to the idiosyncratic productivity. As firms review their idiosyncratic state infrequently, prices adjust infrequently and in particular the frequency of price changes coincides with the frequency of reviews. Thus the assumption $\gamma = \mu + (2\eta - 1)\sigma^2/2$ allows this specification of the model to be consistent with the fact that prices change infrequently. A version of this model was first formulated by Caballero (1989), then extended by Reis (2006), while Bonomo and Carvalho (2004) studied a version where the observation cost is associated with an adjustment cost. In Table 3 we collect the results of this calibration exercise for our baseline economy and the two polar cases with observation and menu cost only, respectively. As already mentioned above, the values of the calibrated adjustment cost parameters are comparable to the (scant) direct evidence on the costs of price adjustments documented by Levy et al. (1997); Zbaracki et al. (2004).

Table 3: Calibrated parameters in different specifications of the model

Model specification	ψ	θ	σ
Observation + Menu cost	0.27%	0.75%	0.11
Observation cost only	0.00%	2.10%	0.12
Menu cost only	0.53%	0.00%	0.10

5 The propagation of the monetary shock

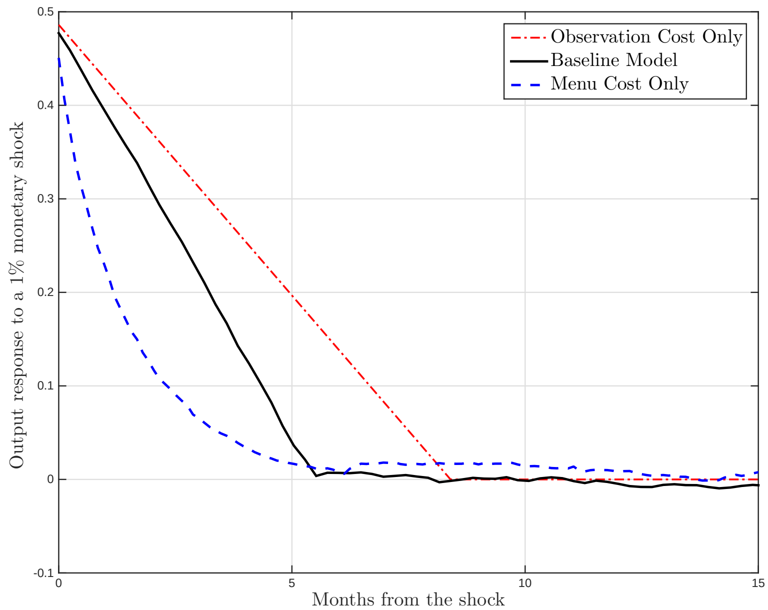
In this section we report the impulse response of aggregate output to the once and for all monetary shock described in Section 3.4, using the parameter values of Section 4. We compare the predictions of our baseline model with both adjustment and observation costs to the impulse responses predicted by the menu-cost and the observation-cost models. We solve numerically for the equilibrium aggregate consumption as defined in Section 3.3 by discretizing the model to a one week period (see the Online Appendix B for a detailed description of our algorithm). As a measure of the real effects of a monetary shock of a given size δ, we use the cumulated output response $\mathcal{M}(\delta)$ defined as

$$\mathcal{M}(\delta) \equiv \int_{t_0}^{\infty} (\log(\mathcal{C}_t(\delta)) - \log(\bar{c}))\ dt \ , \tag{15}$$

where $\mathcal{C}_t(\delta)$ is the equilibrium path of consumption c_t for all $t \geq t_0$, after a monetary shock of size δ at $t = t_0$. As consumption coincides with aggregate output in our economy, we will refer to $\mathcal{C}_t(\delta)$ as the response of output to a monetary shock of size δ.

Figure 2 displays the main result of the calibration. The figure plots the output response to a monetary shock of size $\delta = 1\%$ in deviation from the steady state, for the three different economies described in Section 4. Despite being characterized by the same average frequency and size of price adjustments, these economies have different implications about the aggregate output response to the same monetary shock. Our baseline model with both frictions predicts a significantly larger and more persistent real response than the menu-cost model, getting closer to the response of the model with observation cost only. Moreover, the shape of the

Figure 2: Response of $\log c_t$ to a 1% monetary shock

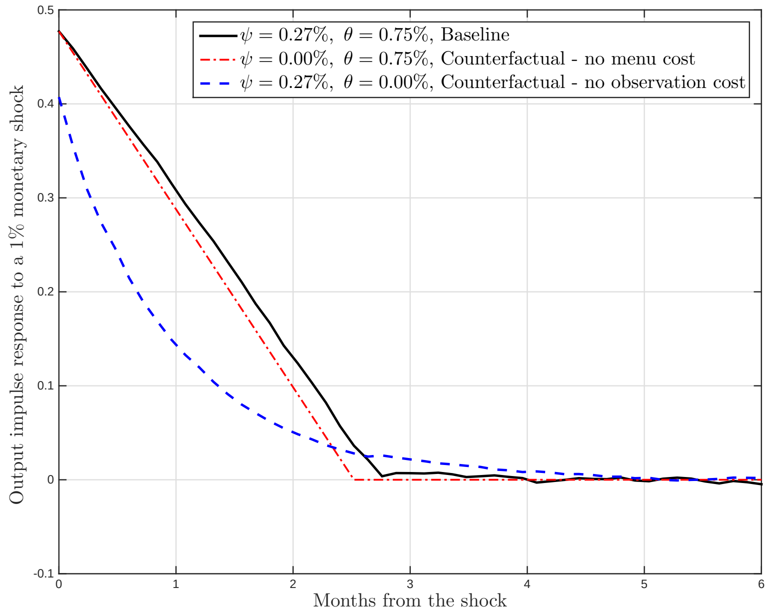


Note: All models are calibrated so that they are observationally equivalent with respect to the average frequency of price changes, $n_a = 1.4$, and to the mean absolute size of price changes, $e_{\Delta p} = 8.5\%$.

impulse response in our model is quasi linear, thus more similar to the one predicted by the observation-cost model than the menu-cost model.

The reason for the different behavior of these models, which are identical in their steady-state behavior (e.g. frequency and size of price changes), stems from the different adjustment rules followed by the individual firms. The price setting rule is state-dependent in the menu cost model: firms adjust prices whenever the price gap g_t crosses the thresholds $\{\underline{g}_t, \bar{g}_t\}$. As emphasized by Golosov and Lucas (2007), a positive monetary shock reduces all price gaps on impact by the same amount, making firms closer to the threshold to exit the inaction region and adjust their price. The larger the mass of firms that is moved outside of the inaction region on impact, the larger the increase of the aggregate price, and the smaller the output response to the monetary shock. Moreover, the response of the aggregate price to the monetary shock takes place both through a larger fraction of firms adjusting after the shock, and through a *selection* effect by which firms with the highest price gap will adjust.

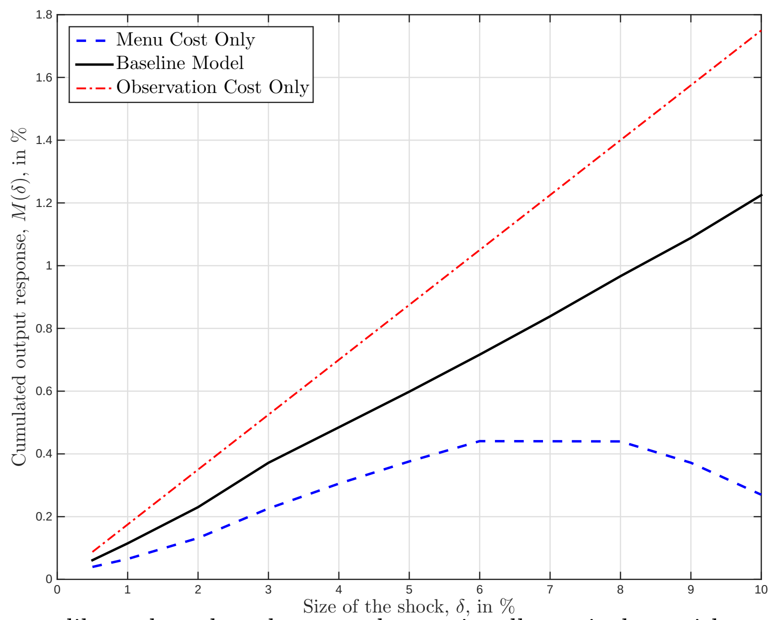
Figure 3: Counterfactuals: combined vs. dissociated frictions



Note: The baseline model is the same as in Figure 2, with an average frequency of price changes, $n_a = 1.4$, and a mean absolute size of price changes, $e_{\Delta p} = 8.5\%$. The menu-cost (observation-cost) model maintains the friction of the baseline model and set the observation-cost (menu-cost) friction to zero. The frequency of price adjustments in these model is slightly higher since the overall adjustment cost is smaller.

In the model with observation cost only, instead, the price setting policy follows a time-dependent rule where firms observing the state and adjusting the price are selected as a function of the time elapsed since the last adjustment, and not of the size of their current price gap. Thus, as it is known in the literature, models with time dependent price adjustment rules predict larger real effects than models with state dependent price adjustment rules because the former are characterized by a weaker *selection* effect than the latter. Our model with both frictions is characterized by a price adjustment policy that has both a time- and a state- dependent element. The time dependent element is due to the fact that firms adjust prices only upon review of the state, which is a function of the time elapsed since the last review. The state dependent element is instead due to the fact that, conditional on observing the state, firms decide whether to adjust the price and when to observe the state again as a function of the current price gap. We emphasize that, even though our model inherits both the time- and state- dependent elements of the polar cases, the shape of the predicted impulse responses resembles the one of models with *only* time dependent adjustment policies. Finally, to see why the effect in the baseline model is smaller than in the observation-cost model, notice that not all reviews lead to a price adjustment, so that the fraction of firms observing the state every period has to be larger than in the observation cost model for the two models to match the same average frequency of price adjustments. Conditional on observing the state, the vast majority of firms in the baseline model finds it optimal to adjust the price in response to the monetary shock because such shock moves almost all the firms outside of the inaction region. These considerations explain why the shape of the output response in the baseline economy is similar, and only slightly below, the response of the observation-cost model.

Figure 4: Cumulated output response as a function of the size of the shock, $\mathcal{M}(\delta)$



Note: All models are calibrated so that they are observationally equivalent with respect to the average frequency of price changes, $n_a = 1.4$, and to the mean absolute size of price changes, $e_{\Delta p} = 8.5\%$.

In Figure 3 we analyze the role of observation and menu costs in accounting for the output impulse response to the 1% monetary shock analyzed above. In particular, we consider a counterfactual exercise in which, starting from the parametrization of the baseline economy, we set the observation cost and the menu cost equal to zero one at the time, leaving all other parameters unchanged. Figure 3 shows that the output impulse response with observation cost only (i.e. $\theta = 0.75\%$ and $\psi = 0$) is very close to the output response in the Baseline economy where the menu cost is instead $\psi = 0.27\%$, so that the cumulated output response with observation cost approximately 90% as large as in the Baseline economy with both costs (see Figure 4). The counterfactual economy with menu cost only (i.e. $\theta = 0$ and $\psi = 0.27\%$), predicts instead much smaller real effects than the Baseline economy, with a cumulated output response that is about 30% of the cumulated effect of the Baseline economy. Thus we conclude that the observation cost accounts for the most part of the real effects of monetary shocks.[17]

Finally, in Figure 4 we quantify the size of the cumulative real effects of the monetary shock for different sizes of the monetary shock δ in each of the 3 economies. This question is of interest since the output response to the monetary shock *depends* on the size of the shock with state-dependent decision rules (an effect that is absent in purely time dependent models). For all values of δ, the cumulated output response $\mathcal{M}(\delta)$ predicted by the Baseline economy is in between the values predicted by the observation cost only and menu cost only models. For instance, the Baseline economy predicts a cumulated output response that is about 70% of the observation-cost only economy and about 1.5 times larger than predicted by the menu-cost model for shocks smaller than 5%. The difference gets much larger for larger shocks since the impulse response function of the menu cost economy is highly sensitive to the size of the shock (obviously due to the state dependent nature of adjustment) whereas the effect is linear, i.e. proportional to the monetary shock, in the observation cost-only model.

[17]We notice that the cumulated output responses in the observation cost only and in the menu cost only sum up to a larger number than the cumulated output response in the Baseline economy with both costs because of the interaction of the two frictions in the price adjustment policy.

6 Concluding remarks

We extended the general equilibrium model of Golosov and Lucas (2007) to include two types of frictions: a menu cost and an observation cost. This economy serves as a laboratory to evaluate the impact of different microeconomic mechanisms behind price rigidity using data on the firms' frequency of price adjustments and price reviews. It is shown that economies that display identical firms' patterns concerning the average frequency and size of price adjustments are nonetheless characterized by very different responses to nominal shocks, depending on the ratio of observation to menu costs. The reason for this result lies in the nature of the optimal decision rule: the menu cost gives rise to state dependent policy rules, which imply a rather quick response of the economy to aggregate shocks because of a "selection effect", namely the fact that firms that are far away from their optimal production level will adjust sooner than others. The observation cost instead gives rise to time-dependent rules, which mutes this selection effect increasing the persistence of the real effects. The main contribution of this paper is to solve the general equilibrium problem in the presence of both frictions and to present a quantitative analysis of the propagation of a small unexpected monetary shock. We quantify the menu-cost and observation-cost frictions using data from the OECD countries and find that the observation-cost friction is non-negligible, and that its presence significantly increases the real effects of monetary shocks compared to the canonical menu-cost model.

While the results in the main body of the paper rely on several simplifying assumptions and on the calibration for the US, many extensions were explored confirming the robustness of the findings. Appendix E shows that alternative parametrizations for other developed economies deliver results that are similar to those discussed for the US economy. Appendix F shows that for small shocks (smaller than 2%), the results in the model with both observation and menu costs are roughly independent of the assumption of whether firms know the realization of the monetary shock on impact. We see this property as a sign of robustness of the approach proposed by Mankiw and Reis (2002), and as a response to the common criticism that easily observable monetary shocks would substantially weaken the imperfect information hypothesis. Finally, our baseline model assumed perfectly correlated productivity and preference shocks to simplify the analytics of the firm optimal decision problem. Appendix G solves the model relaxing this assumption in a model with productivity shocks only. While the solution of this problem is more involved, the results on the propagation of the monetary shocks are virtually unchanged.

The prominent role for the observation friction that we end up estimating is not inherent to the model we propose but follows by matching the model with the firms' data. In Alvarez, Guiso, and Lippi (2012) we study a household portfolio problem in which households face both an observation cost and an adjustment (transaction) cost, along the lines of Abel, Eberly, and Panageas (2013). While the context is different the nature of the problem is similar, and the investors' frequency of review and adjustment depends, as in this paper, on the relative costs of each of these actions. Opposite to the firms' data Alvarez, Guiso and Lippi find that large transactions costs and small observation costs are needed to account for the investors' portfolio-review and portfolio-adjustment frequencies.[18]

Other authors have identified different mechanisms affecting the *level* of the output effects of a monetary shocks, for a given microeconomic friction in price changes: Nakamura and Steinsson (2010), Midrigan (2011) and Alvarez and Lippi (2014) focussed on, respectively, decreasing returns to scale in production, different assumptions about the distribution of idiosyncratic shocks and firms setting prices for multiple products. Likewise recent work by Carvalho and Schwartzman (2012) and Alvarez, Lippi, and Paciello (2014) shows how the real effect of monetary policy are increase with the volatility of adjustment times (keeping the mean frequency constant). Studying how such assumptions affect the output responses for given combination of observation and menu costs is an interesting avenue for future research.

[18]We find the two sets of empirical findings reasonable: the information cost for the investor involves a simple task, namely checking the value of her portfolio. Instead, the observation cost for firms is plausibly larger, as it involves finding out the value of marginal cost. Also the menu cost for firms is likely small compared with the typically large costs of portfolio recompositions.

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Online Appendix - Not for Publication

Phillips curves with Observation and Menu Costs

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A Discussion of the survey evidence

Several recent studies measure two distinct dimensions of the firm's price management: the frequency of price reviews, or the decision of assessing the appropriateness of the price currently charged, and the frequency of price changes, i.e. the decision to adjust the price. The typical survey question asks firms: "In general, how often do you review the price of your main product (without necessarily changing it)?"; with possible choices yearly, semi-yearly, quarterly, monthly, weekly and daily. The same surveys contain questions on frequency of price changes too. Fabiani et al. (2007) survey evidence on frequencies of reviews and adjustments for different countries in the Euro area, and Blinder et al. (1998), Amirault, Kwan, and Wilkinson (2006), and Greenslade and Parker (2008) present similar evidence for US, Canada and UK. We believe that the level of both frequencies, especially the one for reviews, are measured very imprecisely. For instance, the "bins" used to report the frequency of price adjustments and reviews are too coarse, making it impossible to accurately compute statistics such as the mean or the standard deviation.

The surveys in Fabiani et al. (2007), Amirault, Kwan, and Wilkinson (2006) and Greenslade and Parker (2008) collect a wealth of information on many dimensions of price setting, well beyond the ones studied in this paper. Yet, for the questions that we are interested in, the survey data from several countries have some drawbacks. We think that, mostly due to the design of these surveys, the level of the frequencies of price review and price adjustments are likely subject to a large amount of measurement error. One reason is that in most of the surveys firms were given the following choices for the frequency of price reviews: yearly, quarterly, monthly and weekly (in some also semi-yearly and less than a year). It turns out that these bins are too coarse for a precise measurement, given where the medians of the responses are. For example, consider the case where in the population the median number of price reviews is exactly one per year, but where the median number of price changes is strictly larger than one per year. Then, in a small sample, the median for reviews will be likely 1 or 2 reviews a year, with similar likelihood. Instead the sample median for number of adjustments per year is likely to be one. From this example we remark that the median for price reviews is imprecisely measured, as its estimates fluctuates between two values that are one hundred percent apart. The configuration described in this example is likely to describe several of the countries in our surveys.[19] Another reason is that in some cases the sample size is small. While most surveys are above one thousand firms, the surveys for Italy has less than 300 firms and the one for the US has about 200 firms. Yet another difficulty with these measures is that several surveys use different bins to classify the frequency of price reviews and that one price changes. For instance in France and Italy firms are asked the average number of changes, instead of being given a set of bins, as is the case for the frequency of reviews.[20]

[19] For example in Portugal the median frequency of review is 2, but the fraction that reviews at one year or less is 47%, while for price adjustment the median is one and the fraction of firms adjusting exactly once a year is 49.5%, see Martins (2005). In the UK for 1995 the median price review is 12 times a year, but the fraction that reviews at most 4 times a years is about 46%, while for price adjustment the median is 2 and the fraction of firms adjusting 2 times or less a year is 66%, see Hall, Walsh, and Yates (2000). Indeed, consistent with our hypothesis of measurement error, in a similar survey for the UK for year 2007-2008, the median price review is 4 and the median price adjustment is 4, see Greenslade and Parker (2008).

[20] Furthermore for Germany firms where asked whether or not they adjusted the price in each of the

B Computation of Impulse Response

We discretize time into intervals of length Δ, so now calendar dates, and hence choices of times to review $t \in \mathbb{T} \equiv \{0, \Delta, 2\Delta, ...\}$. Let ϕ_0 be the initial distribution, which is set to the stationary distribution for constant money growth rate, and whose computation is described below.

The numerical procedure to compute the path for equilibrium quantities consists of iterating on the following two steps until convergence.

1. Solve for the path for aggregate consumption $\{c_t\}_{t\in\mathbb{T}}$ assuming the economy starts with distribution ϕ_0, receives a monetary shock ϵ_m at $t = 0$, and the firms use decision rules $\{\underline{g}_t, \hat{g}_t, \bar{g}_t, \mathrm{T}_t\}_{t\in\mathbb{T}}$.
2. Solve the problem of the firm for all times $t \in \mathbb{T}$ to obtain the decision rules $\{\underline{g}_t, \hat{g}_t, \bar{g}_t, \mathrm{T}_t\}_{t\in\mathbb{T}}$ given a path for the aggregate consumption $\{c_t\}_{t\in\mathbb{T}}$.

The initial condition for this process is given by $c_t = \bar{c}$ for all $t \in \mathbb{T}$. Also we impose that the equilibrium path of consumption $c_t = \bar{c}$ for $t \geq \hat{T}$ where $\hat{T}$ is a large number. Below we give more details about steps 1 and 2.

B.1 Computation of the Stationary Equilibrium

First we discuss the computation of the decision rules. We use standard value function iterations in the problem described in equation (13), for $c_t = \bar{c}$, with the restriction that time elapsed between reviews $T \in \mathbb{T}$. In this case, the decision rules and value function do not depend on calendar time. In each of the value function iterations we solve the value function in a grid of price gaps $g \in G$ containing n points, and we interpolate to all $g \in \mathbb{R}$. This gives the time independent decision rules $\underline{g}, \hat{g}, \bar{g}, \mathrm{T}(\cdot)$ and the corresponding, time independent, value function $v(\cdot)$.

Using the time independent optimal decision rules we compute the stationary distribution ϕ_0 by simulating the discrete time analog of g for N firms for $\bar{T}/\Delta$ model periods. For each firm i we simulate:

$$g_{t+\Delta,i} = g_{t,i} + \gamma\Delta + \sigma\sqrt{\Delta} \tag{16}$$

in the inaction region, where x is a standard normal and where the starting date is $t = -\bar{T}/\Delta$ up to $t = 0$. Upon observation dates we set $g_{i,t} = \hat{g}$ if $g_{t,i} \notin (\underline{g}, \bar{g})$ or leave $g_{i,t}$ unchanged otherwise. We then choose the next observation date according to $\mathrm{T}(g_{i,t})$ The outcome of this process is to have N time series, each of length $\bar{T}/\Delta$ of g_{it}. The number of firms N and the number of periods $\bar{T}$ are chosen to be large enough so that the cross section of this triplets at time $t = 0$ stabilizes, and hence represents ϕ_0 accurately.

B.2 Computing the consumption path

We use the time dependent decision rules to simulate the price gaps for each of the N firms for $\hat{T}$ periods. Each of the N firms is initialized from the sample of initial distribution ϕ_0 of

preceding 12 months; this places an upper bound of 12 on the frequency of adjustments, while no such restriction applies to the number of reviews.

g. Then for each cross section at times $t \geq 0$ we compute the implied aggregate consumption as:

$$c_t = \left(\sum_{i=1}^{N} \left(\frac{\eta \ \xi}{\eta - 1} \ e^{g_{it}} \right)^{1-\eta} \right)^{\frac{1}{\epsilon \ (\eta-1)}} \tag{17}$$

The decision rules are the solution of the dynamic programming problem described in equation (13) where time elapsed between reviews are restricted to $T \in \mathbb{T}$. The value function and decision rules are computed by backward induction, taking as given the stationary value function v at $t = \hat{T}$. In each of the value function iterations we solve for the value function on a grid of price gaps $g \in G$ containing n points, and we interpolate to all $g \in \mathbb{R}$. This gives a path of decisions rules $\{\underline{g}_t, \hat{g}_t, \bar{g}_t, \mathrm{T}_t(\cdot)\}_{t \in \mathbb{T}, t \leq \hat{T}}$.

We then simulate again the path of $g_{i,t}$ for all i and t, using as decision rules $\{\underline{g}_t, \hat{g}_t, \bar{g}_t, \mathrm{T}_t(\cdot)\}_{t \in \mathbb{T}, t \leq \hat{T}}$ obtained in the last iteration, and obtain a new path of c_t. We iterate until convergence.

C Relationship to Alvarez, Lippi and Paciello (2011)

We show that the problem we studied in Alvarez, Lippi, and Paciello (2011) can be seen as a second order approximation of the profit function discussed in Section 3. The period return function described in Proposition 1 is given by the function $f(g, s)$. The second order approximation of $f(g, s)$ around $(g, s) = (0, 0)$ is given by

$$f(g, s) = 1 + f_s(0, 0)\sigma^2 \ s + \frac{1}{2} \ f_{gg}(0, 0) \ (g)^2 + \frac{1}{2} \ f_{ss}(0, 0) \ s^2 + f_{gs}(0, 0) \ g \ s + o(||\sigma^4 s^2, \sigma^2 \ g^2||),$$

where

$$f_g(0, 0) = 0, \quad f_s(0, 0) = -\frac{\eta(\eta - 1)}{2} \ \sigma^2, \quad f_{gg}(0, 0) = -\eta(\eta - 1),$$
$$f_{ss}(0, 0) = -\eta(\eta - 1) \ \left[(\mu - \gamma)^2 + (2\eta - 1)(\mu - \gamma)\sigma^2 + (3\eta^2 - 3\eta + 1)\frac{\sigma^4}{4} \right],$$
$$f_{g,s}(0, 0) = \ \eta(\eta - 1)^2 \ \left[(\mu - \gamma) + (2\eta - 1) \ \frac{\sigma^2}{2} \right].$$

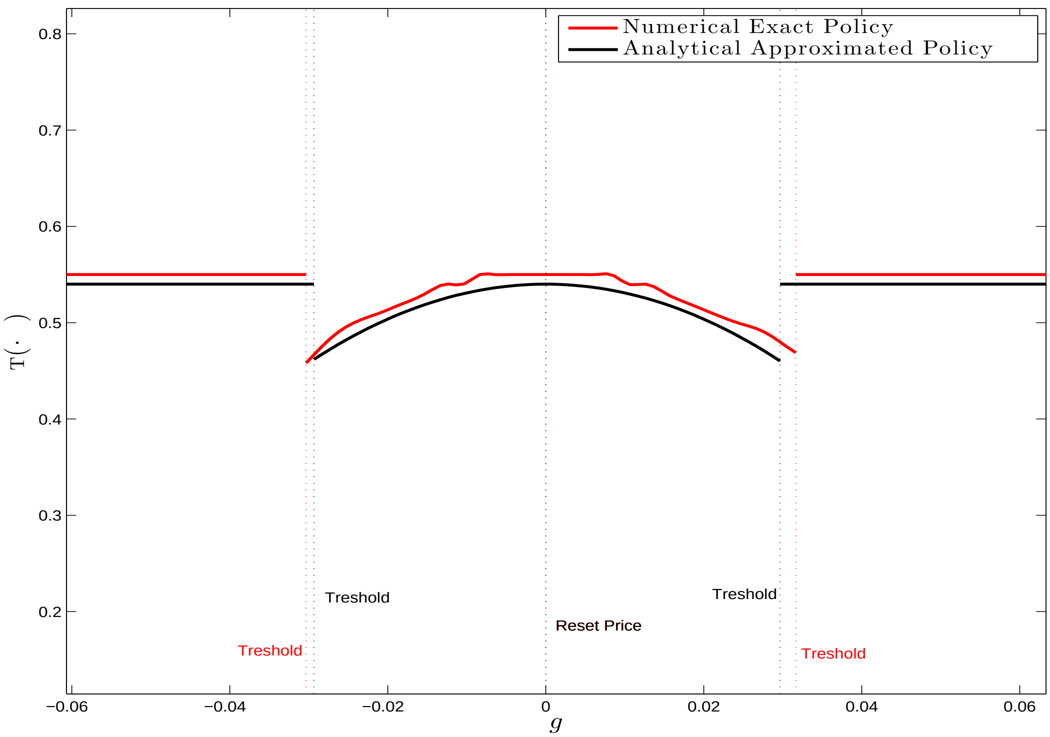
In the case in which $\mu - \gamma = -(2\eta - 1) \ \frac{\sigma^2}{2}$, it follows that $f_{g,s}(0, 0) = 0$ and $f_{ss}(0, 0) = (f_s(0, 0))^2$. In this case the problem is similar to the one we studied in Alvarez, Lippi, and Paciello (2011), in the sense that the value function for the firm problem with zero inflation (equation 7 on page 1928 in our paper) is obtained if the function $f(g, s)$ is approximated by $\sigma^2 s$ which is obviously a good approximation for small values of t around the optimal return point $g = 0$.[21]

To compare the accuracy in the two cases, we plot in Figure V the optimal decision rules in steady state, described by three values for the price gap $\underline{g} < \hat{g} < \bar{g}$, and a function $\mathrm{T}(\cdot)$. The function $\mathrm{T}(g)$ is inverse-U shaped, as firms that find themselves closer to the inaction

[21]For clarity of comparison note that the value function in Alvarez, Lippi, and Paciello (2011) is expressed as a difference from the maximized frictionless profits, i.e. $f(g, s) - 1$.

region are more likely to get a shock that will push them outside of such region, so observing sooner is optimal. Figure V shows that the analytical solution to the approximated problem only slightly understates the time to the next observation and the width of the inaction region.[22]

Figure V: Steady state decision rules from approximated vs numerical solution method



Note: Parameters are such that the model matches the average frequency of price changes, i.e. 1.4 adjustments per year, to the ratio of average number of price reviews to price adjustments per year equal to 1.4, and to the average size of absolute price changes equal to 8.5%. See Section 4 for more details on parameters choices.

D Relationship to Demery (2012)

Demery (2012) solves a problem and addresses questions similar to ours. This appendix discusses important differences in the results obtained by his paper compared to ours. We claim that his results are flawed in two important dimensions. First, we object his conclusion that the size of the output effects of monetary shocks in the economy with both observation and menu costs (our *Baseline* economy) is larger than in the alternative economy with

[22]In this exercise we use parameter values that are representative of the U.S. economy, and that we describe in detail in Section 4. See Appendix C for a systematic comparison of the analytical approximation and numerical exact solution, and for the equations describing the analytical solution.

observation cost only (keeping both economies with an identical frequency and size of price changes). We obtain the opposite result. Second we object his conclusion that the *shape* of the output response is exponentially decaying in the time elapsed since the monetary shock, independently of the combination of observation and menu costs. We find that the output response decays linearly in our *Baseline* economy. Finally we show that the conclusion by Demery is inconsistent with other results reported in his own paper.

We start by discussing the exponential profile of the output response to the monetary shock, which is a direct consequence of the assumed functional form in Demery's equation 5. This assumption is a problem because, as we showed, it is not a good approximation of the shape of the impulse response for a wide range of combinations of observation and menu costs. In particular it is not a good approximation for the range of parameters he considered, where instead the output response should decay linearly as a function of time.

We next discuss the different predictions about the size of the real effects of the monetary shock. We argue that the impulse responses reported in Demery's Figure 2 are flawed, and that this flaw is the source of the difference between his and our results. Our argument builds on comparing his estimates for the (log-) consumption and "output-gap" processes reported in his Table 3. The estimates are obtained by running regressions over the model generated data. Demery reports estimates of the following process for the "output-gap",

$$g_t = \beta_1 g_{t-1} + \beta_2 \log(w_t/w_{t-1}) + u_t, \tag{18}$$

where $\log(w_t/w_{t-1}) = \mu + v_t$ is the growth rate in nominal wages, $g_t \equiv \log(y_t) - \log(\tilde{y}_t)$ denotes the (log-) output-gap, with y_t denoting aggregate output, and $\tilde{y}_t$ being the efficient level of output. The error term u_t captures the approximation error with respect to true output-gap dynamics. Demery also reports estimates of the following AR(1) process for log-consumption at the top of his Table 3,

$$\log(c_t) = \lambda_0 + \lambda_1 \log(c_{t-1}) + \lambda_2 v_t + e_t, \tag{19}$$

which he uses to approximate firm's beliefs about the path of aggregate consumption. The error term e_t captures the approximation error with respect to true consumption dynamics, whereas v_t is the monetary shock. Simple analysis reveals that equation (18) and equation (19) are completely equivalent within his model (remember that these equations are estimated on model generated data). To see this notice that (log-) consumption and output-gap are equal up to a constant in his model: given the absence of capital, consumption is equal to output in this model, i.e. $c_t = y_t$. Given that monetary shocks have no effect on the efficient level of output, and the absence of other aggregate shocks, the efficient level of output is constant, i.e. $\tilde{y}_t = \tilde{y}$ for all t. Thus, we have $\log(c_t) = g_t + \log(\tilde{y})$. Using this result, the process in equation (18) can be rewritten as:

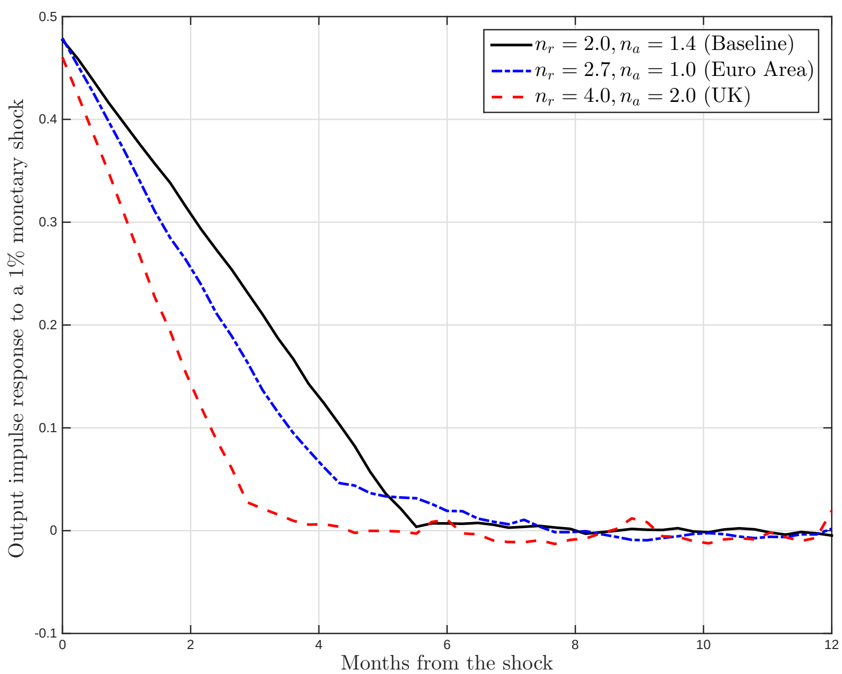
$$\log(c_t) = (1-\beta_1)\log(\tilde{y}) + \beta_1 \log(c_{t-1}) + \beta_2(\mu + v_t) + u_t \quad .$$

The latter and equation (19) are equivalent, requiring $\beta_1 = \lambda_1$, $\beta_2 = \lambda_2$ and $\lambda_0 + e_t = u_t + (1-\beta_1)\log(\tilde{y}) + \beta_2 \mu$. The key point is that the estimates for $\beta_1 = \lambda_1$, $\beta_2 = \lambda_2$ from equations (18) and equation (19) should be identical. Table 3 in Demery's paper does not pass this consistency test. The estimated values of λ_2 and β_2, as well as of λ_1 and β_1, are substantially different (that is, taking into account the estimates' standard errors), suggesting that at least one of the estimated processes is wrong. Importantly, the quantitative properties of the output response implied by the estimated consumption process (but not the shape as discussed in point (ii)) are comparable to the ones obtained in our paper, suggesting that the estimated output-gap process is wrong. Thus we conclude that Demery draws his main conclusions (see his Figure 2) from an estimated output-gap process that is flawed.

E Output responses at alternative parameter choices

In this section we compute impulse responses for alternative parametrizations of the model, and compare them to the predictions obtained under our baseline parametrization. We choose θ and ψ so that the frequencies of price changes and reviews predicted by the model match those reported in Table 1 for the Euro area and the U.K.; in both cases we set σ so that the average size of price changes is 0.085 as in our Baseline economy. This exercise implies $\theta = 0.16\%$, $\psi = 0.56\%$ and $\sigma = 0.09$ for the Euro area, and $\theta = 0.18\%$, $\psi = 0.30\%$ and $\sigma = 0.13$ for the U.K.

Figure VI: Response of $\log c_t$ to an unexpected 1% increase in m_t, for different values of ψ/θ



Note: All models are calibrated so that they are observationally equivalent with respect to the average frequency of price changes, i.e. $n_a = 1.4$, and to the average size of price changes equal to $e_{\Delta p} = 8.5\%$.

Figure VI plots impulse responses of output to a $\delta = 1\%$ increase in money supply in our Baseline economy, as well as in the two alternative economies calibrated to the Euro area and U.K data. We obtain the following results. First, the cumulated output response in the Euro economy is 81% as large as in the Baseline economy. This result is particularly interesting because it showcases the importance of accounting both for infrequent reviews and adjustments in the micro data to assess the real effects of monetary shocks. In fact, according to a model with menu cost only, the real effects of monetary shocks should be higher in the Euro economy than in the Baseline economy because the former has lower frequency of price adjustments than the latter (1 versus 1.4 adjustments per year). Our model with both observation and menu costs obtains the opposite predictions precisely because it is able to match the higher frequency of price reviews in the Euro area with respect to the Baseline economy (2.7 versus 2 reviews per year). Thus, despite firms in the Euro economy adjust prices on average less frequently to the idiosyncratic productivity shock, they will adjust faster in response to the monetary shock as they will be aware of such shock on average sooner than in the Baseline economy.

Moreover, we notice that the Euro economy is the one with the largest value of the ratio n_r/n_a and in fact the shape of its impulse response is closer to a menu cost only model, displaying more persistent and non linear dynamic. Finally, not surprisingly, the U.K. economy predicts a cumulated output response that is 49% as large as in the Baseline economy because it is characterized by higher frequencies of price changes and reviews than the Baseline economy.

F The case of a perfectly observed monetary shock

In this section we assume that paying attention to the monetary shock is costless so that the firms immediately and freely learn its realization, independently of the time(s) at which they acquire information about their state.

F.1 The firm's problem at $t = t_0$

Next we describe the firms' problem at the time of the aggregate shock $t = t_0$. The time of the aggregate shock, $t = t_0$, is a somewhat special date because due to the aggregate shock the cost, and hence the price gap, changes discretely but the exact values of the price gap is not known, given that most firms are between observation dates. Thus, we need to characterize the firm' problem as a function of beliefs about its price gap. We use that the belief about g is normal with parameters $(\tilde{g}, \tilde{\sigma}^2)$, while beliefs about aggregate output are denoted by $\{\tilde{c}_t\}_{t \geq 0}$.

Definition 1. A firm that at time $t = 0$ has beliefs that its own price gap g is normally distributed with parameters $(\tilde{g}, \tilde{\sigma}^2)$, and beliefs on the path of aggregate consumption given by $\{\tilde{c}_t\}_{t\geq 0}$, solves:

$$\max \quad \Bigg\{ \max_{\tau_0 \geq 0} \int_0^{\tau_0} e^{-r\,s} \left(\frac{\tilde{c}_s}{\bar{c}}\right)^{1-\epsilon\eta} f_0\left(\tilde{g}, s, \tilde{\sigma}^2\right)\, ds + \\ + e^{-r\,\tau_0} \int_{-\infty}^{\infty} v_{\tau_0}\left(e^{\tilde{g}+(\gamma-\mu)\tau_0 + \sigma\sqrt{\tau_0 + \frac{\tilde{\sigma}^2}{\sigma^2}}\, x}\right)\, dN(x)\ , \\ -\psi + \max_{\hat{g}_0, \tau_0 \geq 0} \int_0^{\tau_0} e^{-r\,s} \left(\frac{\tilde{c}_s}{\bar{c}}\right)^{1-\epsilon\eta} f_0\left(\hat{g}_0, s, \tilde{\sigma}^2\right)\, ds\ + \\ + e^{-r\,\tau_0} \int_{-\infty}^{\infty} v_{\tau_0}\left(e^{\hat{g}_0+(\gamma-\mu)\tau_0 + \sigma\sqrt{\tau_0 + \frac{\tilde{\sigma}^2}{\sigma^2}}\, x}\right)\, dN(x) \Bigg\} \tag{20}$$

$$\text{where } f_0(\tilde{g}, s, \tilde{\sigma}^2) \equiv \int_{-\infty}^{\infty} \frac{F\left(e^{\tilde{g}+(\gamma-\mu)s + \sigma\sqrt{s + \frac{\tilde{\sigma}^2}{\sigma^2}}\,x}\right)}{F(1)} dN(x).$$

The first term in equation (20) refers to the case where only the first observation date is chosen, while the second term refers to the case where both the first observation date is chosen and the price is adjusted. Notice that the problem in equation (20) differs from the problem in equation (13) because $\tilde{\sigma}^2 \neq 0$. This has two consequences. First, the function $f_{t_0}(\cdot)$ replaces the function $f(\cdot)$. Second, log productivity will accumulate normal innovations with variance σ^2 per unit of time, starting from a variance equal to $\tilde{\sigma}^2$.[23] After the first observation date occurring τ_{t_0} periods from $t = t_0$, the firms' problem is described by Proposition 1 so the continuation value is given by the function $v_{\tau_{t_0}}(\cdot)$.

In equation (20) the firm has two possibilities at $t = t_0$: (i) it pays the menu cost ψ, adjusts the price without observing the state, and decides to observe the state in τ_0 periods; (ii) it does not adjust the price without observing the state, and decides to observe the state in τ_0 periods. Notice that when the firm chooses $\tau_{t_0} = 0$, it is choosing to observe the state immediately at $t = t_0$, which means that at $t = t_0$ the firm solves the problem described in Proposition 1, and can eventually adjust the price after observing the state.

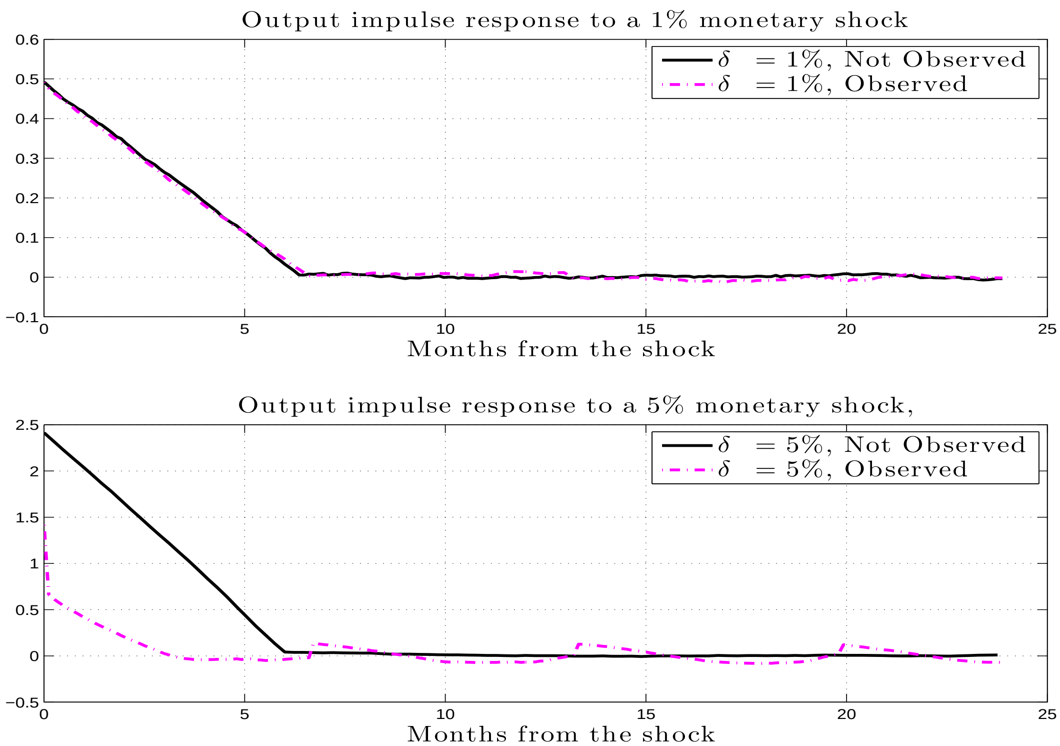
The firm' s optimal decision rule is described by two functions determining the inaction region as a function of the beliefs, defined by $\underline{\mathbf{g}}_{\mathbf{t_0}}(\tilde{\sigma}^2) < \bar{\mathbf{g}}_{\mathbf{t_0}}(\tilde{\sigma}^2)$, a function determining the optimal return point for the belief of the price gap when adjusting prices without observing the state, defined by $\hat{\mathbf{g}}_{\mathbf{t_0}}(\tilde{\sigma}^2)$, and a function for the time to the first observation depending on the beliefs about the price gap, defined by $\mathrm{T}_0(\tilde{g}, \tilde{\sigma}^2)$. At $t = t_0$, a firm that has beliefs about its price gap equal to $(\tilde{g},\, \tilde{\sigma}^2)$ leaves its price unchanged, if $\tilde{g} \in (\underline{\mathbf{g}}_{\mathbf{t_0}}(\tilde{\sigma}^2)\,,\, \bar{\mathbf{g}}_{\mathbf{t_0}}(\tilde{\sigma}^2))$, and otherwise changes its price so that the beliefs about its price gap are given by $(\hat{\mathbf{g}}_{\mathbf{t_0}}(\tilde{\sigma}^2), \tilde{\sigma}^2)$.

[23]For instance, the term $\sigma\sqrt{\tau_{t_0} + \tilde{\sigma}^2/\sigma^2}$ is the standard deviation of the distribution of the log of the price gap after T periods, which includes both the cumulative effect of the innovations, as well as the uncertain initial level of the price gap.

F.2 Impulse responses for observed monetary shocks

In this section we compute impulse responses in the model with both observation and menu costs under the assumption that the monetary shock is immediately observed upon impact. In our economy, upon realization of the monetary shock at $t = t_0$, all firms in the economy will revise their beliefs about the new level of nominal wages and about the path of real aggregate consumption. Figure VII plots the consumption impulse responses to monetary shocks of size $\delta = 1\%$ (top panel) and $\delta = 5\%$ (bottom panel), in the cases of such shocks being immediately observed by all firms (dashed line) or not being observed until the first observation date (solid line).

Figure VII: Impulse response of $\log c_t$ to an unexpected increase in m_t



Note: impulse responses of $\log c_t$ to a one-time permanent increase in m_t, in % deviation from steady state in our baseline model, and in the case where monetary shocks are perfectly observed upon impact. All models are calibrated so that they are observationally equivalent with respect to the average frequency of price changes, i.e. $n_a = 1.4$, and to the average size of price changes equal to $e_{\Delta p} = 8.5\%$.

The output impulse response to a monetary shock of size $\delta = 1\%$ for the case of freely observed monetary shock is roughly identical to the case of unobserved monetary shock. In contrast, for a monetary shock of size $\delta = 5\%$, the output response is much smaller on impact and much less persistent under the assumption that the monetary shock is freely and immediately observed than under the assumption that the monetary shock is unobserved. Similarly to the menu cost only model, the cumulated output response $\mathcal{M}(\delta)$ is hump-shaped in δ: the value of $\mathcal{M}(\delta)$ is larger than in the menu cost only model for $\delta \leq 5\%$, and smaller for values of $\delta > 5\%$. In addition, the cumulated output response has a peak at at $\delta = 3\%$, thus before the peak of the menu cost only model. The cumulated output response in the case of observed monetary shocks is always smaller than in the case of unobserved shocks. However, the difference between the predictions of $\mathcal{M}(\delta)$ under observed and unobserved monetary shocks is negligible for values of $\delta \leq 2\%$. As δ increases further, the cumulated output response with observed monetary shocks departs more and more from the case of unobserved monetary shocks.

The next proposition showcases the different behavior of the impulse responses to a monetary shock comparing the two assumptions about the acquisition of the information about the aggregate monetary shock that occur at date $t = t_0$. In the first case we assume that information about the aggregate state of the economy is known only after the first observation at times $s \geq t$. In the second case we assume that the aggregate shock is known at time $t = t_0$.

Proposition 2. Let $t = t_0$ be the time at which an aggregate monetary shock $\delta > 0$ occurs.

1. Assume that the aggregate shock is not observed at $t = t_0$. All firms make their *first* observation at the date that was planned before $t = 0$. Moreover, all firms that will adjust at time $s \geq t$ do so immediately after observing their idiosyncratic state at time s.

2. Assume that the aggregate shock is observed at $t = t_0$. At time $t = t_0$ firms can be divided into two groups:

   (a) Firms that *immediately* increase their prices without observing their idiosyncratic state, or

   (b) Firms that change the date of their *first* observation relative to the date that was planned before $t = t_0$, including a strictly positive fraction that observes immediately. Firms that have negative expected price change will postpone their *first* observation. Moreover, for small δ, almost all firms that have a positive expected price change will anticipate their *first* observation.

The proof of point 1 follows immediately from the assumption that all firms are unaware about the realization of the aggregate shock until their first observation. Given they are unaware of the shock, their decision rule will not be affected by the shock until then. After the first observation, firms' beliefs about the evolution of the price gap are identical to the ones in steady state, so that price adjustments occur only upon observation. The proof of point 2.a follows from the fact that there is a non-zero mass of firms that observed the state at least an instant before the realization of the aggregate shock, and have uncertainty about the price gap small enough that it is not worth observing the state. Among those firms, a fraction of them found their price gap upon the last observation of the state to be inside the inaction region, with the distance from the adjustment threshold $\underline{g}_t$ being smaller than the size of the monetary shock δ. Thus the monetary shock causes the price gap of these firms to exit the inaction region, i.e. $g < \underline{g}_t$, leading to a price increase without observation. On the other side, another fraction of firms found their price gap close to the adjustment threshold $\bar{g}_t$ at their last observation, so to update their beliefs of price gap further to the left of such threshold. Thus there will be no firm decreasing price without observing the state after a positive monetary shock. The proof of point 2.b follows from the fact that the monetary shock causes an equal shift to the expected price gaps of all firms. If the monetary shock is not too large, not all firms will adjust their price on impact. Among those firms that do not adjust their price immediately, their next observation time will be the same that would have been chosen at the last observation date if the price gap was smaller by an amount δ. Thus, given the inverse-U shape of the optimal time to the next observation, those firms that had a negative price gap (thus a positive expected price change) before the monetary shock, and do not adjust their price immediately, will anticipate the next observation date. At least a fraction of those firms that had a positive price gap upon the last observation (thus a positive expected price change) before the monetary shock, and do not adjust their price immediately, will postpone their next observation date. For instance, those firms that upon the last observation date found their price gap equal to $\bar{g}_t$.

When the information of the monetary shock is observed at $t = t_0$ at no cost, the model responds to a monetary shock in a way that is closer to the menu cost model on two dimensions. First, the monetary shock has an *impact* effect on the price level, i.e. it causes a jump in the fraction of firms that adjust immediately, some of them adjusting without observing the idiosyncratic state and some of them anticipating their observation of the state and adjusting immediately after.

Second the model displays a *selection* effect: right after the unexpected increase in money supply, the fraction of firms that increase prices is larger than the fraction that decrease prices. The last effect originates from the different behavior of firms with positive and negative expected price gap with respect to the planning of their first observation date (and possibly adjustment) after the monetary shock. In fact, firms with positive expected price change (i.e. negative price gap) before the monetary shock anticipate their first observation as the positive monetary shock increases the probability of the actual price gap being outside of the inaction region. Firms with negative expected price change (i.e. positive price gap) postpone their first observation as the positive monetary shock decreases the probability of the actual price gap being outside of the inaction region.

In contrast, the model where the monetary shock is only learned after the idiosyncratic shock is observed predicts no immediate jump in the price level and no selection of the type described above, as the fraction of firms adjusting largely depends on the fraction of firms observing, which has not changed with respect to steady state. As however Figure VII shows, the quantitative implications for the real effects of monetary shocks of such *impact* and *selection* effects depend on the size of the monetary shock. When the monetary shock is smaller, fewer firms find optimal to adjust the price in response to the monetary shock without observing, and less asymmetry will be present in the decision of when to observe next between firms positive and negative expected price gaps. Thus, unless the monetary shock is large, the impact of such effects on the impulse responses is negligible, so that the assumptions about the availability of information on monetary shocks is quantitatively irrelevant. Bonomo and Carvalho (2004) highlights a similar finding in the context of a disinflation experiment. When firms are inattentive, an announced disinflation policy has similar effects to a unannounced one, so long as the disinflation is small enough.

G The model without preference shocks

This appendix considers the problem where preferences shocks are shut down so that the Spence-Dixit-Stiglitz consumption aggregate is given by

$$c_t = \left[\int_0^1 (C_{i,t})^{(\eta-1)/\eta} di\right]^{\eta/(\eta-1)} \quad \text{with} \quad \eta > 1 \ .$$

The price setting equation is the same one as in equation (10), but profits are now given by

$$\Pi_t(p/p^*, z) \quad = \quad c_t^{1-\epsilon\eta} z^{\eta-1} \ F(p/p_t^*) \qquad \text{and} \qquad \Pi_t^*(z) = c_t^{1-\epsilon\eta} \ z^{\eta-1} \ F(1) \tag{21}$$

where, letting $g \equiv \log(p/p^*)$, the function $F(e^g) \equiv \ R \ \xi^{1-\eta} \ \left(e^g \ \frac{\eta}{\eta-1}\right)^{-\eta} \left(e^g \ \frac{\eta}{\eta-1} - 1\right)$. We assume that the observation cost is and the adjustment cost are a constant fraction of the frictionless profit: $\theta_t(z) = \theta \ \Pi_t^*(z)$ and $\psi_t(z) = \psi \ \Pi_t^*(z)$. Here, however, $\Pi_t^*(z)$ is a function of the level of productivity of each firm. Technically, the purpose of having the observation (and menu) cost to scale up with profits is to induce stationarity in the firm's problem, which is a common feature in many models. In order to have a stationary distribution of price gaps and productivities, we assume that products can be replaced at a rate λ large enough to ensure that there exists an invariant distribution of profits, output and productivities. Upon replacement, the new product starts with productivity $z = 1$.

Proposition 3. Consider the normalized value function: $v_t(g) \equiv \frac{V_t(p,z,m)}{m \, \Pi_t^*(z)}$ where g is the price gap. Let $r = \rho + \lambda$. The normalized, single-state, value function solves:

$$\begin{aligned}
v_t(g) \quad &= \quad \max\left\{\hat{v}_t \ , \ \bar{v}_t(g)\right\} \ \text{ where} \\
\hat{v}_t \quad &= \quad -(\theta+\psi) + \max_{T,\hat{g}} \int_0^T e^{-r\,s} \left(\frac{c_{t+s}}{c_t}\right)^{1-\epsilon\eta} f(s,\hat{g})\, ds \ + \\
&+ \quad e^{-r\,T} \left(\frac{c_{t+T}}{c_t}\right)^{1-\epsilon\eta} \int_{-\infty}^{\infty} e^{(\eta-1)\left(\gamma T + \sigma\sqrt{T}\, x\right)} \ v_{t+T}\left(e^{\hat{g} \ +(\gamma-\mu)T + \sigma\sqrt{T}\, x}\right) \ dN(x) \ , \\
\bar{v}_t(g) \quad &= \quad -\theta + \max_{T} \int_0^T e^{-r\,s} \left(\frac{c_{t+s}}{c_t}\right)^{1-\epsilon\eta} f(s,g)\, ds \ + \\
&+ \quad e^{-r\,T} \left(\frac{c_{t+T}}{c_t}\right)^{1-\epsilon\eta} \int_{-\infty}^{\infty} e^{(\eta-1)\left(\gamma T + \sigma\sqrt{T}\, x\right)} \ v_{t+T}\left(e^{g \ +(\gamma-\mu)T + \sigma\sqrt{T}\, x}\right) \ dN(x) \ , \\
&\text{where} \quad f(s,g') \equiv \int_{-\infty}^{\infty} e^{(\eta-1)\left(\gamma s + \sigma\sqrt{s}\, x\right)} \ \frac{F\left(e^{g' \ +(\gamma-\mu)s + \sigma\sqrt{s}\, x}\right)}{F(1)} dN(x) \ ,
\end{aligned} \tag{22}$$

and where $N(\cdot)$ is the CDF of a standard normal.

Proof. (of Proposition 3) We let p_{τ_n} be the price chosen in the case of an adjustment. We let $\chi_{\tau_n} = 1$ be an indicator of price adjustment and let $\chi_{\tau_n} = 0$ be an indicator of no price adjustment at τ_i. The optimal value for a firm that maximizes expected discounted profits net of observation and menu costs at the first observation date after the aggregate shock,

$t = \tau_0$, and up to the time the product is replaced is

$$V_t(p, z, m) = \max_{\{\tau_{n+1},\ p_{\tau_n},\ \chi_{\tau_n}\}_{i=0}^{\infty}} \mathcal{V}_t\left(\{\tau_n, p_{\tau_n}, \chi_{\tau_n}\}_{i=0}^{\infty};\, p, z, m\right) \ , \tag{23}$$

where

$$\mathcal{V}_t\left(\{\tau_n, p_{\tau_n}, \chi_{\tau_n}\}_{i=0}^{\infty};\, p, z, m\right) \equiv \mathbb{E}\Bigg\{ \sum_{i=0}^{\infty} e^{-(\mu+r)(\tau_n - t)} \Big[m_{\tau_n}\ \theta\, \Pi_t^*(z) \ + \tag{24}$$
$$(1-\chi_{\tau_n}) \int_{\tau_n}^{\tau_{n+1}} e^{-(\mu+r)(s-\tau_n)}\ \mathbb{E}\left[m_s\ \Pi\left(p_{\tau_{n-1}}/p_s^*, c_s\right) \big| z_{\tau_n}\right] ds \ \ +$$
$$\chi_{\tau_n} \left(\ m_{\tau_n}\ \psi\, \Pi_t^*(z) + \int_{\tau_n}^{\tau_{n+1}} e^{-(\mu+r)(s-\tau_n)}\ \mathbb{E}\left[m_s\ \Pi\big(p_{\tau_n}/p_s^*, c_s\big) \big| z_{\tau_n}\right]\ ds \right) \Big] \ \Big| z_t = z \Bigg\} \ ,$$

where $m_t = m$, $p_{-1} = p$ and $\tau_0 = t \geq 0$. The firm price at all dates $s \geq t$ is thus given by

$$\begin{aligned} p_s &= p_{\tau_{n-1}} \text{ for } s \in [\tau_n, \tau_{n+1}) \ \text{ if } \chi_{\tau_n} = 0 \ \text{ no adjustment takes place at } \tau_n \ , \\ p_s &= p_{\tau_n} \quad \text{ for } s \in [\tau_n, \tau_{n+1}) \ \text{ if } \chi_{\tau_n} = 1 \ \text{ an adjustment takes place at } \tau_n \ , \end{aligned}$$

where p_s^* is given by equation (10), z_s follows equation (1). Profits, observation cost and menu cost are scaled by the stock of money supply and hence the value function V_t is expressed in nominal terms. The discounting uses the result that the nominal interest rate equals $\rho + \mu$. Since no information is gathered between observation dates, expectations of profits are conditional on the information gathered at the last observation date. No pricing decision takes place between observation dates, and hence the nominal price is constant between observation dates.

The optimal value of the sequence problem in equation (23) solves the following functional equation:

$$\begin{aligned} \frac{V_t(p,z,m)}{m} &= \max\left\{ \frac{\hat{V}_t(z,m)}{m} \ , \ \frac{\bar{V}_t(p,z,m)}{m} \right\} \\ \frac{\hat{V}_t(z,m)}{m} &= -\theta_t(z) - \psi_t(z) + \max_{T,\hat{p}} \int_0^T e^{-r\,s}\ c_{t+s}^{1-\epsilon\eta}\ \mathbb{E}\left[z_s^{\eta-1}\ F\left(\frac{\hat{p}}{p_{t+s}^*}\right) \big| z_0 = z \right] ds + \\ &+ \ e^{-r\,T}\ c_{t+T}^{1-\epsilon\eta}\ \mathbb{E}\left[\frac{V_{t+T}\left(\hat{p}, z_T\right)}{m\, e^{\mu T}} \ \big| z_0 = z \right] \ , \\ \frac{\bar{V}_t(p,z,m)}{m} &= -\theta_t(z) + \max_{T} \int_0^T e^{-r\,s}\ c_{t+s}^{1-\epsilon\eta}\ \mathbb{E}\left[z_s^{\eta-1}\ F\left(\frac{p}{p_{t+s}^*}\right) \ \big| z_0 = z \right] ds + \\ &+ \ e^{-r\,T}\ c_{t+T}^{1-\epsilon\eta}\ \mathbb{E}\left[\frac{V_{t+T}\left(p, z_T\right)}{m\, e^{\mu T}} \ \big| z_0 = z \right] \ , \end{aligned}$$

From this recursion we can show the following homogeneity for the nominal expected discounted net profits:

$$V_t(p,z,m) \;\; = \;\; m \; z^{\eta-1} \; V_t\left(\frac{p\;z}{m},1,1\right) \quad \text{for all} \;\; p,z,m,t \geq 0 \tag{25}$$

and hence we define the expected discounted profits relative to the economy money supply, for a normalized shock and a price relative to the flexible optimal price. This value function, scaled by an index of real aggregate demand, turns out to be equal to the ratio of the nominal value function to the nominal the frictionless profit, which gives an economic interpretation to the normalization:

$$v_t(g) \;\; \equiv \;\; \frac{V_t\left(R\alpha\frac{\eta}{\eta-1}\; e^g,1,1\right)}{c_t^{1-\epsilon\eta}\; F(1)} = \frac{V_t(p,z,m)}{m\;\Pi_t^*(z)} \tag{26}$$

We can write a one state Bellman equation associated to the real net profits immediately after the time of an observation as:

$$\begin{aligned}
v_t(g) &= \max\{\hat{v}_t \;,\; \bar{v}_t(g)\} \;\; \text{where} \\
\hat{v}_t &= -(\theta+\psi) + \max_{T,\hat{g}} \int_0^T e^{-r\,s}\left(\frac{c_{t+s}}{c_t}\right)^{1-\epsilon\eta} \mathbb{E}\left[\left(\frac{z_s}{z_0}\right)^{\eta-1} \frac{F\left(e^{g_s}\right)}{F(1)} \mid g_0 = \hat{g}\right] ds + \\
&+ \; e^{-r\,T}\left(\frac{c_{t+T}}{c_t}\right)^{1-\epsilon\eta} \mathbb{E}\left[\left(\frac{z_T}{z_0}\right)^{\eta-1} v_{t+T}\left(g_T\right) \mid g_0 = \hat{g}\right] \;, \\
\bar{v}_t(g) &= -\theta + \max_{T} \int_0^T e^{-r\,s}\left(\frac{c_{t+s}}{c_t}\right)^{1-\epsilon\eta} \mathbb{E}\left[\left(\frac{z_s}{z_0}\right)^{\eta-1} \frac{F\left(e^{g_s}\right)}{F(1)} \mid g_0 = g\right] ds + \\
&+ \; e^{-r\,T}\left(\frac{c_{t+T}}{c_t}\right)^{1-\epsilon\eta} \mathbb{E}\left[\left(\frac{z_T}{z_0}\right)^{\eta-1} v_{t+T}\left(g_T\right) \mid g_0 = g\right] \;,
\end{aligned}$$

The dynamics of the state g in the inaction region are given by $dg = (\gamma-\mu)dt + \sigma dB$.

Finally, we define the function $f(\cdot)$ as

$$\begin{aligned}
f(s,g') &\equiv \mathbb{E}\left[\left(\frac{z_s}{z_0}\right)^{\eta-1} \frac{F\left(e^{g_s}\right)}{F(1)} \mid g_0 = g'\right] \\
&= \int_{-\infty}^{\infty} e^{(\eta-1)\left(\gamma s + \sigma\sqrt{s}\,x\right)} \; \frac{F\left(e^{g' \; + (\gamma-\mu)s + \sigma\sqrt{s}\,x}\right)}{F(1)} dN(x),
\end{aligned}$$

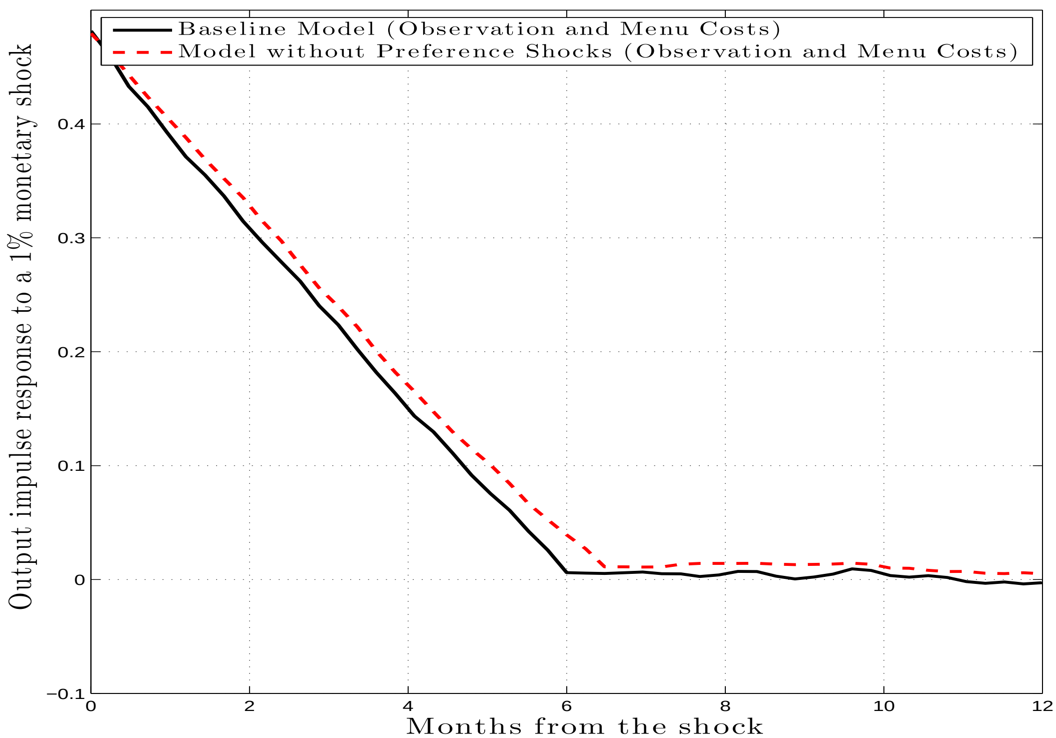
which after simple algebra is given by

$$f(s,g') = \eta \; e^{\;(1-\eta)g' + (\eta-1)\mu s} - (\eta-1)\; e^{-\eta g' + (\eta\mu - \gamma + 0.5\sigma^2)s}. \tag{27}$$

□

Finally, we solve numerically for the equilibrium dynamics following an unexpected $\delta = 1\%$ permanent increase in m_t at $t = 0$, in the model of this section and we compare it to the equilibrium dynamics predicted by our baseline model. Figure VIII displays output response to the monetary shock in the two models.

Figure VIII: Impulse response of $\log c_t$ to an unexpected 1% increase in m_t, $\delta = 1\%$



Note: All models are calibrated so that they are observationally equivalent with respect to the average frequency of price changes, i.e. 1.4 adjustments per year, and to the average size of price changes equal to 8.5%.